Standard Model of Particle Physics

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The QCD Lagrangian

$$\mathscr{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\psi}(i\not{D} - m)\psi + \mathscr{L}_{\mathrm{GF}}$$

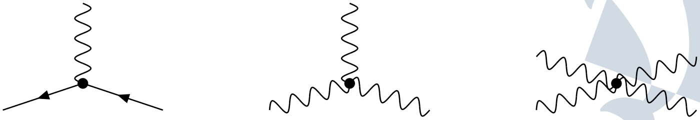
where $a = 1,8$ is an adjoint label.

Each flavour of quark transforms under the fundamental representation of $SU(3)$ and the gluons transform under the adjoint representation (as do all gauge bosons).

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

f^{abc} are the structure constants of $SU(3)$

$$[T^a, T^b] = i f^{abc} T^c, \qquad D_\mu = \partial_\mu - i g A^a_\mu T^a$$



Asymptotic Freedom

- Asymptotic Freedom $\Rightarrow$ we can make perturbative predictions in QCD for a large range of important hard processes.

$$\beta(g) \equiv \mu \frac{\partial g}{\partial \mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2} - \beta_2 \frac{g^7}{(16\pi^2)^3} - \cdots$$

where (β_2 is in the $\overline{\mathrm{MS}}$ scheme)

$$\beta_0 = 11 - \frac{2}{3} n_f\,, \quad \beta_1 = 102 - \frac{38}{3} n_f\,, \quad \beta_2 = \frac{2857}{2} - \frac{5033}{18} n_f + \frac{325}{54} n_f^2\,,$$

where n_f is the number of quarks with mass less than the scale μ .

- β_3 is also known. S.A.Larin et al.(1997)
- The key feature is that the first term is negative $\Rightarrow$ the coupling constant decreases with the scale μ.

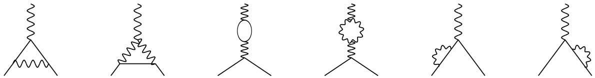
$$\alpha_s(\mu) \equiv \frac{g^2(\mu)}{4\pi} = \frac{4\pi}{\beta_0 \log(\mu^2/\Lambda^2)} \left\{ 1 - \frac{\beta_1}{\beta_0^2} \frac{\log\left[\log(\mu^2/\Lambda^2)\right]}{\log(\mu^2/\Lambda^2)} + \frac{\beta_1^2}{\beta_0^4 \log^2(\mu^2/\Lambda^2)} \times \left(\left(\log[\log(\mu^2/\Lambda^2)] - \frac{1}{2} \right)^2 + \frac{\beta_2 \beta_0}{\beta_1^2} - \frac{5}{4} \right) \right\}.$$

Running Coupling Constant

$$\alpha_s(\mu) \equiv \frac{g^2(\mu)}{4\pi} = \frac{4\pi}{\beta_0 \log(\mu^2/\Lambda^2)} \left\{ 1 - \frac{\beta_1}{\beta_0^2} \frac{\log\left[\log(\mu^2/\Lambda^2)\right]}{\log(\mu^2/\Lambda^2)} + \frac{\beta_1^2}{\beta_0^4 \log^2(\mu^2/\Lambda^2)} \times \left(\left(\log[\log(\mu^2/\Lambda^2)] - \frac{1}{2} \right)^2 + \frac{\beta_2 \beta_0}{\beta_1^2} - \frac{5}{4} \right) \right\}.$$

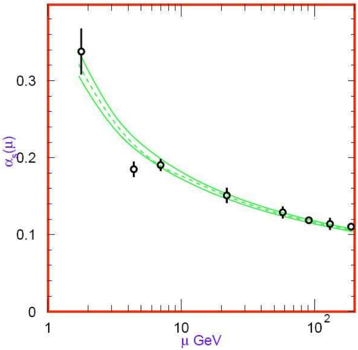
- The constant of integration, Λ, can be considered as a parameter of QCD (equivalent to g)

Dimensional Transmutation $\qquad g \Leftrightarrow \Lambda.$



$+$ ghosts

The Running Coupling Cont



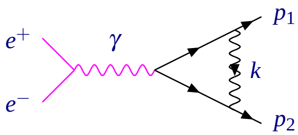
$\alpha_s(\mu)$ at values of μ where they are measured, (τ-width, Υ-decays, Deep Inelastic Scattering, e^+e^- Event Shapes at 22 and 59 GeV, Z–Width, e^+e^- Event Shapes at 135 and 189 GeV (PDG(2005)).

PDG(2005) result:

$$\alpha_s(M_Z) = 0.1176 \pm 0.002\,.$$

Infrared Safety

Consider the following diagram contributing to the $e^+e^- \to q\bar{q}$ amplitude:



To illustrate the behaviour at small momenta ($|k_\mu| \ll \sqrt{p_1 \cdot p_2}$) consider the integral:

$$I \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + i\varepsilon)((p_1+k)^2 - m^2 + i\varepsilon)((p_2-k)^2 - m^2 + i\varepsilon)} .$$

(For small momenta the numerator is a constant and so we simply neglect it here.)

- For $p_1^2 = p_2^2 = m^2$, at small momenta

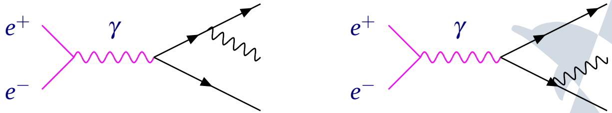
$$I \sim \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(2p_1 \cdot k)(-2p_2 \cdot k)} \quad \text{and is logarithmically divergent.}$$

Infrared Safety Cont.

- The presence of *infrared divergences* $\Rightarrow$ long-distance/low-momentum contributions are important and therefore there is a danger that asymptotic freedom may not be sufficient to calculate predictions in perturbation theory.
- For *inclusive* reactions, such as $e^+e^- \to$ hadrons, the infrared divergences cancel between diagrams with virtual and real gluons.

  Generalization of Bloch-Nordsieck (1937) Theorem from QED to QCD.

  For example, at $O(\alpha_s)$ the following diagrams contribute to $\sigma(e^+e^- \to \text{hadrons})$:



  These also contribute infrared divergent terms to $\sigma(e^+e^- \to \text{hadrons})$ at $O(\alpha_s)$.

  $\sigma(e^+e^- \to \text{hadrons})$ at any order of perturbation theory is free of infrared divergences.

Infrared Safety Cont.

- The standard physical interpretation in QED is that in any experiment we cannot distinguish e from $e+$soft γs, where the γs are too soft to be detected. It is therefore not unreasonable to have to sum over all experimentally indistinguishable contributions.
- Infrared divergences are not the only source of *mass singularities*.
  Consider two massless particles moving parallel to each other (in the z-direction say).

  $$q_1 = \omega_1\,(1,0,0,1)\,,\; q_2 = \omega_2\,(1,0,0,1) \quad \Rightarrow \quad (q_1+q_2)^2 = 0\,.$$

  When internal particles are *collinear* with external ones we get *collinear divergences*.
- The Kinoshita-Lee-Naunberg theorem $\Rightarrow$ collinear divergences cancel when we sum over all degenerate final and initial states.
  For QCD perturbative corrections to $\sigma(e^+e^- \to \text{hadrons})$ only the sum over final states has to performed and the collinear divergences cancel.
- The standard physical interpretation is that we cannot distinguish q from $q+$a collinear gluon (for example), where the collinearity is below the angular resolution. Again it is therefore not unreasonable to have to sum over all experimentally indistinguishable contributions.

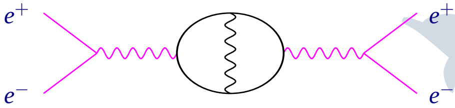
Infrared Safety Cont.

- To give you some confidence in the statements above consider the consequences of unitarity:

$$SS^\dagger = I \quad \Rightarrow \quad (I + iT)(I - iT^\dagger) = I \quad \Rightarrow \quad 2\text{Im}\langle i\,|\,T\,|\,i\rangle = \sum_n |\langle i\,|\,T\,|n\rangle|^2$$

Optical Theorem

- Thus $\sigma(e^+e^- \to \text{hadrons})$ is proportional to the imaginary part of the e^+e^- forward amplitude. But when we look at diagrams such as:



power counting $\Rightarrow$ there are no mass singularities $\Rightarrow$ the mass singularities cancel between the separate contributions to the cross section.

$\sigma(e^+e^- \to \textbf{hadrons})$

Consider now the cross-section for $e^+e^- \to \gamma^* \to \text{hadrons}$. It takes the form

$$\sigma = \sigma_0 \left(3\sum_f Q_f^2\right)\left(1 + \frac{\alpha_s(\mu)}{\pi} + \frac{\alpha_s^2(\mu)}{(4\pi)^2}\left\{4\beta_0 \log\left(\frac{\mu^2}{Q^2}\right) + c_2\right\} + \cdots\right)$$

where

- σ_0 is the lowest order $e^+e^- \to \mu^+\mu^-$ cross section.
- μ is the renormalization scale at which the coupling is defined;
- the form of the logarithms is fixed by the renormalization group (i.e. independence of σ of μ) and the absence of mass-singularities;
- c_2 is a constant.
- In order to avoid *Large Logarithms* we should choose $\mu^2 \simeq Q^2$

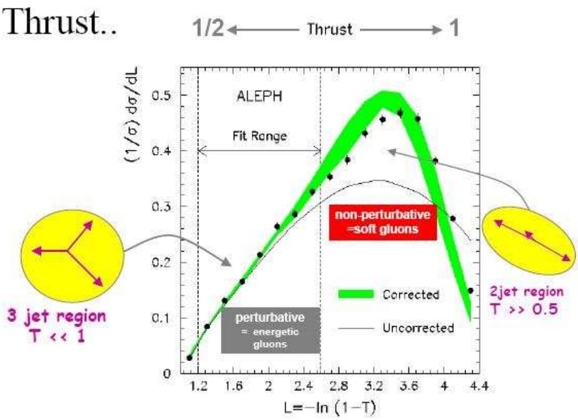
$$\sigma = \sigma_0 \left(3\sum_f Q_f^2\right)\left(1 + \frac{\alpha_s(Q)}{\pi} + 1.411\,\frac{\alpha_s^2(Q)}{(\pi)^2} - 12.8\,\frac{\alpha_s^3(Q)}{(\pi)^3} + \cdots\right)$$

Event Shape Variables

- For about 30 years now we have been trying to get fundamental information about quark and gluon interactions from the observed hadrons in e^+e^- annihilation.
- An instructive example of a measurable quantity which is not calculable because it is not infrared safe is *Sphericity*, proposed by the SLAC group in 1974:
$$\hat{S} \equiv \frac{3}{2} \min_{\text{axes}} \frac{\sum_i |p_\perp^i|^2}{\sum_i |\vec{p}^{\,i}|^2} .$$
The expectation was that $\hat{S} = 0$ for a two-jet event and 1 for an isotropic event.
$\hat{S}$ is experimentally measurable but is not calculable, since $(p_\perp^1)^2 + (p_\perp^2)^2 \neq (p_\perp^1 + p_\perp^2)^2$.
- Today many infrared-safe event shape variables are being used. A classic example is *thrust*
$$T = \max_{\text{axes}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} ,$$
so that $T = 1$ for a two-jet event and $1/2$ for a spherical event.

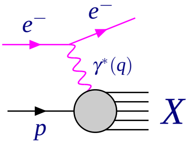
Thrust

LEP QCD Working Group - Roger Jones 6/3/2003 .



Deep Inelastic Scattering

Consider the process $ep \to e + X$:



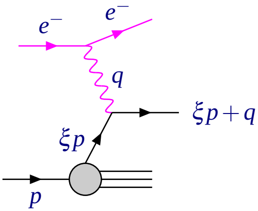
- The incoming lepton can also be a μ or a ν .
- The exchanged boson can also be a Z^0, or in the case of charged-current interactions a W .
- The kinematic region we will be interested in has $-q^2$ and $2p \cdot q$ large (where *large* means w.r.t. Λ) and

$$x \equiv \frac{-q^2}{2p \cdot q} \sim O(1) .$$

x is called *Bjorken* x and is experimentally measurable for each event .

- $(p+q)^2 > 0 \Rightarrow q^2 + 2p \cdot q(+p^2) > 0 \Rightarrow 0 \leq x \leq 1$.

Deep Inelastic Scattering Cont.

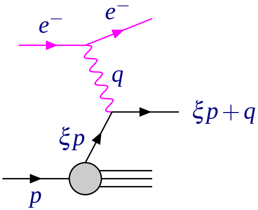


Much intuition was gained from the Feynman-Bjorken parton picture. Noting that the typical scale of strong-interactions is 1 fm or 200 MeV, consider a frame in which $|\vec{p}\,|$ is large

$$(\xi p+q)^2 \simeq 0 \;\Rightarrow\; 2\xi p\cdot q+q^2 \simeq 0 \;\Rightarrow\; \xi = x\,.$$

The experimentally measurable quantity x gives the fraction of the proton's momentum carried by the struck quark (in the *infinite momentum* frame).

Deep Inelastic Scattering Cont.



- Let the probability density of finding the (anti-)quark f with longitudinal fraction x of the proton's momentum be $f_{q_f}(x)$.
  $f_{q_f}(x)$ is called the *parton distribution function*.
- In the parton model:

$$\sigma(e^-(k)p(p) \to e^-(k')X) = \int_0^1 d\xi \sum_f f_{q_f}(\xi)\, \sigma(e^-(k)q_f(\xi p) \to e^-(k')\, q_f(\xi p+q))$$

- We will consider the QCD corrections later.

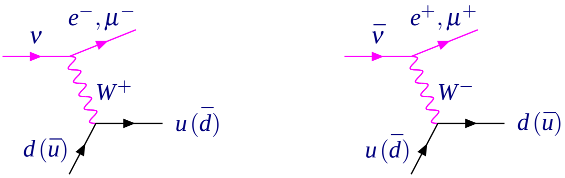
Deep Inelastic Scattering Cont.

- In the parton model

$$\frac{d^2\sigma}{dx\,dy} = \left(\sum_f x f_{q_f}(x)\, Q_f^2\right) \frac{2\pi\alpha^2 s}{q^4}\left[1+(1-y)^2\right],$$

  where $s=(p+k)^2 \simeq 2p\cdot k$ and $y=(2p\cdot q)/s$.
  - In the rest-frame of the proton, y is the fraction of the electron's energy which is transferred to the proton.
- DIS $\Rightarrow$ information about momentum distribution of quarks in the proton.
- Information about different linear combinations of the distribution functions can be obtained from ν scattering (and by including the Z^0 contribution).

Deep Inelastic Scattering Cont.



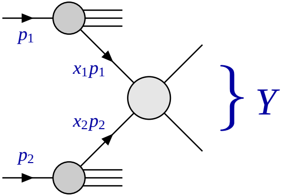
$$\frac{d^2\sigma(\nu p \to \mu^- X)}{dx\,dy} = \frac{G_F^2 s}{\pi}\left[x f_d(x) + x(1-y)^2 f_{\bar{u}}(x)\right]$$

$$\frac{d^2\sigma(\bar{\nu} p \to \mu^+ X)}{dx\,dy} = \frac{G_F^2 s}{\pi}\left[x(1-y)^2 f_u(x) + x f_{\bar{d}}(x)\right]$$

By combining information from e, μ and ν DIS (and more) we get information about each of the distribution functions.

Hard Scattering Processes in Hadronic Collisions

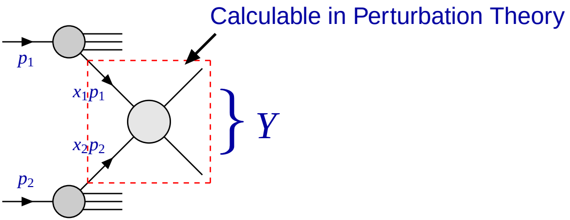
- Before leaving the parton model, consider some hard scattering process in hadron-hadron collisions.



- For example, Y can be a heavy particle (resonance, Higgs, i.e. Drell-Yan Processes) or two (or more) jets at large transverse momentum.

$$\sigma(h_1(p_1)+h_2(p_2)\to Y+X)=\int_0^1 dx_1\int_0^1 dx_2\sum_{f_1,f_2} f_{f_1}(x_1)f_{f_2}(x_2)\sigma(f_1+f_2\to Y)\,.$$

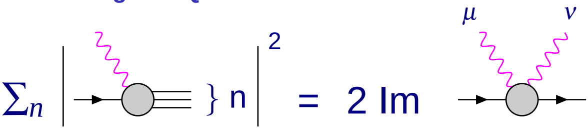
Hard Scattering Processes in Hadronic Collisions



$$\sigma(h_1(p_1)+h_2(p_2)\to Y+X)=\int_0^1 dx_1\int_0^1 dx_2\sum_{f_1,f_2} f_{f_1}(x_1)f_{f_2}(x_2)\sigma(f_1+f_2\to Y)\,.$$

- The f_{f_i} s are "known" from Deep Inelastic Scattering.
- It is in this way (modified to take QCD corrections into account) that we were able to make predictions for the cross sections for W and Z production at the SPS or are able to make predictions for Higgs Boson production at the LHC.

Deep Inelastic Scattering and QCD



- Using the optical theorem, we need to evaluate the virtual forward Compton amplitude:

$$W^{\mu\nu}(x,q^2) = i \int d^4x\, e^{iq\cdot x} \langle p \,|\, T\{J^\mu(x) J^\nu(0)\} \,|\, p \rangle$$

- Lorentz & Parity Invariance and Current Conservation $\Rightarrow$

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1(x,q^2) + \left(p^\mu - q^\mu \frac{p\cdot q}{q^2}\right)\left(p^\nu - q^\nu \frac{p\cdot q}{q^2}\right) W_2(x,q^2),$$

  where $W_{1,2}$ are scalar functions.

- With weak interactions, so that parity is no longer a good symmetry, there is a third *structure function* W_3 multiplying the tensor $\varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta$.

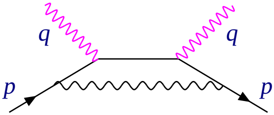
Deep Inelastic Scattering and QCD Cont.

- In the parton model

$$\text{Im}\,W_1(x) = \pi \sum_f Q_f^2 f_f(x) \quad \text{and} \quad \text{Im}\,W_2(x) = \frac{4\pi}{ys} \sum_f Q_f^2 x f_f(x) \quad \text{so that} \quad \text{Im}\,W_1 = \frac{ys}{4x}\,\text{Im}\,W_2 .$$

In a commonly used notation $\nu = p \cdot q$, $F_1 \equiv \text{Im}\,W_1$, $F_2 \equiv \text{Im}\,\nu W_2$ so that in the parton model $F_2 = 2xF_1$.

- In QCD there are diagrams such as

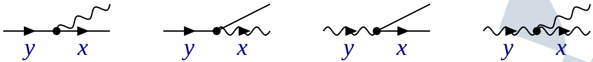


- These one-loop diagrams give a contribution proportional to $\alpha_s \log(q^2/p^2)$.
- The (collinear) mass singularities do not cancel, in spite of the KLN theorem, because we do not sum over all degenerate initial states.
- Thus the structure functions (and parton distribution functions) are functions of q^2 as well as x.

Deep Inelastic Scattering and QCD Cont.

- I refer to the standard textbook for the use of the Operator Product Expansion (OPE) to determine the q^2 behaviour of the structure functions.
- The same results can be obtained from the DGLAP equations (let $t = \log(q^2/q_0^2)$):

$$
\begin{aligned}
\frac{dq^{\rm NS}(x,t)}{dt} &= \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\, q^{\rm NS}(y,t)\, P_{q\to q}\left(\frac{x}{y}\right)\\
\frac{dq^{\rm S}(x,t)}{dt} &= \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\left\{q^{\rm S}(y,t)\, P_{q\to q}\left(\frac{x}{y}\right) + g(y,t)\, P_{g\to q}\left(\frac{x}{y}\right)\right\}\\
\frac{dg(x,t)}{dt} &= \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\left\{q^{\rm S}(y,t)\, P_{q\to g}\left(\frac{x}{y}\right) + g(y,t)\, P_{g\to g}\left(\frac{x}{y}\right)\right\}
\end{aligned}
$$

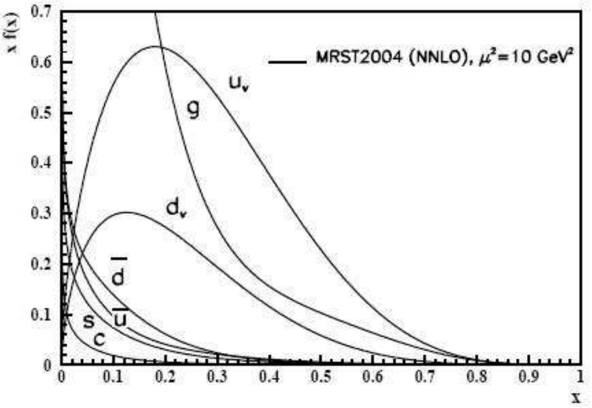


$P_{q\to q}(x/y)$ $P_{q\to g}(x/y)$ $P_{g\to q}(x/y)$ $P_{g\to g}(x/y)$

Deep Inelastic Scattering and QCD – Comments

- We can calculate the (logarithmic) scaling violations, i.e. the behaviour of the structure functions with q^2. We cannot calculate the structure functions themselves.
- The behaviour of the distribution functions with q^2 is the intuitive one – as q^2 increases there are fewer partons at large x and more at small x.
- By measuring the behaviour of the structure functions with q^2 we are able to determine the gluon distribution in the proton (even though the γ, Z^0 and the $W's$ do not couple to the gluons).
- The *factorization* of hadron-hadron hard-scattering cross sections into a convolution of parton distribution functions (as measured in DIS experiments) and perturbatively calculable parton scattering cross sections is also valid in QCD.
- We can therefore make predictions for specific cross sections (such as that for Higgs production) at the LHC.

Quark Distribution Functions (PDG2005)



Scaling Violations (PDG2005)