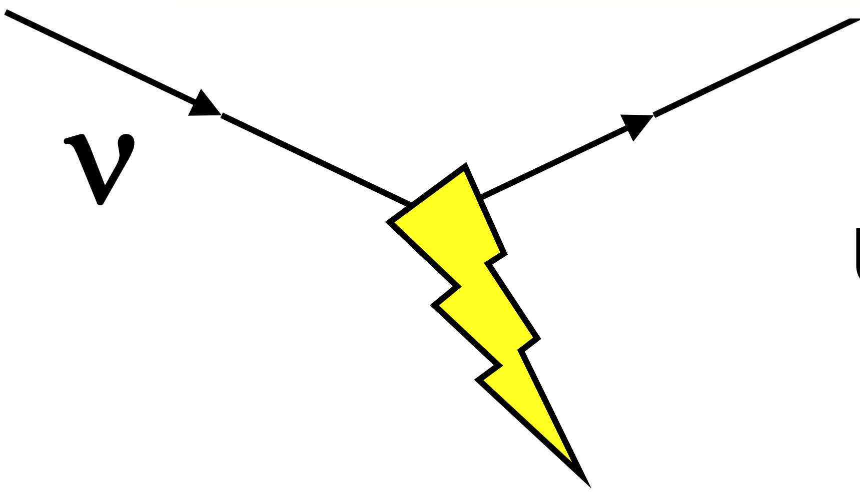
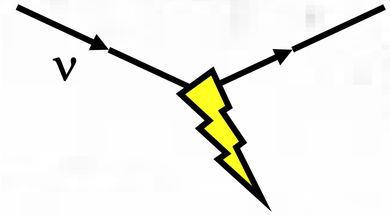
Interactions of Neutrinos at High and Low Energies



Kevin McFarland
University of Rochester
Neutrinos at SUSSP
12-15 August 2006

Neutrino Interaction Outline

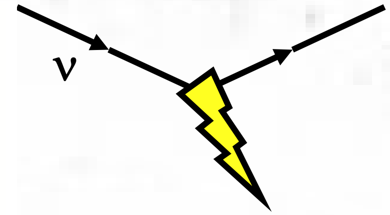


- Motivations for and History of Measuring Neutrino Interactions
- Weak interactions and neutrinos
  - Elastic and quasi-elastic processes, e.g., νe scattering
  - Deep inelastic scattering, (νq scattering)
  - The difficulties of being in near thresholds…
- Current & future cross-section knowledge
  - What we need to learn and how to learn it

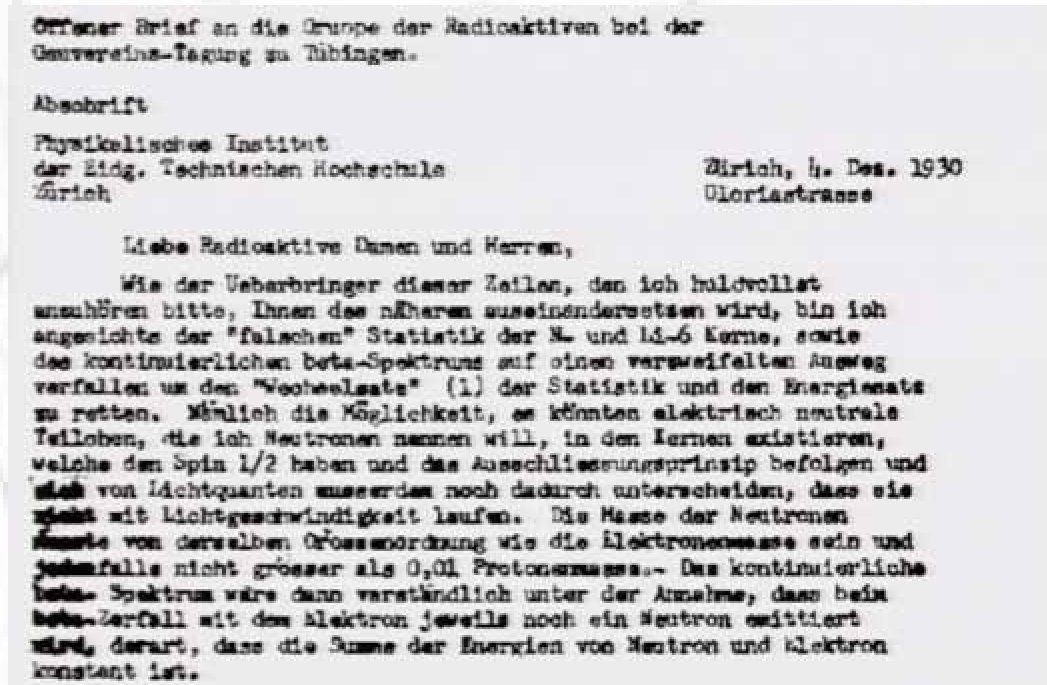
Tone of These Lectures

- Focus will be on
  - Cross-sections useful for experiments
  - Estimating cross-sections
  - Understanding qualitatively the key effects
- Therefore, it should therefore go without saying that I am the *second* experimentalist lecturing at SUSSP…

The Birth of the Neutrino



Wolfgang Pauli



Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

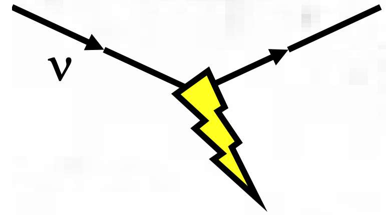
Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Des. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grössenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

Translation from the German, Please?



4th December 1930

Dear Radioactive Ladies and Gentlemen,

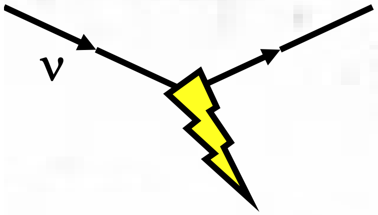
As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and ^{6}Li nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass (and in any event not larger than 0.01 proton masses). The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

From now on, every solution to the issue must be discussed. Thus, dear radioactive people, look and judge. Unfortunately I will not be able to appear in Tübingen personally, because I am indispensable here due to a ball which will take place in Zürich during the night from December 6 to 7….

Your humble servant,

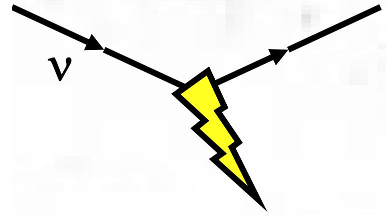
W. Pauli

The True Source of Slow Progress in Neutrino Physics

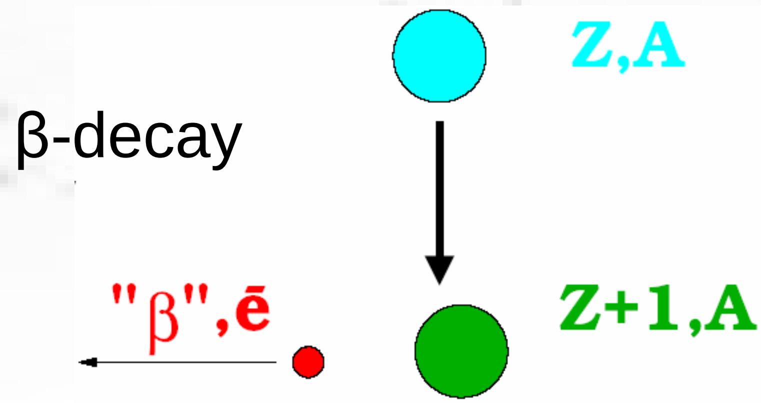


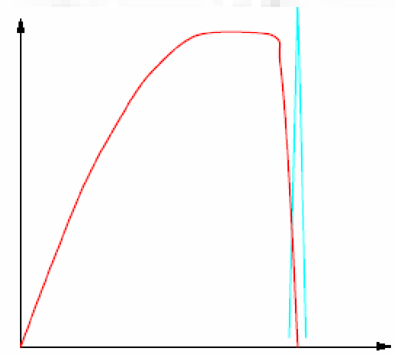
From now on, every solution to the issue must be discussed. Thus, dear radioactive people, look and judge. Unfortunately I will not be able to appear in Tübingen personally, because I am indispensable here due to a ball which will take place in Zürich during the night from December 6 to 7….
Your humble servant,
W. Pauli

Translation from the Archaic Physics Terms, Please?



- To save the law of conservation of energy?

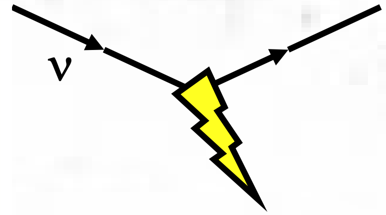




The Energy of the "β"

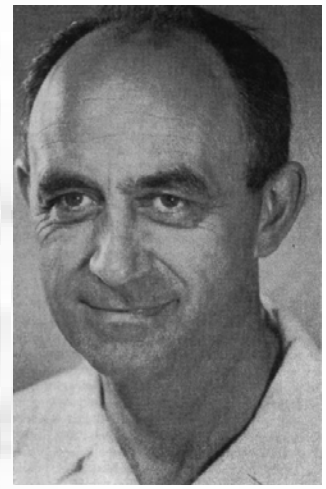
- If the above picture is complete, conservation of energy in this two body decay predicts monochromatic β
  - but a continuous spectrum had been observed (since 1914)
- Pauli suggests "neutron" takes away energy!
- *"The exchange theorem of statistics", by the way, refers to the fact that a spin½ neutron can't decay to an spin½ proton + spin½ electron*

Weak Interactions



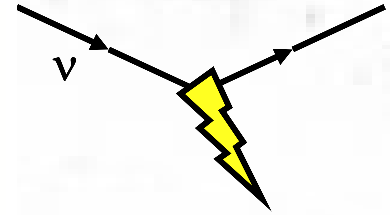
- Current-current interaction
  Fermi, Z. Physik, 88, 161 (1934)

$$\mathcal{H}_w = \frac{G_F}{\sqrt{2}} \mathcal{J}^\mu \mathcal{J}_\mu$$



  - Paper rejected by *Nature* because "it contains speculations too remote from reality to be of interest to the reader"
- Prediction for neutrino interactions
  - If $n \to p e^- \bar{\nu}$, then $\bar{\nu} p \to e^+ n$
  - Better yet, it is robustly predicted by Fermi theory
    - o Bethe and Peirels, Nature 133, 532 (1934)
  - For neutrinos of a few MeV from a reactor, a typical cross-section was found to be $\sigma_{\bar{\nu} p} \sim 5 \times 10^{-44}\,\mathrm{cm}^2$
  - (Actually wrong by a factor of two (parity violation)

How Weak is This?

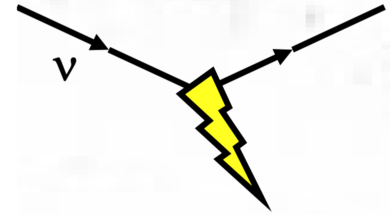


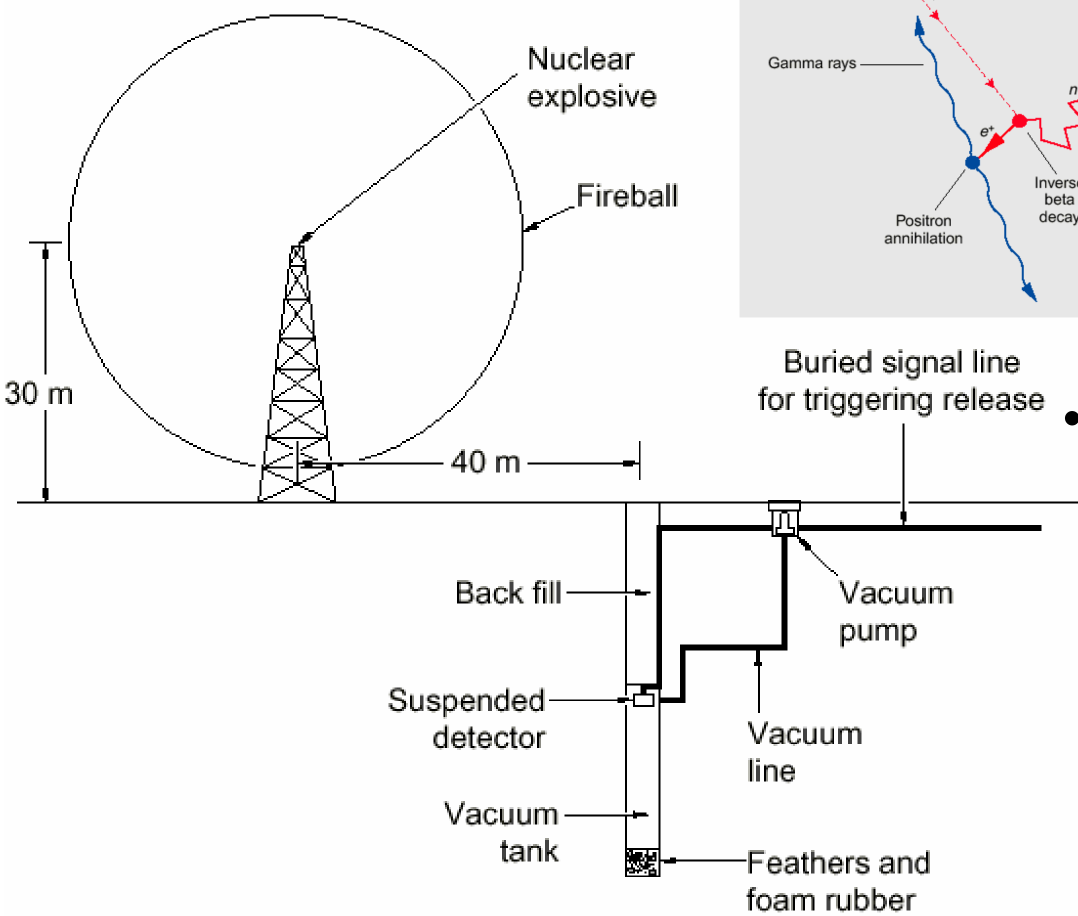
- $\sigma \sim 5 \times 10^{-44}$cm^2 compared with
  - $\sigma_{\gamma p} \sim 10^{-25}$ cm^2 at similar energies, for example
- The cross-section of these few MeV neutrinos is such that the mean free path in steel would be 10 light-years

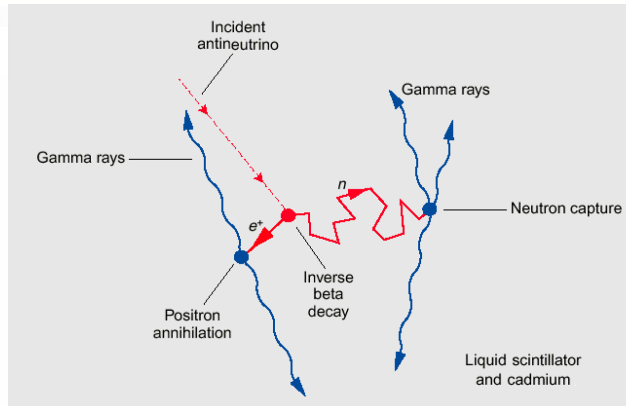
"I have done something very bad today by proposing a particle that cannot be detected; it is something no theorist should ever do."

Wolfgang Pauli

Extreme Measures to Overcome Weakness (Reines and Cowan, 1946)



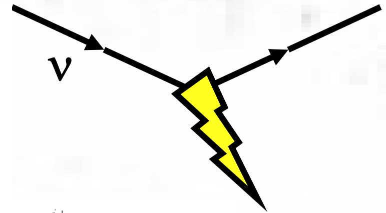




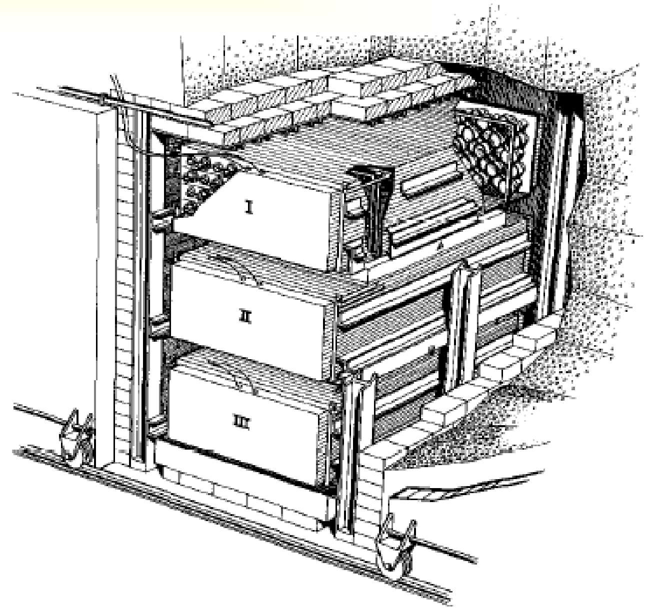
$$\bar{\nu}\, p \rightarrow e^{+} n$$

- Why inverse neutron beta decay?
  - clean prediction of Fermi weak theory
  - clean signature of prompt gammas from e^{+} plus delayed neutron signal.
    - Latter not as useful with bomb source.

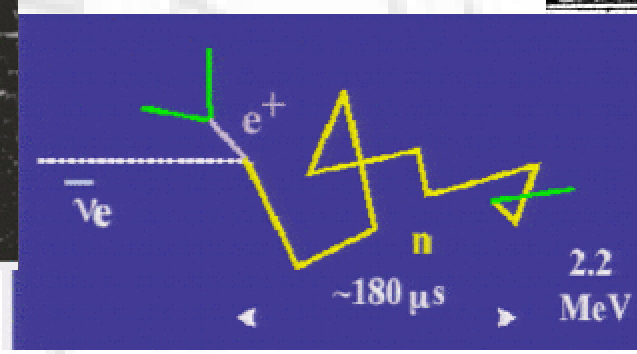
Discovery of the Neutrino

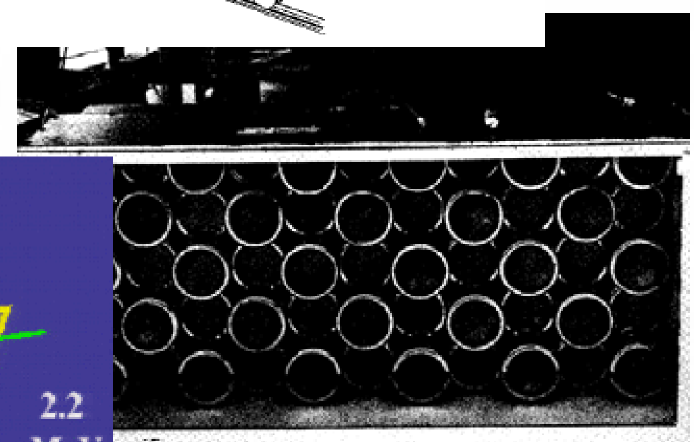


- Reines and Cowan (1955)
  - Chose a constant source, nuclear reactor (Savannah River)
  - 1956 message to Paul: "We are happy to inform you [Pauli] that we have definitely detected neutrinos…"
  - 1995 Nobel Prize for Reines

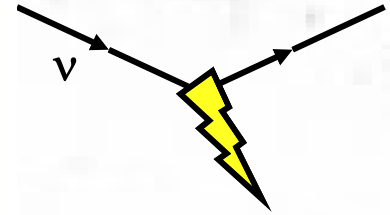


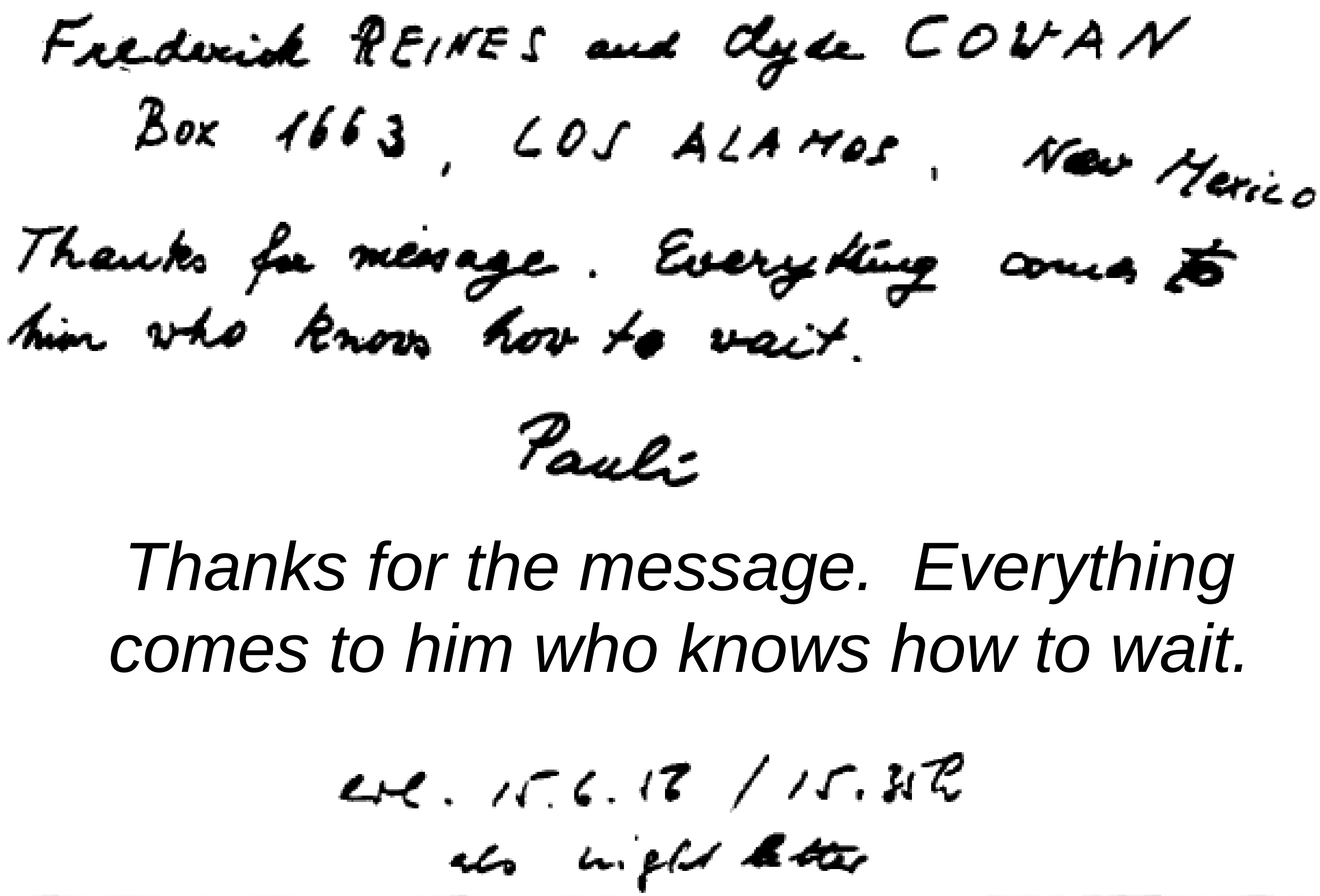
$$\bar{\nu}\, p \rightarrow e^{+} n$$





Better than the Nobel Prize?





Fredrick REINES and Clyde COWAN
Box 1663, LOS ALAMOS, New Mexico

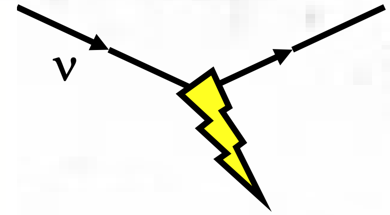
Thanks for message. Everything comes to him who knows how to wait.

Pauli

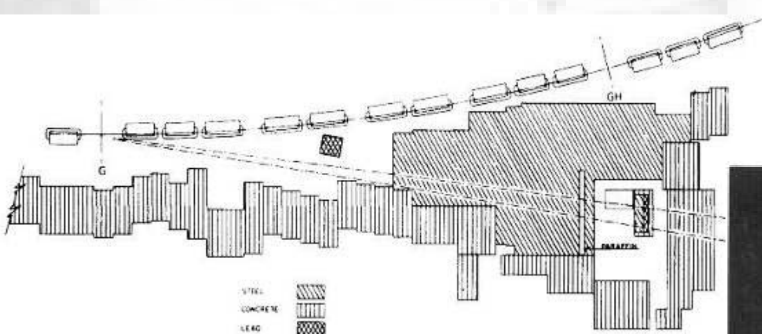
[illegible]

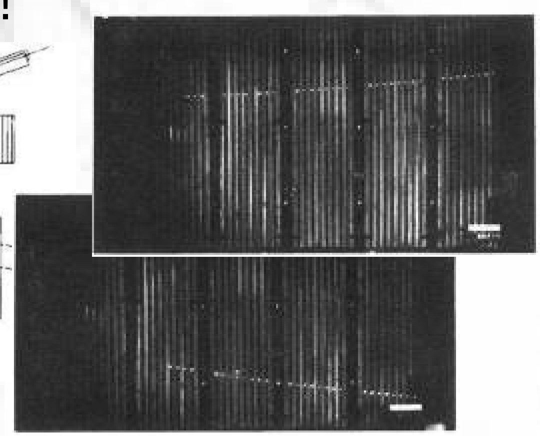
Thanks for the message. Everything comes to him who knows how to wait.

Interactions and Flavor



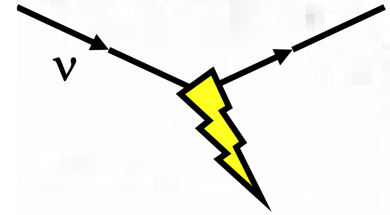
- 1962 Lederman, Schwartz, Steinberger at Brookhaven Nat'l Lab
- One neutrino was known (beta decay)
  - Question: if $\mu^+ \rightarrow e^+ \nu \bar{\nu}$, why not $\mu^+ \rightarrow e^+ \gamma$?
- First accelerator neutrino beam
  - 5 GeV protons on Be Target (3.5×10^{17} of them)
  - $\pi^+ \rightarrow \mu^+ \nu_\mu$ in a 21m decay region
  - Found 34 single-μ events, 5 background, but NO e-like events!





1988 Nobel citation: "for the neutrino beam method and the demonstration of the doublet structure of the leptons through the discovery of the muon-neutrino"

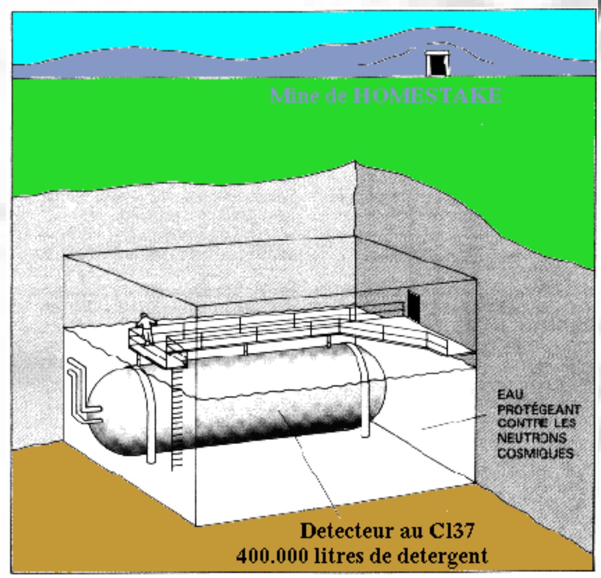
Another Flavor Example



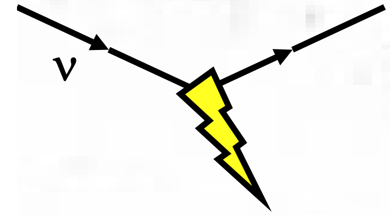
- Radiochemical Solar Neutrino Detector

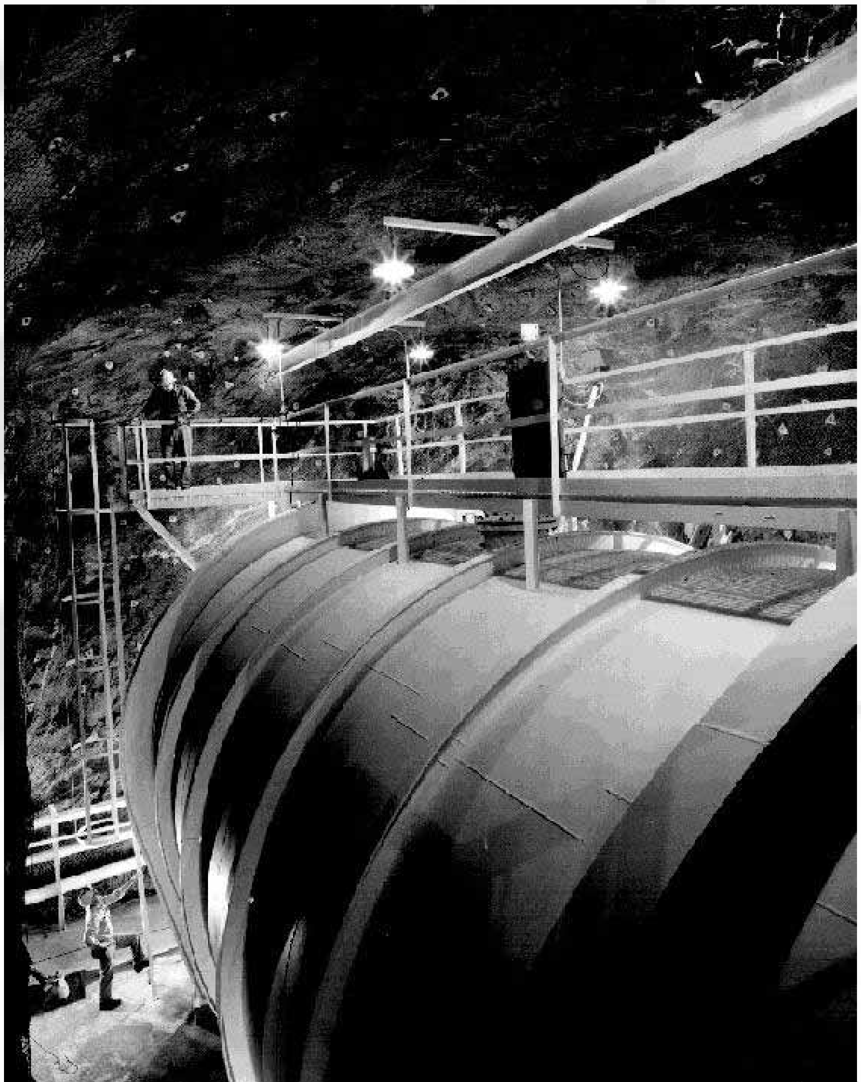
  Ray Davis (Nobel prize, 2002)

  - $\nu + n \rightarrow p + e^-$ (stimulated β-decay)
  - Use this to produce an unstable isotope, $\nu + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$, which has 35 day half-life
  - Put 615 tons of Perchloroethylene in a gold mine
    - o expect one ^{37}Ar atom every 17 hours.

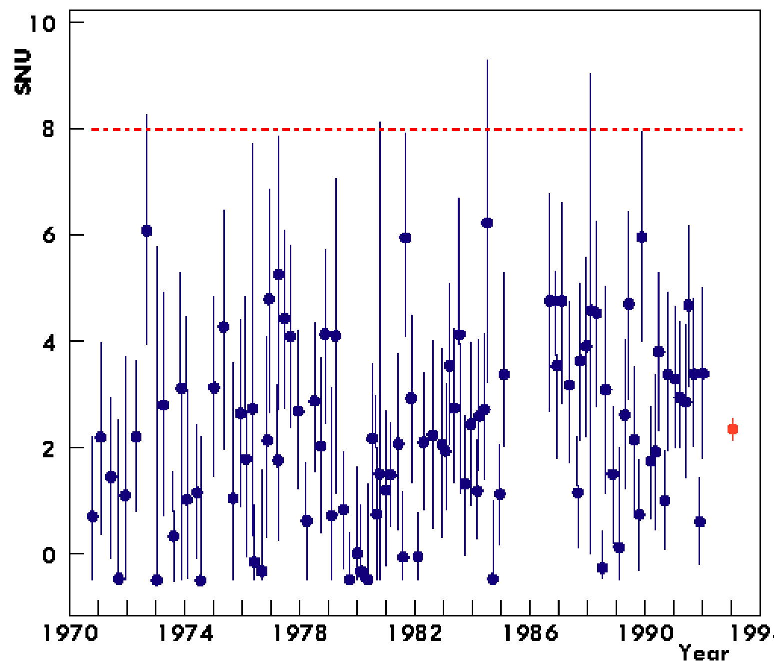


Another Flavor (cont'd)

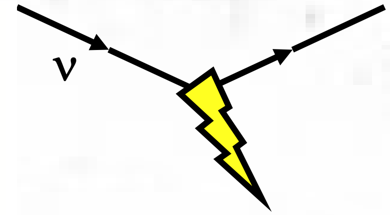




- Confirmed that sun shines from fusion, but 1/3 of ν !
- Of course this is oscillation and flavor selection of interaction $\nu + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$

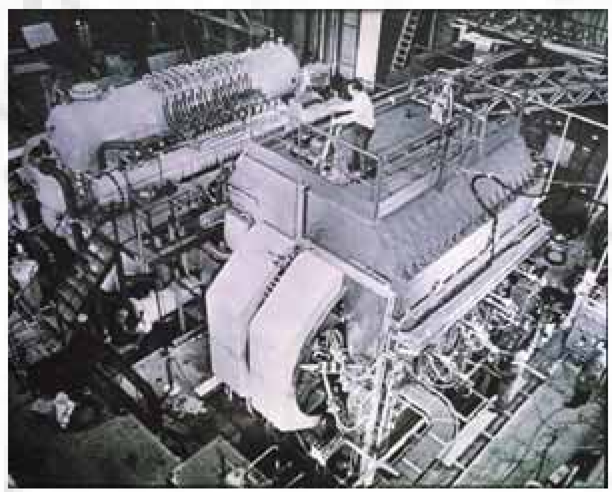


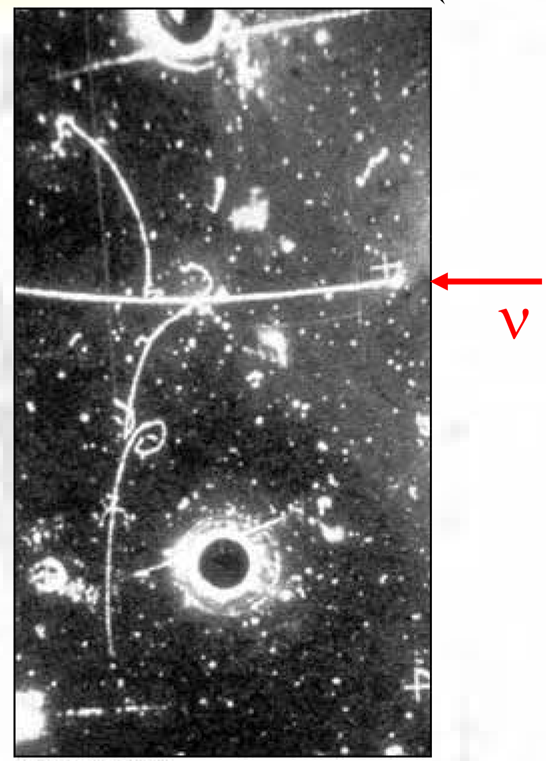
Another Neutrino Interaction Discovery



- Neutrinos only feel the weak force
  - a great way to study the weak force!
- Search for neutral current
  - arguably the most famous neutrino interaction ever observed is shown at right

$$\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$$

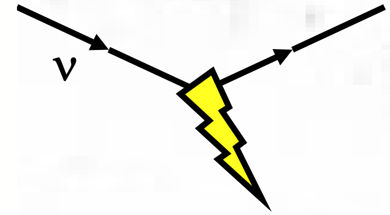




AEROMETRIC photo

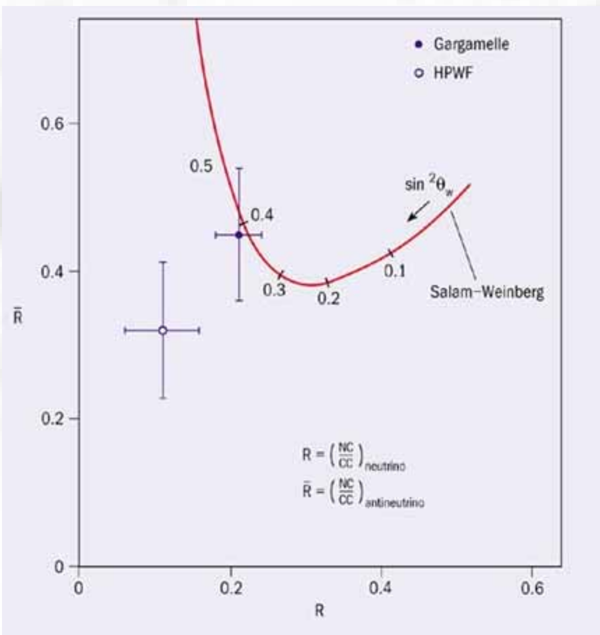
Gargamelle, event from neutral weak force

An Illuminating Aside



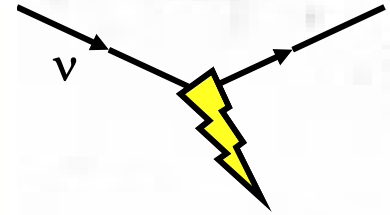
- The "discovery signal" for the neutral current was really neutrino scattering from nuclei
  - usually quoted as a ratio of muon-less interactions to events containing muons

$$R^{\nu} = \frac{\sigma(\nu_{\mu} N \rightarrow \nu_{\mu} X)}{\sigma(\nu_{\mu} N \rightarrow \mu^{-} X)}$$

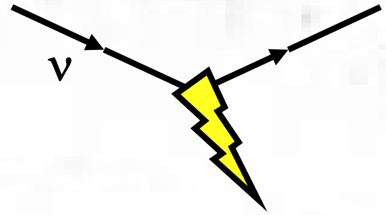


- But this discovery was complicated for 12-18 months by a lack of understanding of neutrino interactions
  - backgrounds from neutrons induced by neutrino interactions outside the detector
  - not understanding probability of fragmentation to high E hadrons which then "punched through" to fake muons

The Future: Interactions and Oscillation Experiments

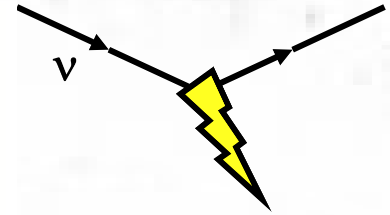


- Boris has elegantly described a situation where muon oscillation appearance experiments at L/E~300 km/GeV have rich physics potential
  - mass hierarchy, CP violation, τ appearance (sterile ν's)
- What Boris hasn't worried about (at least in front of you)
  - transition probabilities are small, must be precisely measured for mass hierarchy and CP violation
  - the neutrinos must be at difficult energies of 1-few GeV for electron appearance, many GeV (> charm threshold) for τ
- *We are not looking for neutrino flavor measurements in which distinguishing 1 from 0 or 1 from 1/3 buys a ticket to Stockholm*
  - Difficulties are akin to neutral current experiments
  - Is there a message for us here?



Present View of Weak Interactions

Weak Interactions Revisited

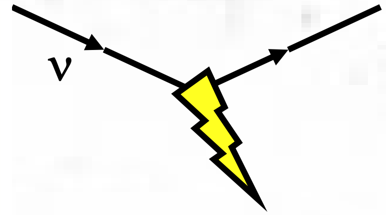


- Current-current interaction (Fermi 1934) $\mathcal{H}_w = \frac{G_F}{\sqrt{2}} \mathcal{J}^\mu \mathcal{J}_\mu$
  - Paper rejected by *Nature* because "it contains speculations too remote from reality to be of interest to the reader"
- Modern version:

$$H_{weak} = \frac{G_F}{\sqrt{2}} \left[\bar{l} \gamma_\mu (1-\gamma_5) \nu \right] \left[\bar{f} \gamma^\mu (V - A\gamma_5) f \right] + h.c.$$

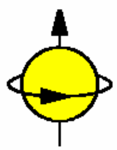
- $P_L = 1/2(1-\gamma_5)$ is a projection operator onto left-handed states for fermions and right-handed states for anti-fermions

Helicity and Chirality



- ***Helicity*** is projection of spin along the particles direction
  - Frame dependent (if massive)

The operator: $\sigma \cdot \mathbf{p}$

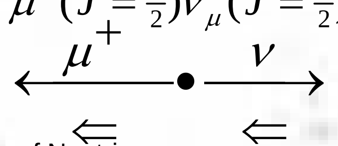
right-helicity  left-helicity 

- Neutrinos only interact weakly with a (V-A) interaction
  - All **neutrinos** are **left-handed**
  - All **antineutrinos** are **right-handed**
    - o **because of production!**
  - Weak interaction **maximally** violates parity

- However, ***chirality*** ("handedness") is Lorentz-invariant
  - – Only same as helicity for massless particles.

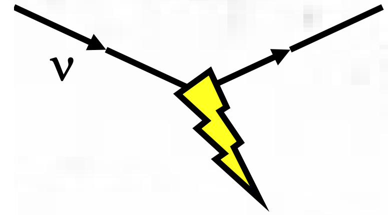
- If neutrinos have mass then left-handed neutrino is:
  - – Mainly left-helicity
  - – But also small right-helicity component $\propto$ *m/E*
- Only left-handed charged-leptons (e^-,μ^-,τ^-) interact weakly but mass brings in right-helicity:

$$\pi^+(J=0) \rightarrow \mu^+(J=\tfrac{1}{2})\nu_\mu(J=\tfrac{1}{2})$$



$$R_{theory} = \frac{\Gamma(\pi^\pm \to e^\pm \nu_e)}{\Gamma(\pi^\pm \to \mu^\pm \nu_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.23 \times 10^{-4}$$

Two Weak Interactions



- W exchange gives Charged-Current (CC) events and Z exchange gives Neutral-Current (NC) events

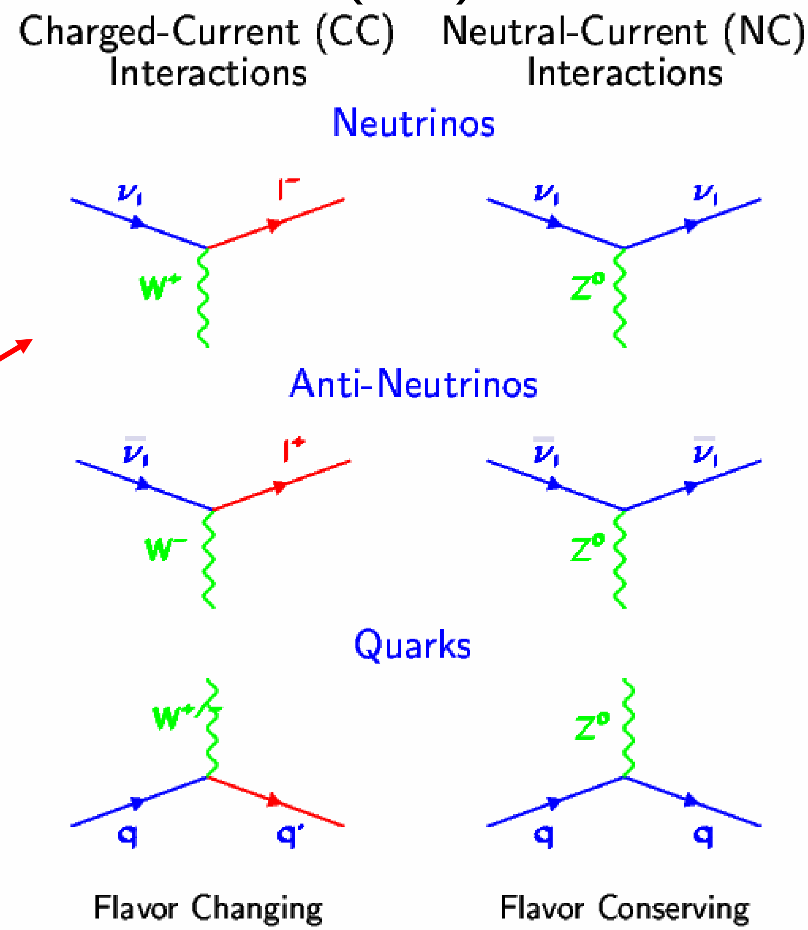
In charged-current events,

Flavor of outgoing lepton tags flavor of neutrino

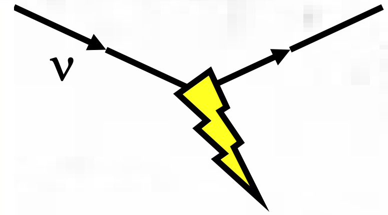
Charge of outgoing lepton determines if neutrino or antineutrino

$$l^- \Rightarrow \nu_l$$

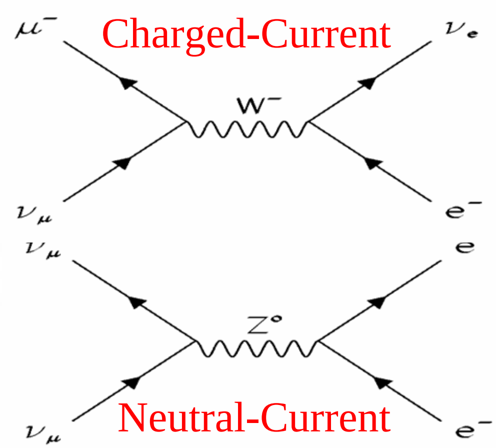
$$l^+ \Rightarrow \overline{\nu}_l$$



Electroweak Theory



- Standard Model
  - SU(2) ⊗ U(1) gauge theory unifying weak/EM
    $\Rightarrow$ weak NC follows from EM, Weak CC
  - Measured physical parameters related to mixing parameter for the couplings, $g' = g \tan\theta_W$



Fermion Lagrangian

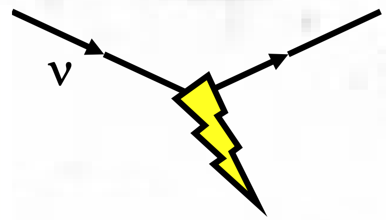
The terms in the Lagrangian involving the fermions then take the form:

$$
\begin{aligned}
\mathscr{L} \quad = \quad & \bar{E}_L(i\,\partial\!\!\!/)E_L + \bar{e}_R(i\,\partial\!\!\!/)e_R + \bar{Q}_L(i\,\partial\!\!\!/)Q_L + \bar{u}_R(i\,\partial\!\!\!/)u_R + \bar{d}_R(i\,\partial\!\!\!/)d_R \\
& + g\left(W_\mu^+ J_W^{\mu\,+} + W_\mu^- J_W^{\mu\,-} + Z_\mu^0 J_Z^\mu\right) + eA_\mu J_{\mathrm{EM}}^\mu ,
\end{aligned}
$$

where

$$
\begin{aligned}
J_W^{\mu\,+} \quad &= \quad \frac{1}{\sqrt{2}}\left(\bar{\nu}_L\gamma^\mu e_L + \bar{u}_L\gamma^\mu d_L\right); \\
J_W^{\mu\,-} \quad &= \quad \frac{1}{\sqrt{2}}\left(\bar{e}_L\gamma^\mu \nu_L + \bar{d}_L\gamma^\mu u_L\right); \\
J_Z^\mu \quad &= \quad \frac{1}{\cos\theta_W}\left\{\frac{1}{2}\,\bar{\nu}_L\gamma^\mu\nu_L + \left(\sin^2\theta_W - \frac{1}{2}\right)\bar{e}_L\gamma^\mu e_L + \sin^2\theta_W\,\bar{e}_r\gamma^\mu e_R\right. \\
& \qquad + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)\bar{u}_L\gamma^\mu u_L - \frac{2}{3}\sin^2\theta_W\,\bar{u}_R\gamma^\mu u_R \\
& \qquad \left. + \left(\frac{1}{3}\sin^2\theta_W - \frac{1}{2}\right)\bar{d}_L\gamma^\mu d_L + \frac{1}{3}\sin^2\theta_W\,\bar{d}_R\gamma^\mu d_R\right\}; \\
J_{\mathrm{EM}}^\mu \quad &= \quad -\bar{e}\gamma^\mu e + \frac{2}{3}\,\bar{u}\gamma^\mu u - \frac{1}{3}\,\bar{d}\gamma^\mu d\,.
\end{aligned}
$$

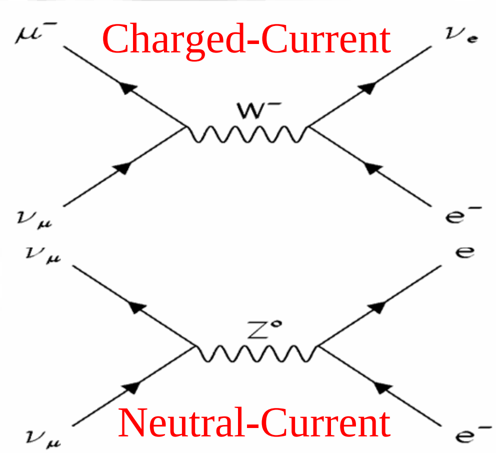
Electroweak Theory



- Standard Model
  - SU(2) ⊗ U(1) gauge theory unifying weak/EM
    $\Rightarrow$ weak NC follows from EM, Weak CC
  - Measured physical parameters related to mixing parameter for the couplings, $g' = g \tan\theta_W$

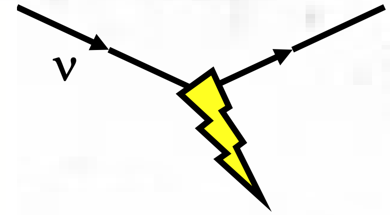
Z Couplings	g_L	g_R
ν_e , ν_μ , ν_τ	1/2	0
e , μ , τ	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
u , c , t	$1/2 - 2/3\ \sin^2\theta_W$	$-\ 2/3\ \sin^2\theta_W$
d , s , b	$-1/2 + 1/3\ \sin^2\theta_W$	$1/3\ \sin^2\theta_W$

$$e = g\sin\theta_W,\ G_F = \frac{g^2\sqrt{2}}{8M_W^2},\ \frac{M_W}{M_Z} = \cos\theta_W$$



- Neutrinos are special in SM
  - Right-handed neutrino has ***NO*** interactions!

Why "Weak"?

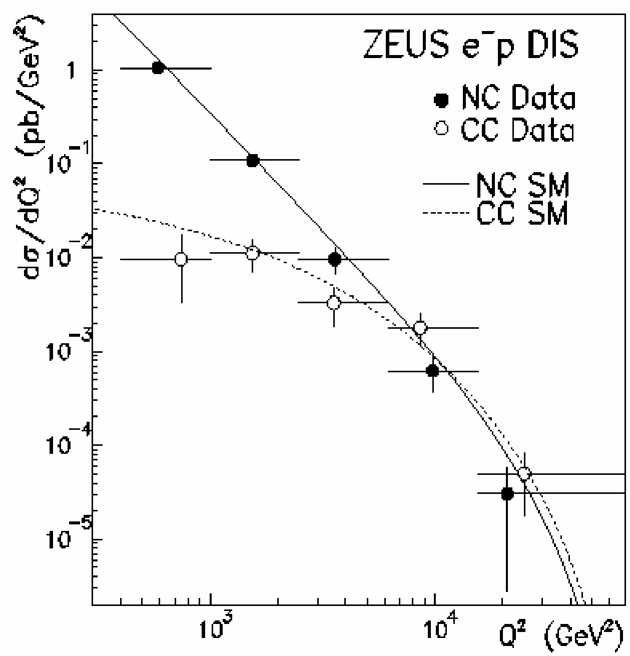


- Weak interactions are weak because of the massive W and Z bosons exchange

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

q is 4-momentum carried by exchange particle
M is mass of exchange particle

At HERA see W and Z propagator effects
- Also weak ~ EM strength



- Explains dimensions of Fermi "constant"

$$G_F = \frac{\sqrt{2}}{8}\left(\frac{g_W}{M_W}\right)^2$$
$$= 1.166 \times 10^{-5} / GeV^2 \quad (g_W \approx 0.7)$$

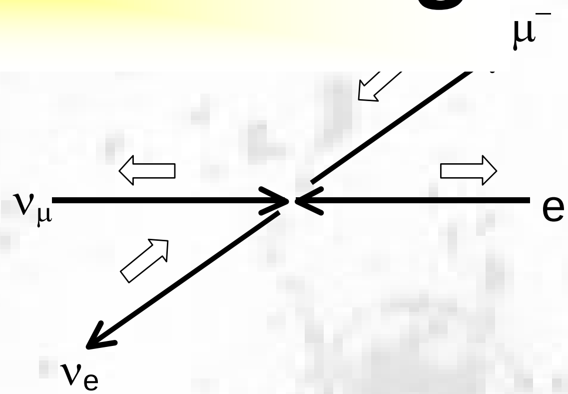
Neutrino-Electron Scattering

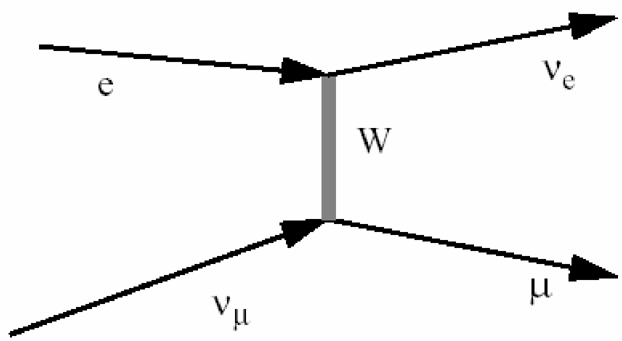
- Inverse μ–decay:

  $$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

  - Total spin J=0

    (*Assuming massless muon*, helicity=chirality)

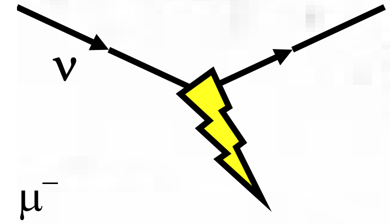




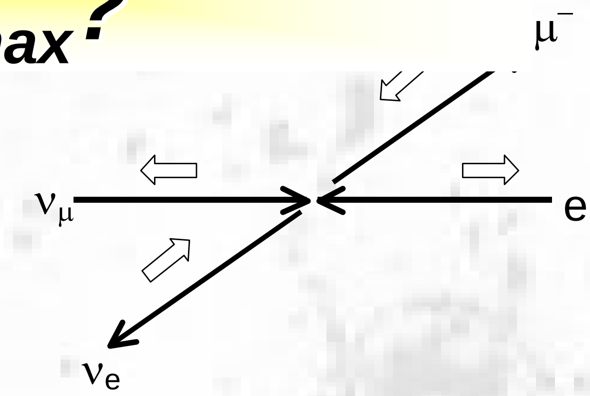
$$\sigma_{TOT} \propto \int_0^{Q^2_{\max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2}$$

$$\approx \frac{Q^2_{\max}}{M_W{}^4}$$

Touchstone Question #1
What is Q^2_{max}?



$$\nu_\mu + e^- \to \mu^- + \nu_e$$



$$Q^2 \equiv -\left(\underline{e} - \underline{\nu}_e\right)^2$$

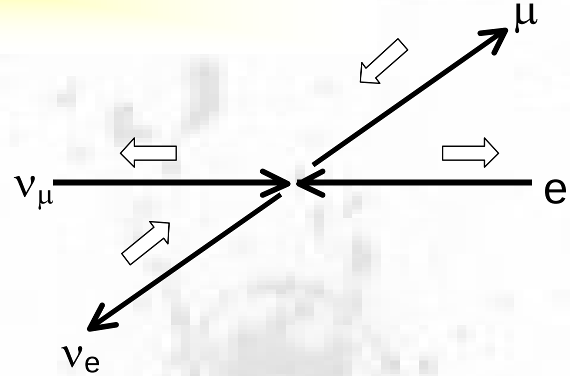
*Let's work in the center-of-mass frame. Assume, **for now**, we can neglect the masses*

Neutrino-Electron (cont'd)

$$\sigma_{TOT} \propto Q^2_{\max} = s$$

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}$$

$$= 17.2 \times 10^{-42} cm^2 / GeV \cdot E_\nu (GeV)$$

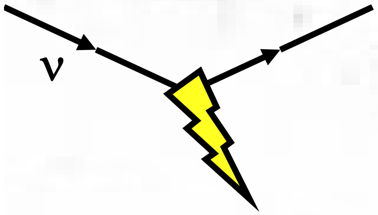


- Why is it proportional to beam energy?

$$s = (\underline{p}_{\nu_\mu} + \underline{p}_e)^2 = m_e^2 + 2 m_e E_\nu \text{ (e}^- \text{ rest frame)}$$

- Proportionality to energy is a generic feature of point-like scattering!
  - because $d\sigma/dQ^2$ is constant (at these energies)

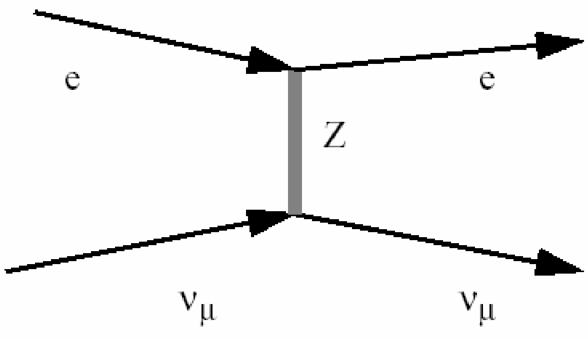
Neutrino-Electron (cont'd)



- Elastic scattering:

$$\nu_\mu + e^- \to \nu_\mu + e^-$$

  - Coupling to left *or* right-handed electron
  - Total spin, J=0,1



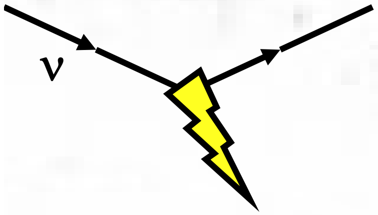
- Electron-Z^0 coupling
  - (LH, V-A): $-1/2 + \sin^2\theta_W$

$$\sigma \propto \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \sin^4\theta_W\right)$$

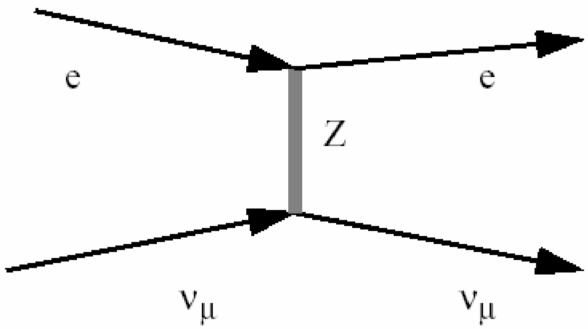
  - (RH, V+A): $\sin^2\theta_W$

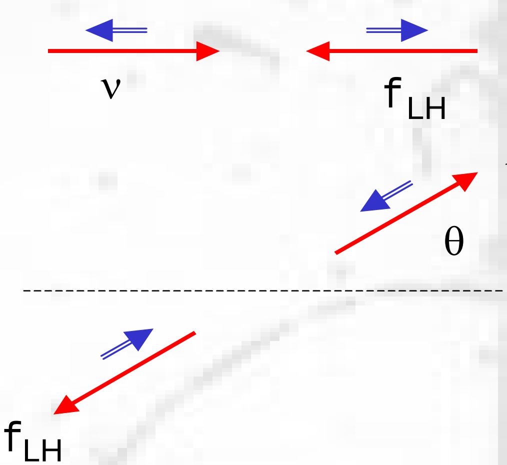
$$\sigma \propto \frac{G_F^2 s}{\pi}\left(\sin^4\theta_W\right)$$

Neutrino-Electron (cont'd)

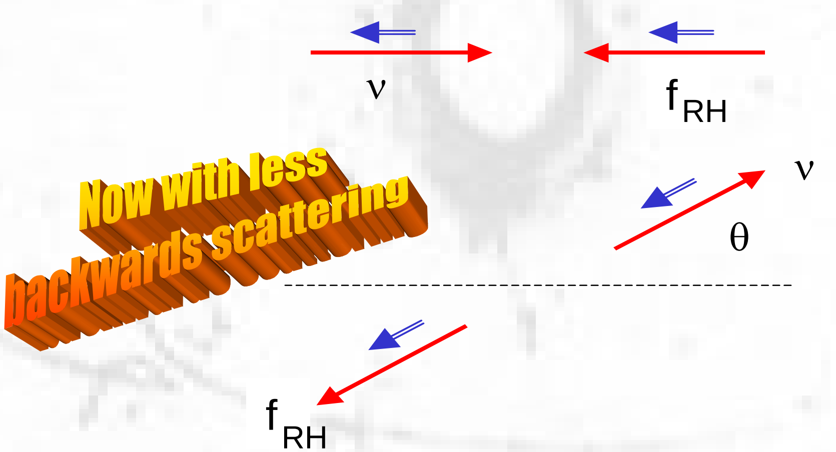


- What are relative contributions of left *and* right-handed scattering from electron?



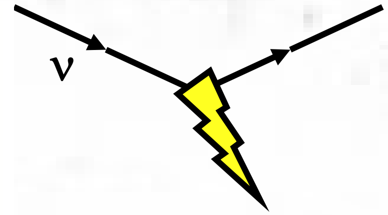


$$\frac{d\sigma}{d\cos\theta} = \text{const}$$



$$\frac{d\sigma}{d\cos\theta} = \text{const} \times \left(\frac{1+\cos\theta}{2}\right)^2$$

Neutrino-Electron (cont'd)



- Electron-Z^0 coupling $\sigma \propto \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \sin^4\theta_W\right)$
  - (LH, V-A): $-1/2 + \sin^2\theta_W$
  - (RH, V+A): $\sin^2\theta_W$ $\sigma \propto \frac{G_F^2 s}{\pi}\left(\sin^4\theta_W\right)$

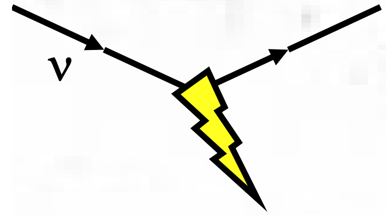
Let y denote inelasticity.
Recoil energy is related to
CM scattering angle by

$$y = \frac{E_e}{E_\nu} \approx 1 - \tfrac{1}{2}(1-\cos\theta)$$

$$\int dy \frac{d\sigma}{dy} = \begin{cases} \text{LH:} & \int dy = 1 \\ \text{RH:} & \int (1-y)^2 dy = 1/3 \end{cases}$$

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \frac{4}{3}\sin^4\theta_W\right) = 1.4\times 10^{-42}\, cm^2/GeV \cdot E_\nu(GeV)$$

Touchstone Question #2: Flavors and νe Scattering



The reaction

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

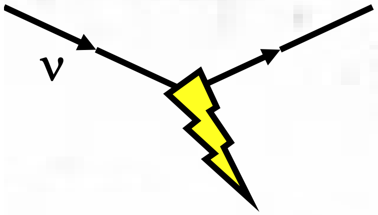
has a much smaller cross-section than

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

Why?

)

Touchstone Question #2: Flavors and νe Scattering



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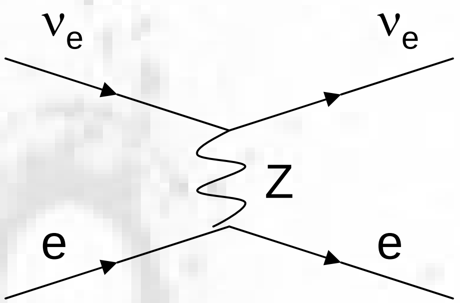
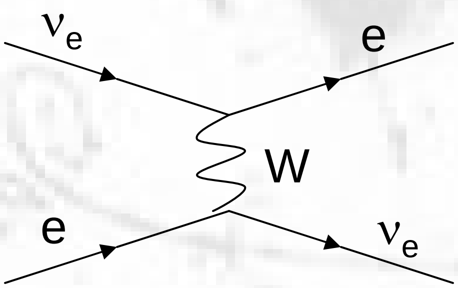
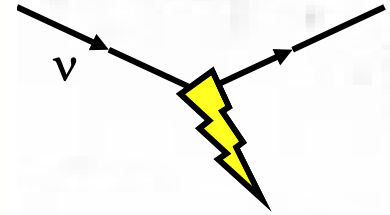


$$\nu_e + e^- \rightarrow \nu_e + e^-$$

has a second contributing reaction, charged current



Touchstone Question #2: Flavors and νe Scattering



Show that this increases the rate

(Recall from the previous pages…

$$\sigma_{TOT} = \int dy \frac{d\sigma}{dy} = \int dy \left[\frac{d\sigma^{LH}}{dy} + \frac{d\sigma^{RH}}{dy} \right] = \sigma_{TOT}^{LH} + \tfrac{1}{3}\sigma_{TOT}^{RH}$$

$$\sigma_{TOT}^{LH} \propto \left| \text{total coupling}_{e^-}^{LH} \right|^2$$

For electron…	LH coupling	RH coupling
Weak NC	$-1/2+ \sin^2\theta_W$	$\sin^2\theta_W$
Weak CC	-1/2	0

)

We have to show the interference between CC and NC is constructive.

The total RH coupling is unchanged by addition of CC because there is no RH weak CC coupling

There are two LH couplings: NC coupling is $-1/2+\sin^2\theta_W \approx -1/4$ and the CC coupling is -1/2. We add the associated amplitudes… and get $-1+\sin^2\theta_W \approx -3/4$

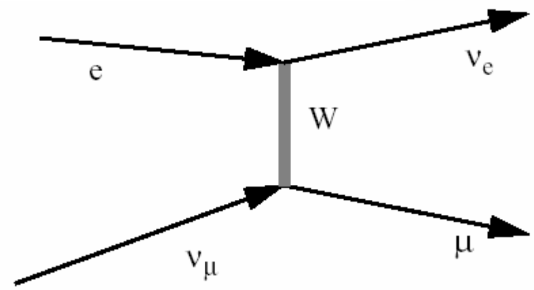
Lepton Mass Effects

- Let's return to
  Inverse μ–decay:

  $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$

  - What changes in the presence of final state mass?
    - o pure CC so always left-handed
    - o BUT there must be finite Q^2 to create muon in final state!

$$Q^2_{\min} = m_\mu^2$$



$$\sigma_{TOT} \propto \int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2}$$

$$\approx \frac{Q^2_{\max} - Q^2_{\min}}{M_W{}^4}$$

$$\sigma_{TOT} = \frac{G_F^2 (s - m_\mu^2)}{\pi}$$

$$= \left[\sigma_{TOT}{}^{\text{(massless)}}\right]\left(1 - \frac{m_\mu^2}{s}\right)$$

  - see a suppression scaling with **(mass/CM energy)^2**
    - o can be generalized…

What about other targets?

- Imagine now a proton target
  - Neutrino-proton elastic scattering:

$$\nu_e + p \rightarrow \nu_e + p$$

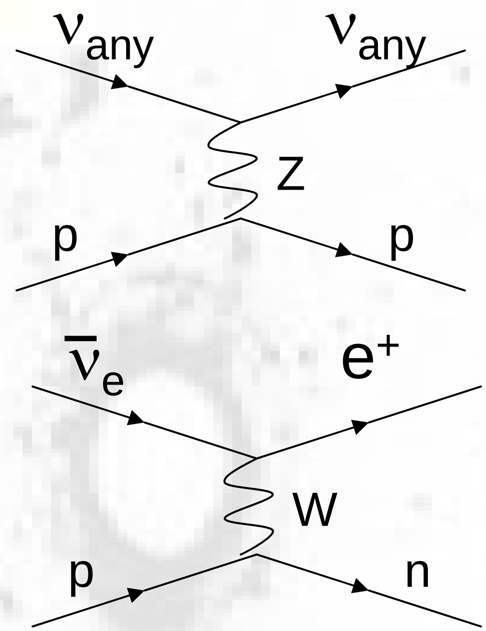
  - "Inverse beta-decay" (IBD):

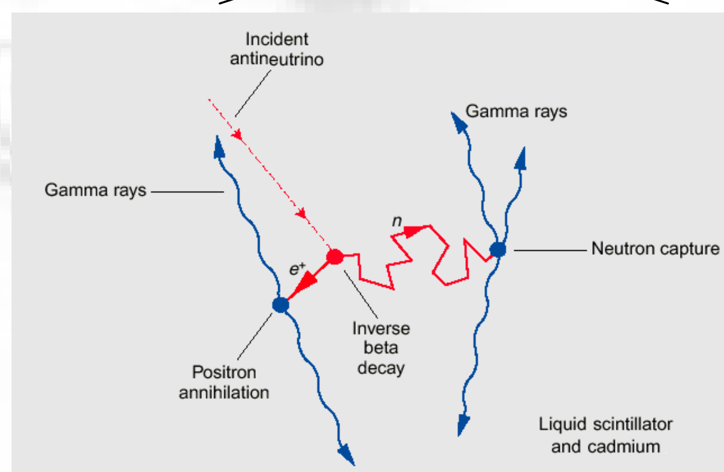
$$\bar{\nu}_e + p \rightarrow e^+ + n$$

  - and its close cousin:

$$\nu_e + n \rightarrow e^- + p$$

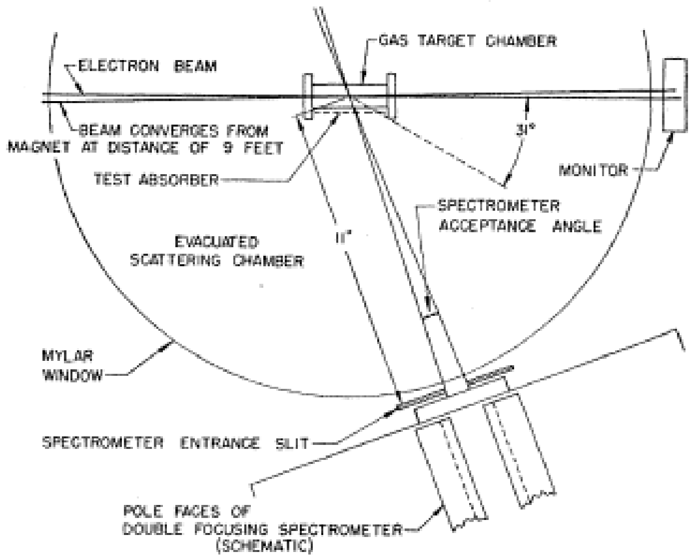
  - Recall that IBD was the Reines and Cowan discovery signal



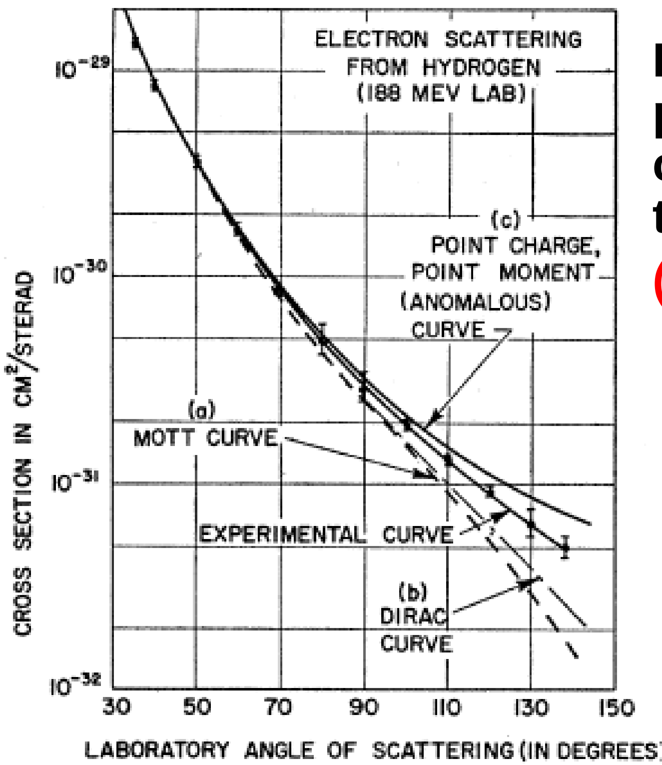


Proton Structure

- How is a proton different from an electron?
  - anomalous magnetic moment, $\kappa \equiv \frac{g-2}{2} \neq 1$
  - "form factors" related to finite size



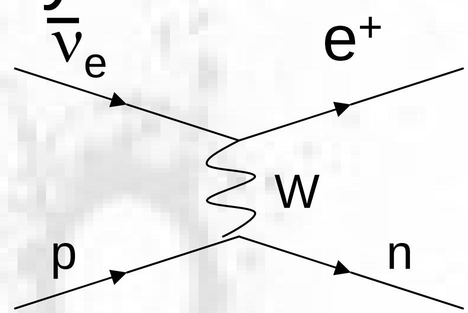
McAllister and Hofstadter 1956
188 MeV and 236 MeV electron beam from linear accelerator at Stanford



Determined proton RMS charge radius to be
(0.7±0.2) x10^{-13} cm

Final State Mass Effects

- In IBD, $\bar{\nu}_e + p \rightarrow e^+ + n$, have to pay a mass penalty *twice*
  - *M_n-M_p≈1.3 MeV, M_e≈0.5 MeV*

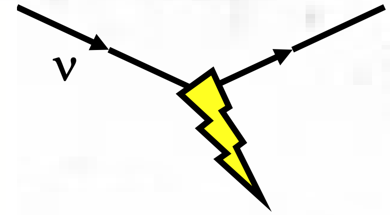


- What is the threshold?
  - kinematics are simple, at least to zeroth order in M_e/M_n → heavy nucleon kinetic energy is zero

$$s_{\text{initial}} = (\underline{p}_\nu + \underline{p}_p)^2 = M_p^2 + 2M_p E_\nu \text{ (proton rest frame)}$$
$$s_{\text{final}} = (\underline{p}_e + \underline{p}_n)^2 \approx M_n^2 + m_e^2 + 2M_n\left(E_\nu - \left(M_n - M_p\right)\right)$$

- Solving… $E_\nu^{\text{min}} \approx \dfrac{\left(M_n + m_e\right)^2 - M_p^2}{2M_p} \approx 1.806 \text{ MeV}$

Final State Mass Effects (cont'd)



- Define δE as E_ν-E_ν^{min}, then

$$\begin{aligned} s_{\text{initial}} &= M_p^2 + 2M_p\left(\delta E + E_\nu^{\text{min}}\right) \\ &= M_p^2 + 2\delta E \times M_p + \left(M_n + m_e\right)^2 - M_p^2 \\ &= 2\delta E \times M_p + \left(M_n + m_e\right)^2 \end{aligned}$$

- Remember the suppression generally goes as

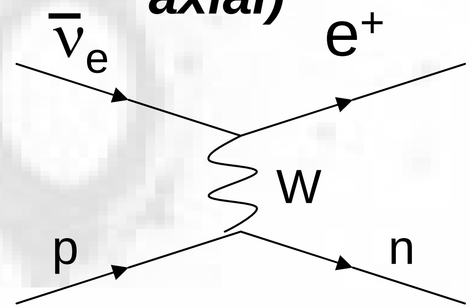
$$\xi_{\text{mass}} = 1 - \frac{m_{\text{final}}{}^2}{\text{s}} = 1 - \frac{\left(M_n + m_e\right)^2}{\left(M_n + m_e\right)^2 + 2M_p \times \delta E}$$

$$= \frac{2M_p \times \delta E}{\left(M_n + m_e\right)^2 + 2M_p \times \delta E} \approx \begin{cases} \delta E \times \dfrac{2M_p}{\left(M_n + m_e\right)^2} & \text{low energy} \\ 1 - \dfrac{\left(M_n + m_e\right)^2}{2M_p{}^2}\dfrac{M_p}{\delta E} & \text{high energy} \end{cases}$$

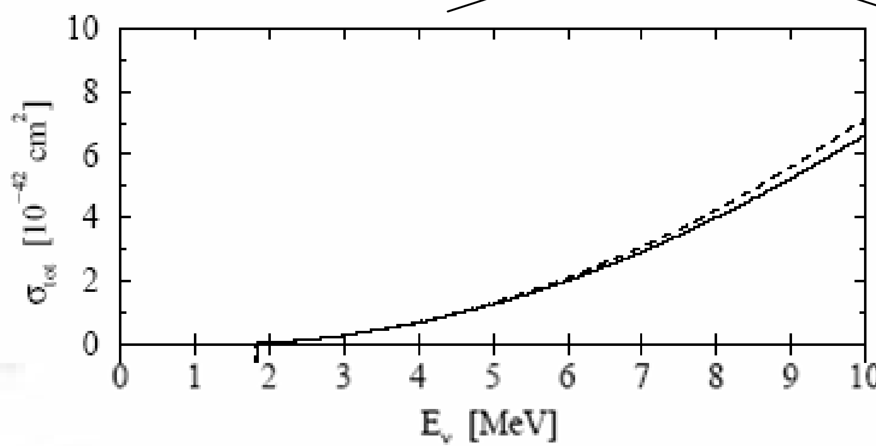
Putting it all together…

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi} \times \cos^2 \vartheta_{\text{Cabibbo}} \times \left(\xi_{\text{mass}}\right) \times \left(g_V{}^2 + 3 g_A{}^2\right)$$

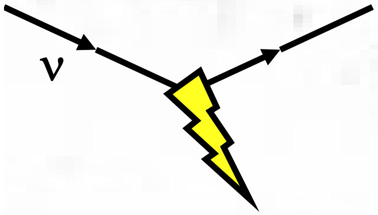
quark mixing! ***final state mass suppression*** ***proton form factors (vector, axial)***



- mass suppression is proportional to δE at low E_ν, so quadratic near threshold
- vector and axial-vector form factors (for IBD usually referred to as f and g, respectively) g_V, $g_A \approx 1, 1.26$.
  - FFs, $\theta_{Cabibbo}$, best known from τ_n

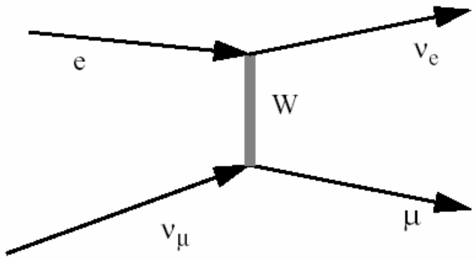


Touchstone Question #3: Quantitative Lepton Mass Effect



- Which is closest to the minimum beam energy in which the reaction

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

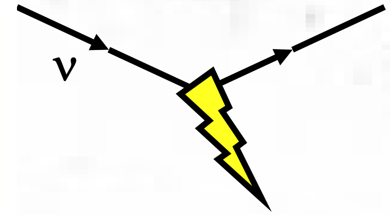


can be observed?

(a) 100 MeV (b) 1 GeV (c) 10 GeV

(It might help you to remember that $Q^2_{\min} = m_\mu^2$
or you might just want to think about the total CM energy required to produce the particles in the final state.)

Summary and Outlook



- We know νe^- scattering and IBD cross-sections!
- In point-like weak interactions, key features are:
  - $d\sigma/dQ^2$ is ≈ constant.
    - o Integrating gives $\sigma \propto E_\nu$
  - LH coupling enters w/ $d\sigma/dy \propto 1$, RH w/ $d\sigma/dy \propto (1-y)^2$
    - o Integrating these gives 1 and 1/3, respectively
  - Lepton mass effect gives minimum Q^2
    - o Integrating gives correction factor in σ of $(1-Q^2_{min}/s)$
  - Structure of target can add form factors
- Deep Inelastic Scattering is also a point-like limit where interaction is ν-quark scattering