Standard Model of Particle Physics

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Lecture 2 — The Electroweak Theory

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Chirality

- Experiment $\Rightarrow$ only the left-handed components of the fermions participate in charged current weak interactions, i.e. the W's only couple to the left-handed components.
- $$\psi_L = P_L\psi = \frac{1}{2}(1-\gamma^5)\psi \qquad \psi_R = P_R\psi = \frac{1}{2}(1+\gamma^5)\psi$$
  Under parity transformations $\psi_L(x_0,\vec{x}) \to \psi_R(x_0,-\vec{x})$ and $\psi_R(x_0,\vec{x}) \to \psi_L(x_0,-\vec{x})$
- P_L and P_R are projection operators
  $$P_L^2 = P_L \quad \text{and} \quad P_R^2 = P_R \qquad (P_LP_R = P_RP_L = 0,\ P_L + P_R = I)$$
- $$\bar{\psi}\gamma^\mu\psi = \bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R \quad \text{and} \quad \bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\,.$$
  (Thus for QCD with N massless fermions we have a $U(N)\times U(N)$ (global) chiral symmetry - I come back to this in later lectures.)
- In order to accommodate the observed nature of the parity violation the left and right-handed fermions are assigned to different representations of $SU(2)\times U(1)$, with the right-handed fields being singlets of $SU(2)$.

Fermions

For a general representation of fermions the covariant derivative takes the form:

$$D_\mu = \partial_\mu - igA_\mu^a T^a - ig' Y B_\mu \,,$$

where the T^a are the corresponding generators of $SU(2)$ and the $Y's$ are the weak-hypercharges. The covariant derivative can be rewritten in terms of the mass-eigenstates as:

$$D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{g^2 T^3 - g'^2 Y}{\sqrt{g^2 + g'^2}} Z_\mu - i\frac{gg'}{\sqrt{g^2 + g'^2}}(T^3 + Y)A_\mu \,.$$

- Thus the electic charge operator is

$$Q = T_3 + Y \quad \text{and} \quad e = \frac{gg'}{\sqrt{g^2 + g'^2}} \,. \qquad (Q = -1 \text{ for the electron}).$$

- The left-handed quarks and leptons are assigned to doublets of $SU(2)$ and the right-handed fermions are singlets.

Assignment of Fermions

$$Q = T_3 + Y$$

- The left handed leptons are assigned to the doublet.

$$E_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L .$$

In order to have the correct charge assignments $Y_{\nu_e} = Y_{e_L} = -1/2$.

- For the right-handed lepton fields $T_3 = 0$ and hence $Y_{e_R} = -1$. In the standard model we do not have a right-handed neutrino!
- For the left-handed quark fields we have the left-handed doublet:

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L .$$

with $Y_{Q_L} = 1/6$.

- The right-handed quark fields therefore have $Y_{u_R} = 2/3$ and $Y_{d_R} = -1/3$.
- Similar assignments are made for the other two generations.

Fermion Lagrangian

The terms in the Lagrangian involving the fermions then take the form:

$$
\begin{aligned}
\mathscr{L} &= \bar{E}_L(i\not\partial)E_L + \bar{e}_R(i\not\partial)e_R + \bar{Q}_L(i\not\partial)Q_L + \bar{u}_R(i\not\partial)u_R + \bar{d}_R(i\not\partial)d_R \\
&\qquad + g\left(W^+_\mu J^{\mu\,+}_W + W^-_\mu J^{\mu\,-}_W + Z^0_\mu J^\mu_Z\right) + eA_\mu J^\mu_{\mathrm{EM}}\,,
\end{aligned}
$$

where

$$
\begin{aligned}
J^{\mu\,+}_W &= \frac{1}{\sqrt{2}}\left(\bar{\nu}_L\gamma^\mu e_L + \bar{u}_L\gamma^\mu d_L\right); \\
J^{\mu\,-}_W &= \frac{1}{\sqrt{2}}\left(\bar{e}_L\gamma^\mu \nu_L + \bar{d}_L\gamma^\mu u_L\right); \\
J^\mu_Z &= \frac{1}{\cos\theta_W}\left\{\frac{1}{2}\,\bar{\nu}_L\gamma^\mu\nu_L + \left(\sin^2\theta_W - \frac{1}{2}\right)\bar{e}_L\gamma^\mu e_L + \sin^2\theta_W\,\bar{e}_r\gamma^\mu e_R\right. \\
&\qquad + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)\bar{u}_L\gamma^\mu u_L - \frac{2}{3}\sin^2\theta_W\,\bar{u}_R\gamma^\mu u_R \\
&\qquad \left. + \left(\frac{1}{3}\sin^2\theta_W - \frac{1}{2}\right)\bar{d}_L\gamma^\mu d_L + \frac{1}{3}\sin^2\theta_W\,\bar{d}_R\gamma^\mu d_R\right\}; \\
J^\mu_{\mathrm{EM}} &= -\bar{e}\gamma^\mu e + \frac{2}{3}\,\bar{u}\gamma^\mu u - \frac{1}{3}\,\bar{d}\gamma^\mu d\,.
\end{aligned}
$$

Fermion Masses - Yukawa Couplings

The standard mass term for the fermions is of the form

$$m\bar{\psi}\psi = m\bar{\psi}_L\psi_R + m\bar{\psi}_R\psi_L\,.$$

It is therefore not invariant under the $SU(2)_L$ gauge-symmetry and can be shown to spoil renormalizability.

- In the SM, mass terms for the fermions are generated through Yukawa Couplings to the Higgs Doublet, for example:

$$\Delta\mathscr{L}_e = -\lambda_e\,(\bar{E}^i_L\phi^i)\,e_R + h.c.$$

where $i = 1,2$ is the $SU(2)$ label. As before, we rewrite the complex doublet ϕ in terms of the fields shifted by $\langle\phi\rangle$, so that

$$\Delta\mathscr{L}_e = -\frac{\lambda_e v}{\sqrt{2}}\,\bar{e}_L e_R + h.c. + \text{ interaction terms}$$

In this picture therefore

$$m_e = \frac{\lambda_e v}{\sqrt{2}}\,.$$

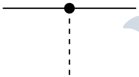
Fermion Masses - Yukawa Couplings Cont.

$$m_e = \frac{\lambda_e v}{\sqrt{2}}\,.$$

- Thus we have generated a mass-term for the electron in a gauge invariant way. We have traded the parameter m_e for the Yukawa coupling λ_e .
- λ_e is very small ($v \simeq 250\,$GeV) and the problem of understanding the pattern of fermion masses becomes the problem of understanding the pattern of Yukawa couplings.
- We can choose a gauge such that the scalar field is written in the form

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}\,,$$

where $h(x)$ is the physical Higgs scalar. The electron Yukawa term now takes the form $\mathscr{L}_e = -m_e\left(1+\frac{h}{v}\right)\bar{e}e\,.$



$= -i\,\frac{m_e}{v}$

Fermion Masses - Yukawa Couplings Cont.

$$\mathscr{L}_e = -m_e \left(1+\frac{h}{v}\right)\bar{e}e\,.$$

- By construction, it is a general feature that the couplings of the Higgs boson h are proportional to the masses (or squares of masses) of the particles it is interacting with.

  This is an important ingredient in the phenomenology of Higgs searches.
- For the down quark we can introduce a similar Yukawa term to that of the electron. For the up quark, this clearly does not work, but we can exploit the existence of the invariant anti-symmetric tensor ε^{ij}.

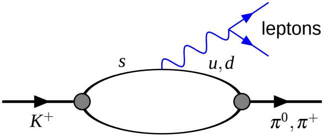
$$\begin{aligned}\Delta\mathscr{L}_q &= -\lambda_d \bar{Q}^i_L \phi^i d_R - \lambda_u \varepsilon^{ij} \bar{Q}^i_L \phi^{\dagger j} u_R + h.c. \\ &= -\frac{\lambda_d v}{\sqrt{2}}\bar{d}_L d_R - \frac{\lambda_u v}{\sqrt{2}}\bar{u}_L u_R + h.c. + \text{ interaction terms} \\ &= -m_d\left(1+\frac{h}{v}\right)\bar{d}d - m_u\left(1+\frac{h}{v}\right)\bar{u}u\,.\end{aligned}$$

(Note that apart from being singlets under $SU(2)$, the terms in the action also have zero net hypercharge.)

Quark Mixing

Two Experimental Numbers:

$$B(K^+ \to \pi^0 e^+ \nu_e) \simeq 5\% \; (K_{e3}^+ \text{ Decay}) \quad \text{and} \quad B(K^+ \to \pi^+ e^+ e^-) < 3 \times 10^{-7}.$$



Measurements like this show that $s \to u$ (charged-current) transitions are not rare, but that *Flavour Changing Neutral Current* (FCNC) transitions, such as $s \to d$ are.

In the picture that we have developed so far, there are no transitions between fermions of different generations. This has to be modified.

The picture which has emerged is the Cabibbo-Kobayashi-Maskawa (CKM) theory of quark mixing which we now consider.

CKM Theory

In the CKM theory the (quark) mass eigenstates are not the same as the weak-interaction eigenstates which we have been considering up to now.

Let

$$U' = \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix} = U_u \begin{pmatrix} u \\ c \\ t \end{pmatrix} = U_u\, U \quad \text{and} \quad D' = \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = U_d \begin{pmatrix} d \\ s \\ b \end{pmatrix} = U_d D$$

where the $'$s denote the weak interaction eigenstates and U_u and U_d are unitary matrices.

- For neutral currents:

$$\bar{U}' \cdots U' = \bar{U} \cdots U \quad \text{and} \quad \bar{D}' \cdots D' = \bar{D} \cdots D$$

  and no FCNC are induced. The $\cdots$ represent Dirac Matrices, but the identity in flavour.

- For charged currents:

$$J_W^{\mu\,+} = \frac{1}{\sqrt{2}}\, \bar{U}'_L \gamma^\mu D'_L = \frac{1}{\sqrt{2}}\, \bar{U}_L U_u^\dagger \gamma^\mu U_d D_L = \frac{1}{\sqrt{2}}\, \bar{U}_L \gamma^\mu (U_u^\dagger U_d) D_L \equiv \frac{1}{\sqrt{2}}\, \bar{U}_L \gamma^\mu V_{\text{CKM}} D_L$$

The CKM Matrix

- The charged-current interactions are of the form

$$J^{+}_{\mu} = (\bar{u}, \bar{c}, \bar{t})_L \gamma_\mu V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L ,$$

- 2005 Particle Data Group summary for the magnitudes of the entries:

$$\begin{pmatrix} 0.9739 - 0.9751 & 0.221 - 0.227 & 0.0029 - 0.0045 \\ 0.221 - 0.227 & 0.9730 - 0.9744 & 0.039 - 0.044 \\ 0.0048 - 0.014 & 0.037 - 0.043 & 0.9990 - 0.9992 \end{pmatrix} .$$

- How many parameters are there?
  - Let N_g be the number of generations.
  - $N_g \times N_g$ unitary matrix has N_g^2 real parameters.
  - $(2N_g - 1)$ of them can be absorbed into unphysical phases of the quark fields.
  - $(N_g - 1)^2$ physical parameters to be determined.

Parametrizations of the CKM Matrix

- For $N_g = 2$ there is only one parameter, which is conventionally chosen to be the Cabibbo angle:

$$V_{\text{CKM}} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}.$$

- For $N_g = 3$, there are 4 real parameters. Three of these can be interpreted as angles of rotation in three dimensions (e.g. the three Euler angles) and the fourth is a phase. The general parametrization recommended by the PDG is

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

  where c_{ij} and s_{ij} represent the cosines and sines respectively of the three angles θ_{ij}, $ij = 12, 13$ and 23. δ_{13} is the phase parameter.

- It is conventional to use approximate parametrizations, based on the hierarchy of values in V_{CKM} ($s_{12} \gg s_{23} \gg s_{13}$).

The Wolfenstein Parametrization

The Wolfenstein parametrization is

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$

- $\lambda = s_{12}$ is approximately the Cabibbo angle.
- A, ρ and η are real numbers that a priori were intended to be of order unity.
- Corrections are of $O(\lambda^4)$.

The Unitarity Triangle

Unitarity of the CKM-matrix we have a set of relations between the entries. A particularly useful one is:

$$V_{ud}V_{ub}^{*}+V_{cd}V_{cb}^{*}+V_{td}V_{tb}^{*}=0\ .$$

In terms of the Wolfenstein parameters, the components on the left-hand side are given by:

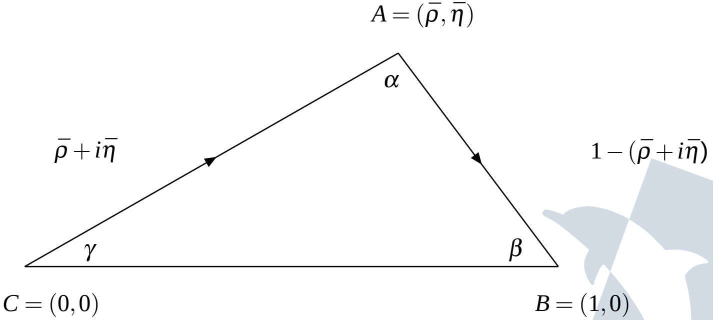
$$\begin{aligned} V_{ud}V_{ub}^{*} &= A\lambda^{3}[\bar{\rho}+i\bar{\eta}]+O(\lambda^{7}) \\ V_{cd}V_{cb}^{*} &= -A\lambda^{3}+O(\lambda^{7}) \\ V_{td}V_{tb}^{*} &= A\lambda^{3}[1-(\bar{\rho}+i\bar{\eta})]+O(\lambda^{7})\ , \end{aligned}$$

where $\bar{\rho}=\rho(1-\lambda^{2}/2)$ and $\bar{\eta}=\eta(1-\lambda^{2}/2)$.

The unitarity relation can be represented schematically by the famous “unitarity triangle” (obtained after scaling out a factor of $A\lambda^{3}$).

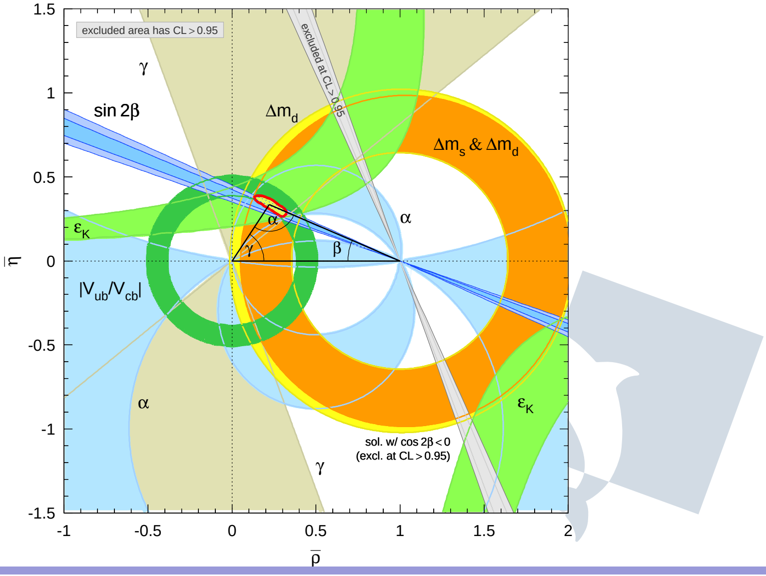
The Unitarity Triangle Cont.

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0 \ .$$



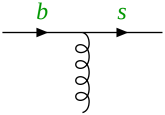
A particularly important approach to testing the *Limits of the SM* is to over-determine the position of the vertex A to check for consistency.

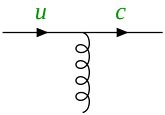
PDG2006 Unitarity Triangle



Flavour Changing Neutral Currents (FCNC)

We have seen that in the SM, unitarity implies that there are no FCNC reactions at tree level, i.e. there are no vertices of the type:

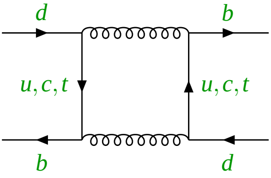


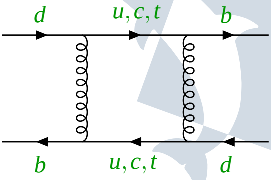


.

Quantum loops, however, can generate FCNC reactions, through *box* diagrams or *penguin* diagrams.

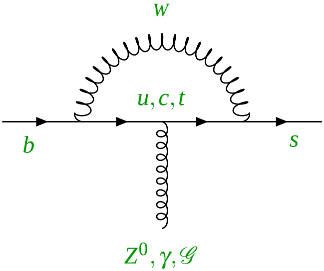
Example relevant for $\bar{B}^0 - B^0$ mixing:

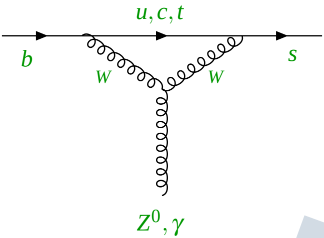




FCNC Cont.

Examples of penguin diagrams relevant for $b \to s$ transitions:





We will discuss several of the physical processes induced by these loop-effects.

The Glashow-Illiopoulos-Maiani (GIM) mechanism $\Rightarrow$ FCNC effects vanish for degenerate quarks ($m_u = m_c = m_t$). For example unitarity implies

$$V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0$$

$\Rightarrow$ each of the above penguin vertices vanish.

The Discrete Symmetries P, C and CP

Parity

$$(\vec{x}, t) \rightarrow (-\vec{x}, t).$$

The vector and axial-vector fields transform as:

$$V_\mu(\vec{x}, t) \rightarrow V^\mu(-\vec{x}, t) \;\; \text{and} \;\; A_\mu(\vec{x}, t) \rightarrow -A^\mu(-\vec{x}, t).$$

- The vector and axial-vector currents transform similarly.

Left-handed components of fermions $\psi_L = (\frac{1}{2}(1-\gamma^5)\psi)$ transform into right-handed ones $\psi_R = (\frac{1}{2}(1+\gamma^5)\psi)$, and vice-versa.

- Since CC weak interactions in the SM only involve the left-handed components, parity is not a good symmetry of the weak force.
- QCD and QED are invariant under parity transformations.

Charge Conjugation – Charge conjugation is a transformation which relates each complex field ϕ with $\phi^\dagger$.

Under C the currents transform as follows:

$$\bar{\psi}_1\gamma_\mu\psi_2 \to -\bar{\psi}_2\gamma_\mu\psi_1 \quad \text{and} \quad \bar{\psi}_1\gamma_\mu\gamma_5\psi_2 \to \bar{\psi}_2\gamma_\mu\gamma_5\psi_1,$$

where ψ_i represents a spinor field of type (flavour or lepton species) i.

CP – Under the combined CP-transformation, the currents transform as:

$$\bar{\psi}_1\gamma_\mu\psi_2 \to -\bar{\psi}_2\gamma^\mu\psi_1 \quad \text{and} \quad \bar{\psi}_1\gamma_\mu\gamma_5\psi_2 \to -\bar{\psi}_2\gamma^\mu\gamma_5\psi_1.$$

The fields on the left (right) hand side are evaluated at $(\vec{x},t)$ ($(-\vec{x},t)$).

CP Cont.

Consider now a charged current interaction:

$$(W_\mu^1 - iW_\mu^2)\,\bar{U}^i\gamma^\mu(1-\gamma^5)V_{ij}D^j + (W_\mu^1 + iW_\mu^2)\,\bar{D}^j\gamma^\mu(1-\gamma^5)V_{ij}^*U^i,$$

U^i and D^j are up and down type quarks of flavours i and j respectively.

Under a CP transformation, the interaction term transforms to:

$$(W_\mu^1 + iW_\mu^2)\,\bar{D}^j\gamma^\mu(1-\gamma^5)V_{ij}U^i + (W_\mu^1 - iW_\mu^2)\,\bar{U}^i\gamma^\mu(1-\gamma^5)V_{ij}^*D^j$$

- CP-invariance requires V to be real
  (or more strictly that any phases must be able to be absorbed into the definition of the quark fields).
- For CP-violation in the quark sector we therefore require 3 generations.

Higgs Mass and Interactions

Imagine that the Higgs potential is

$$V=-\mu^2\,(\phi^\dagger\phi)+\lambda\,(\phi^\dagger\phi)^2 \quad \text{and write} \quad \phi=\begin{pmatrix}0\\ \frac{1}{\sqrt{2}}(v+h(x))\end{pmatrix} \quad \text{where} \quad v^2=\frac{\mu^2}{\lambda}\,.$$

In terms of $h(x)$:

$$V=\mu^2\,h^2+\sqrt{\lambda}\mu\,h^3+\frac{\lambda}{4}\,h^4\,.$$

- We know $v=\mu/\sqrt{\lambda}=250\,$GeV from M_W and other quantities.
- The mass of the Higgs is $\sqrt{2}\mu$. Today, we have no direct way of knowing this.
- The larger that m_h is, the stronger are the Higgs self interactions.
- Finally, I stress that even if the overall picture is correct, the Higgs sector may be more complicated than the simplest picture presented here.