Standard Model of Particle Physics

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Elements of the Standard Model

- Gauge Group –

$$SU(2)_L \times U(1) \times SU(3) \Rightarrow 3{+}1{+}8 \text{ Gauge Bosons}\,.$$

  The 12 gauge bosons are the $W^{\pm}, Z^0, \gamma$ and the eight gluons.

- Fermions – 3 Generations of Quarks and Leptons:

u	d	e^-	ν_e
c	s	μ^-	ν_μ
t	b	τ^-	ν_τ

- The Higgs Field – needed to generate masses.

In spite of the simplicity of these elements, the SM contains a rich and subtle structure which continues to be tested to great precision at experimental facilities, so far with frustrating success.

It does however, leave many unanswered questions and is surely incomplete.

- The purpose of these lecture is to review the structure and status of the SM and the tools being used to make SM calculations.

Immediate Questions

Some immediate questions spring to mind:

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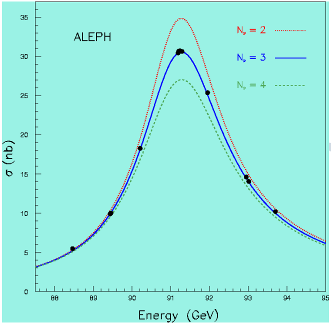
- Why are there three generations?



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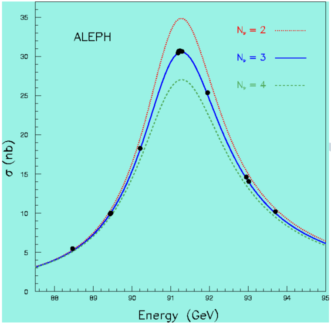

- What is the reason for the huge range of fermion masses?

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1. Weak Decays
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3. Abelian Higgs Model
4. $SU(2) \times U(1)$

Weak Decays and Massive Vector Bosons

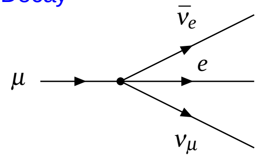
- Short range nature of weak force ($\sim 10^{-18}$ m) together with experimental studies $\Rightarrow$ Fermi Model:

$$\mathscr{L} = -\frac{G_F}{\sqrt{2}}\, j^{\dagger}_{\mu} j^{\mu}$$

where j_μ is the weak $V-A$ current

$$j^\mu = \bar{\nu}_e \gamma^\mu (1-\gamma^5) e + \bar{\nu}_\mu \gamma^\mu (1-\gamma^5)\mu + \bar{\nu}_\tau \gamma^\mu (1-\gamma^5)\tau + \text{ Hadronic Terms}\,.$$

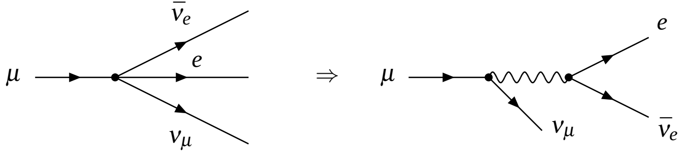
Example – Muon Decay



- G_F is the *Fermi Constant.*
It has dimensions of $[m]^{-2}$ ($G_F \simeq 1.17 \times 10^{-5}\,\text{GeV}^{-2}$) $\Rightarrow$ loop corrections diverge as powers of the cut-off and the 4-Fermion theory is non-renormalizable.

Weak Decays and Massive Vector Bosons – Cont.

- A natural suggestion for the cure of the problem of non-renormalizability is the introduction of an *intermediate vector boson*, which is sufficiently heavy to account for the short-range of the interaction.



It may seem that

$$G_F \propto \frac{g^2}{k^2 - M^2}$$

where g is a dimensionless coupling constant, M is the mass of the boson and k is the momentum flowing through the propagator.

Weak Decays and Massive Vector Bosons – Cont.

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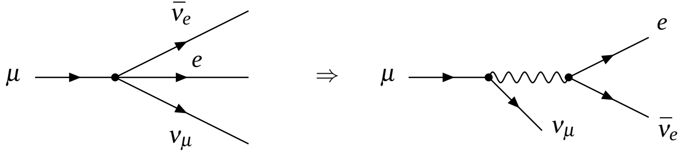


It may seem that

$$G_F \propto \frac{g^2}{k^2 - M^2}$$

where g is a dimensionless coupling constant, M is the mass of the boson and k is the momentum flowing through the propagator.

- However, the propagator of a massive vector boson is

$$\frac{-i}{k^2 - M^2} \left\{ g^{\mu\nu} - \frac{k^\mu k^\nu}{M^2} \right\}$$

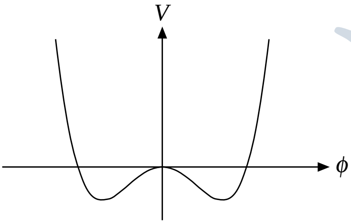
and the problem of non-renormalizability remains (μ, ν are Lorentz indices).

Goldstone Bosons

- Today, the standard approach to the construction of a theory with massive gauge bosons requires the introduction of scalar (Higgs) fields.
- I start however, by reminding you about *Goldstone bosons*.
- Consider a field theory with a single (real) scalar field ϕ with the potential:

$$V(\phi) = -\frac{\mu^2}{2}\,\phi^2 + \frac{\lambda}{4!}\,\phi^4\,.$$

The non-standard feature is that the mass term has the wrong sign.



Goldstone Bosons – Discrete Symmetry Cont.

$$V(\phi) = -\frac{\mu^2}{2}\,\phi^2 + \frac{\lambda}{4!}\,\phi^4 .$$

- The lowest-energy classical field configuration is the translationally invariant field

$$\phi(x) = \phi_0 = \pm v = \pm\sqrt{\frac{6}{\lambda}}\,\mu .$$

v is called the *Vacuum Expectation Value of* ϕ.

- To interpret this theory, imagine quantum fluctuations close to one of the minima, $+v$ say. To study the quantum fluctuations it is convenient to write

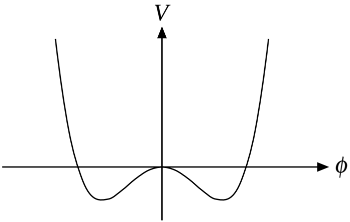
$$\phi(x) = v + \sigma(x)$$

so that

$$V(\sigma) = \frac{1}{2}\,(2\mu^2)\,\sigma^2 + \sqrt{\frac{\lambda}{6}}\,\mu\sigma^3 + \frac{\lambda}{4!}\,\sigma^4 .$$

- This is now a standard field theory of a scalar field, with mass $\sqrt{2}\mu$ and with cubic and quartic interactions.
- Original $\phi \to -\phi$ symmetry is now hidden in the relations between the three coefficients in terms of two parameters λ and μ.

Goldstone Bosons – Discrete Symmetry Cont.



Comments:

- This is a simple example of *spontaneous symmetry breaking*, i.e. of the discrete symmetry $\phi \to -\phi$.
- In quantum mechanics the situation is qualitatively different. Tunnelling$\Rightarrow$ the ground state wave function is symmetric around the two minima.

Goldstone Bosons – Continuous Symmetries

- More interesting features occur when we have the spontaneous symmetry of a continuous symmetry.
- Consider the linear sigma model, describing the interactions of N real scalar fields:

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{\mu^2}{2}(\phi^i)^2 - \frac{\lambda}{4}\left[(\phi^i)^2\right]^2 .$$

  - There are implicit sums over i in each $(\phi^i)^2$ over $1 \le i \le N$.
  - $\mathscr{L}$ is invariant under $O(N)$ rotational symmetry.
  - The potential is minimized for any $\{\phi_0^i\}$ such that

$$(\phi_0^i)^2 = \frac{\mu^2}{\lambda} .$$

    This is a condition only on the length of the vector ϕ_0 and we therefore have a continuous infinity of equivalent vacua.
- Let us choose

$$\phi_0 = (0, 0, \cdots, 0, v) \quad \text{with} \quad v = \frac{\mu}{\sqrt{\lambda}} .$$

Goldstone Bosons – Continuous Symmetries Cont.

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{\mu^2}{2}(\phi^i)^2 - \frac{\lambda}{4}\left[(\phi^i)^2\right]^2 .$$

$$\phi_0 = (0, 0, \cdots, 0, v) \quad \text{with} \quad v = \frac{\mu}{\sqrt{\lambda}} .$$

- We now define the shifted fields:

$$\phi^i(x) = (\pi^j(x), v + \sigma(x)) \qquad j = 1, \cdots, N-1 .$$

in terms of which the Langrangian becomes:

$$\begin{aligned} \mathscr{L} \quad = \quad & \frac{1}{2}(\partial_\mu \pi^j)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}(2\mu^2)\,\sigma^2 - \sqrt{\lambda}\mu\sigma^3 \\ & -\sqrt{\lambda}\mu(\pi^j)^2\sigma - \frac{\lambda}{4}\sigma^4 - \frac{\lambda}{2}(\pi^j)^2\sigma^2 - \frac{\lambda}{4}\left[(\pi^j)^2\right]^2 . \end{aligned}$$

Goldstone Bosons – Continuous Symmetries Cont.

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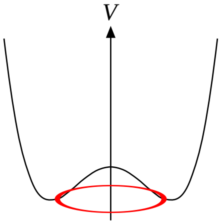
- The most striking feature is that there is no term proportional to $(\pi^j)^2$. Thus we have $N-1$ massless *Goldstone* Bosons and 1 massive boson (σ) with mass $\sqrt{2}\mu$.

Goldstone Bosons – Continuous Symmetries Cont.



- The existence of Goldstone Bosons can be understood in terms of zero modes.
- $O(N)$ has $N(N-1)/2$ generators and the residual symmetry $O(N-1)$ has $(N-1)(N-2)/2$ generators.
- The number of *Broken Symmetries* is therefore

$$\frac{1}{2}\{N(N-1)-(N-1)(N-2)\} = N-1$$

which is the number of Goldstone Bosons .

Goldstone Bosons – Continuous Symmetries Cont.

- The linear sigma model is an example of Goldstone's Theorem which states that the number of broken symmetries is equal to the number of Goldstone Bosons.

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Goldstone Bosons – Continuous Symmetries Cont.

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- There do not appear to be any massless scalar bosons in nature (however, we'll come back below to the pions as the pseudo-Goldstone bosons of the spontaneous breaking of chiral symmetry).
- Spontaneous Symmetry Breaking however, is a central feature in the Higgs Mechanism for mass generation as we will now see.

The Abelian Higgs Model

An instructive example is the theory of a complex scalar field coupled to the electromagnetic field (and itself)

$$\mathscr{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu\phi|^2 - V(\phi)$$

where D is the covariant derivative, $D_\mu = \partial_\mu + ieA_\mu$ and

$$V(\phi) = -\mu^2\phi^*\phi + \frac{\lambda}{2}(\phi^*\phi)^2 .$$

- $\mathscr{L}$ is invariant under the local (Abelian) $U(1)$ gauge transformation:

$$\phi(x) \to e^{i\alpha(x)}\phi(x)\,, \qquad A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x)\,.$$

- For $\mu^2 < 0$ this is simply the quantum electrodynamics of a charged scalar boson.
- For $\mu^2 > 0$ the $U(1)$ symmetry is spontaneously broken.

The Abelian Higgs Model Cont.

$$V(\phi) = -\mu^2\,\phi^*\phi + \frac{\lambda}{2}\,(\phi^*\phi)^2\,.$$

- The minimum of this potential is at:

$$|\langle\phi\rangle| = |\phi_0| = \left(\frac{\mu^2}{\lambda}\right)^{\frac{1}{2}}\,.$$

- Consider the minimum to be in the positive real direction (i.e. ϕ_0 to be real and positive) and define the shifted fields $\phi_{1,2}$:

$$\phi(x) = \phi_0 + \frac{1}{\sqrt{2}}\,(\phi_1(x) + i\phi_2(x))\,.$$

- The potential can be rewritten in terms of the fields $\phi_{1,2}$:

$$V(\phi) = -\frac{1}{2\lambda}\,\mu^4 + \frac{1}{2}\,2\mu^2\phi_1^2 + O(\phi_i^3)\,.$$

- ϕ_1 is a scalar with mass $\sqrt{2}\mu$ and ϕ_2 is the massless Goldstone Boson.

The Abelian Higgs Model Cont.

$$|D_\mu\phi|^2 = \frac{1}{2}\left(\partial_\mu\phi_1\right)^2 + \frac{1}{2}\left(\partial_\mu\phi_2\right)^2 + \sqrt{2}e\phi_0 A_\mu\partial^\mu\phi_2 + e^2\phi_0^2 A_\mu A^\mu + \cdots .$$

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- The photon has acquired a mass

$$m_A^2 = 2e^2\phi_0^2 .$$

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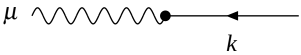
- There is a peculiar two-point term between the Goldstone Boson ϕ_2 and the photon:

$$\sqrt{2} e \phi_0 \, A_\mu \partial^\mu \phi_2 .$$



$$= \; i\sqrt{2} e \phi_0 (-ik^\mu) \; = \; m_A k^\mu$$

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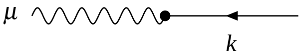


$$= \; i\sqrt{2}e\phi_0\,(-ik^\mu) \; = \; m_A k^\mu$$

- To interpret this it is convenient to go to the *Unitary Gauge*, i.e. to choose a gauge transformation $\alpha(x)$ at each point such that $\phi_2(x) = 0$.
  - Theory with a real scalar field ϕ_1 and a massive vector field A_μ.
  - Degrees of Freedom:

Massless Vector + Complex Scalar	$2+2=4$
Massive Vector + Real Scalar	$3+1=4$.

The Abelian Higgs Model Cont.

Degrees of Freedom:

$$\begin{array}{cc} \text{Massless Vector + Complex Scalar} & 2+2=4 \\ \text{Massive Vector + Real Scalar} & 3+1=4\,. \end{array}$$

- The Goldstone Boson becomes the longitudinal degree of freedom of the massive vector boson.
- It is frequently said that the GB has been *eaten* by the vector.
- In the unitary gauge the propagator of the vector boson is:

$$\frac{-i}{k^2-M_A^2}\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{M_A^2}\right)\,.$$

The spectrum is the physical one, but renormalizability is not manifest.

The Abelian Higgs Model Cont.

- To define the 't Hooft gauges we add a gauge-fixing term to the Lagrangian:

$$\mathscr{L}_{\mathrm{GF}} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu + \xi M_A \phi_2\right)^2 .$$

Now the propagator of the vector boson is

$$\frac{-i}{k^2 - M_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) - \frac{i\xi}{k^2 - \xi M_A^2}\,\frac{k^\mu k^\nu}{k^2} ,$$

and that of ϕ_2 is

$$\frac{i}{k^2 - \xi M_A^2} .$$

Now the power-counting is manifestly correct for a renormalizable theory but unitarity is not manifest.

- 't Hooft and Veltmann's Nobel Prize in 1999 was for *for elucidating the quantum structure of electroweak interactions in physics* which included the demonstration of the renormalizability and consistency of spontaneously broken field theories, particularly non-Abelian ones (to which we now turn).

Towards $SU(2)\times U(1)$

Textbooks give many examples of the spontaneous breaking of non-abelian gauge symmetries.

In general the number of massive vector bosons = the number of *broken generators*, i.e. the number of symmetries of the action which are not symmetries of a vacuum state.

An illustrative example is the Georgi-Glashow Model (1972) in which an $SU(2)$ gauge theory with the Higgs in the adjoint representation $\to U(1)$.

- The $U(1)$ could be electromagnetism and the two massive vectors could have been the $W^{\pm}$.
- 3 Scalars + 3 Massless Vectors have 3+6=9 degrees of freedom.
- 1 (physical) scalar + 1 massless vector + 2 massive vectors have 1+2+6=9 degrees of freedom $\surd$.
- The demonstration of the existence of weak *neutral current* interactions implied that we also need a neutral massive vector (Z^0) and hence a different theory $\Rightarrow SU(2)\times U(1)$.

$SU(2) \times U(1)$

The covariant derivative for the $SU(2) \times U(1)$ theory, with the complex Higgs fields in the fundamental representation (complex doublet = 4 real fields), is:

$$D_\mu \phi = (\partial_\mu - igA^a_\mu \tau^a - \frac{1}{2} g' B_\mu)\, \phi$$

- $\tau^a = \sigma^2/2$ and the σ's are the Pauli spin matrices.
- Gauge transformation:

  $$\phi \rightarrow e^{i\alpha^a \tau^a} e^{i\beta/2} \phi$$

  where a $U(1)$ charge +1/2 has been assigned to the Higgs fields.
- Imagine that the Higgs Potential is such that a minimum occurs at

  $$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}.$$

- Gauge transformation with $\alpha^1 = \alpha^2 = 0$ and $\alpha^3 = \beta$ leaves $\langle\phi\rangle$ unchanged $\Rightarrow$ $SU(2) \times U(1) \rightarrow U(1)$.
- We therefore expect 1 massless vector boson (the photon) and three massive vectors ($W^\pm, Z^0$) and one physical Higgs scalar.

$SU(2)\times U(1)$ Cont.

$$D_\mu\phi=(\partial_\mu - igA^a_\mu\tau^a - \frac{1}{2}g'B_\mu)\,\phi$$

The mass terms for the vector bosons come from $|D_\mu\phi|^2$:

$$\frac{1}{2}\begin{pmatrix}0 & v\end{pmatrix}\left(gA^a_\mu\tau^a+\frac{1}{2}g'B_\mu\right)\left(gA^{\mu\,b}\tau^b+\frac{1}{2}g'B^\mu\right)\begin{pmatrix}0\\ v\end{pmatrix}$$

which, using the properties of the σ-matrices, can readily be rewritten as

$$\frac{1}{2}\frac{v^2}{4}\left\{g^2\,(A^1_\mu)^2+g^2\,(A^2_\mu)^2+(-gA^3_\mu+g'B_\mu)^2\right\}.$$

Thus we have the expected spectrum of vector bosons:

$$W^\pm=\frac{1}{\sqrt{2}}\left(A^1_\mu\mp iA^2_\mu\right) \qquad m_W=g\frac{v}{2}$$

$$Z_\mu=\frac{1}{\sqrt{g^2+g'^2}}\left(gA^3_\mu-g'B_\mu\right) \qquad m_z=\sqrt{g^2+g'^2}\,\frac{v}{2}$$

$$A_\mu=\frac{1}{\sqrt{g^2+g'^2}}\left(g'A^3_\mu+gB_\mu\right) \qquad m_\gamma=0\,.$$

$SU(2) \times U(1)$ Cont.

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g A^3_\mu - g' B_\mu \right) \qquad A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g' A^3_\mu + g B_\mu \right)$$

- It is convenient to define the *weak mixing angle* θ_W:

$$\begin{pmatrix} Z^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A^3 \\ B \end{pmatrix}$$

  so that

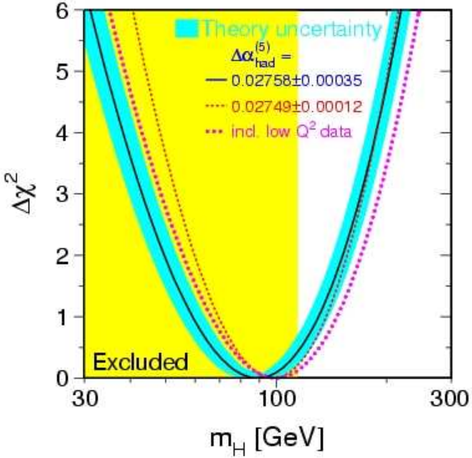
$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}\,, \quad \text{and} \quad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}\,.$$

- At tree level

$$m_W = m_Z \cos\theta_W\,.$$

- Inclusion of Fermions - Lecture 2

What is the likely value of the Higgs' Mass?



What is the Mass of the Higgs Boson?

- Preferred Value $m_h = 89^{+42}_{-30}$ GeV .
- LEP-2 Direct Search Limit $M_h > 114.4$ GeV . hep-ex/0306033
- The relation $m_W = m_z \cos\theta_w$ holds experimentally (up to radiative corrections). Does this imply a single complex Higgs doublet and a single physical Higgs boson? Not necessarily.

  • The sector of the theory responsible for the symmetry breaking has a global $SU(2)\times U(1)$ symmetry (promoted to a local symmetry when couplings to the gauge bosons are introduced).

  • If, as the gauge symmetry is broken, the global $SU(2)$ symmetry remains unbroken and the three Goldstone bosons and the corresponding Noether currents transform as triplets under this *custodial* $SU(2)$ *symmetry* $\Rightarrow$ we recover the above relation.

  • Chiral Symmetry breaking in QCD is one possible example. Custodial $SU(2)$ = vector isospin symmetry. However the value of the decay constant f_π,

$$\langle 0|J^5_\mu|\pi(p)\rangle = i f_\pi p^\mu$$

  is $O(10^3)$ too small ($f_\pi = 132$ MeV).