

Exercise 1 (August 16th, 2006)

- Show that the L/E dependence of the 2 flavor Oscillation probability is given by

$$\sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \sin^2\left(\frac{1.27\Delta m^2(\text{eV}^2)L(\text{km})}{E(\text{GeV})}\right)$$

- hints : the left-hand side of the above equation is presented in the natural unit. Use $\hbar c = 197\text{MeVfm}$ to get the units back.

Superbeam, Beta Beam, and Neutrino Factory (2)

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Osaka University

Second Lecture

61st Scottish Summer School in Physics
17 August, 2006

Outline of the Second Lecture

- What Is A Neutrino Factory ?
- Eight-fold Degeneracies
- Neutrino Factory : Sensitivity and Optimization
- Beta Beam
- Summary

Neutrino Factory



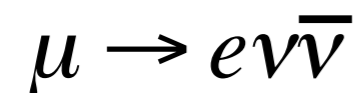
Neutrinos from Pion Decay and Muon Decay

- Pion-decay based neutrino beam

- prompt decays



- backgrounds



- Beam normalization ~ 10%

- Muon-decay based Neutrino beam

- delayed decay after all pions and kaons decay.

- Less beam backgrounds

- Beam normalization can be better known.

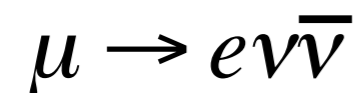
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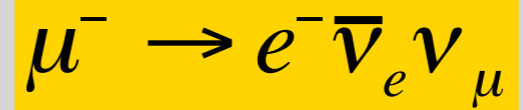
Neutrinos from Muon Decay

- Single (almost 100%) decay mode
- Well defined kinematics

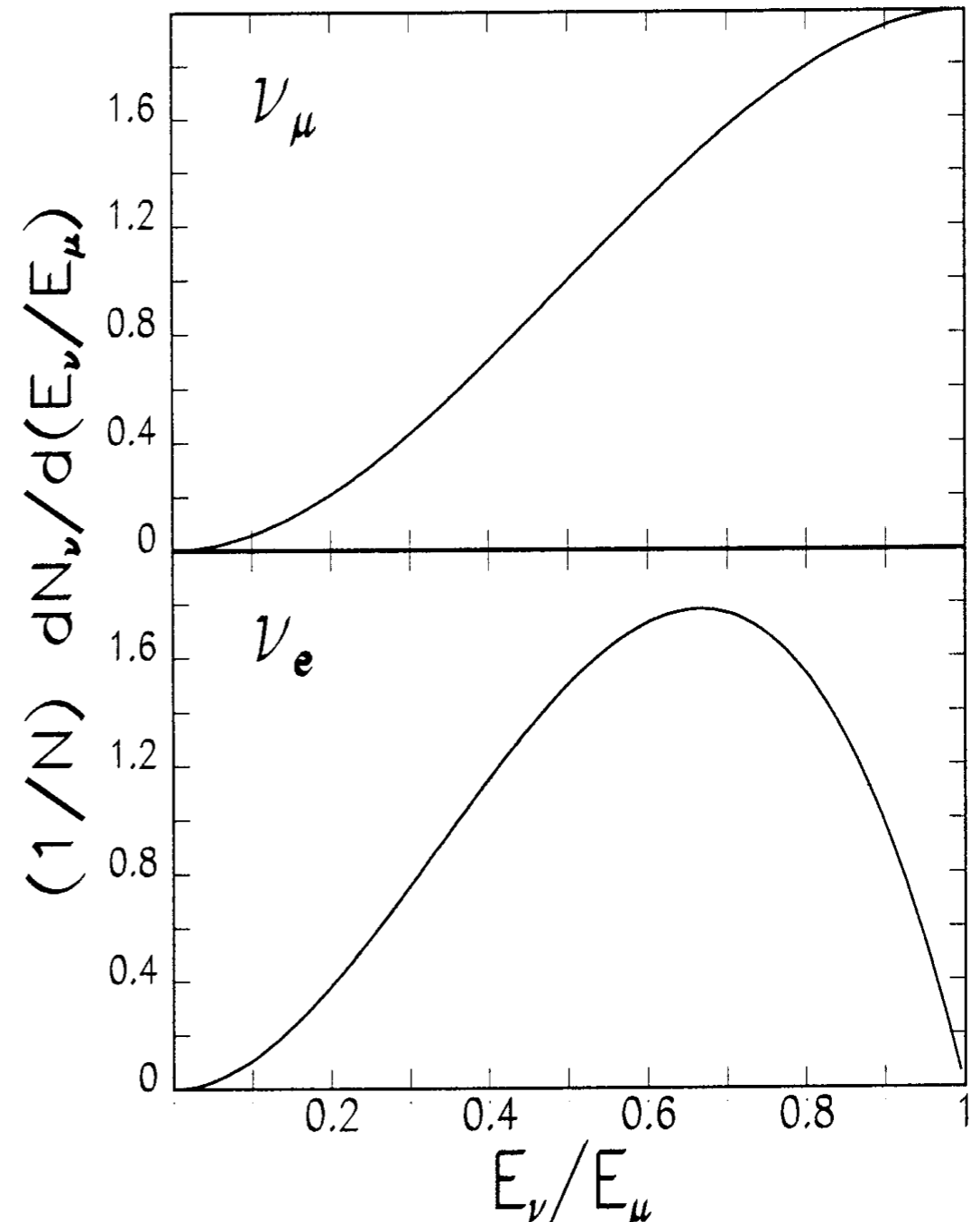
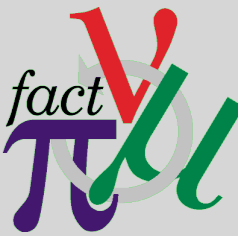
$$\frac{dN(\nu_\mu)}{dx d \cos \theta_{CM}} = 2x^2 \left[(3 - 2x) \mp P_\mu (1 - 2x) \cos \theta_{CM} \right]$$

$$\frac{dN(\nu_e)}{dx d \cos \theta_{CM}} = 6x^2 \left[(1 - x) \mp P_\mu (1 - x) \cos \theta_{CM} \right]$$

$$x = \frac{E_\nu}{E_{\max}}, \text{ where } E_{\max} = m_\mu / 2$$



$$P_\mu = 0$$



Accelerate to Get More Neutrinos !

- Given the proton beam power, numbers of pions and muons are similar.
- Acceleration of the parent particles gives more neutrinos by Lorentz boosting. $N \propto E^2$
- Pion has too short lifetime.
- Only muon live long enough to accelerate.

$$\frac{d^2 N_{\nu_\mu, \bar{\nu}_\mu}}{dy d\Omega} = \frac{4 n_\mu}{\pi L^2 m_\mu^6} E_\mu^4 y^2 (1 - \beta \cos \varphi) \times \left[\left\{ 3m_\mu^2 - 4E_\mu^2 y (1 - \beta \cos \varphi) \right\} \mp P_\mu \left\{ m_\mu^2 - 4E_\mu^2 y (1 - \beta \cos \varphi) \right\} \right]$$

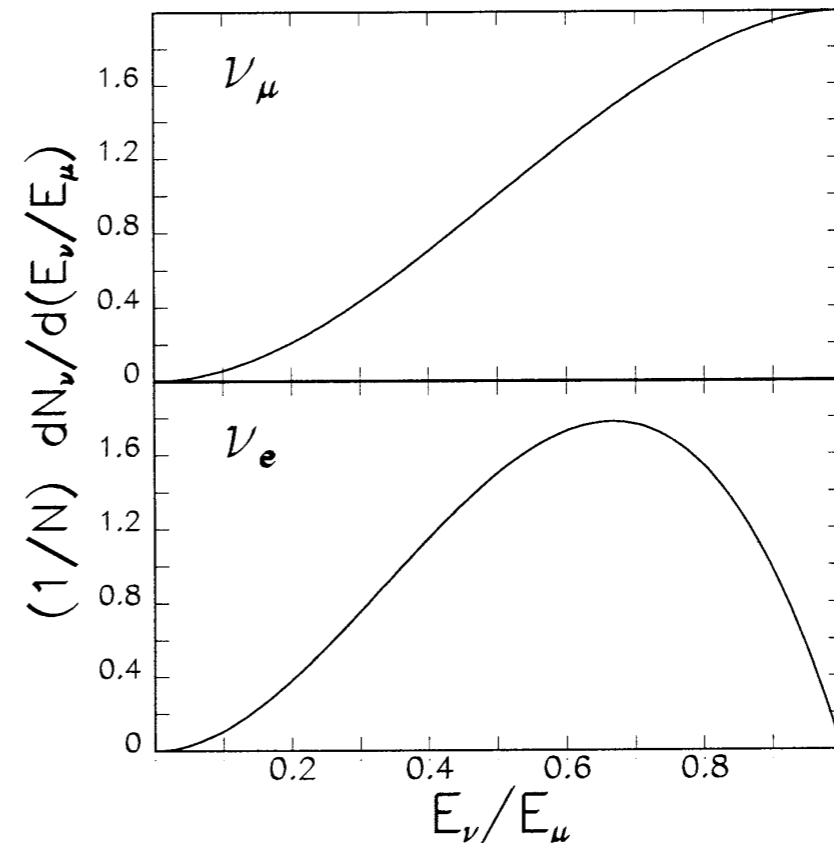
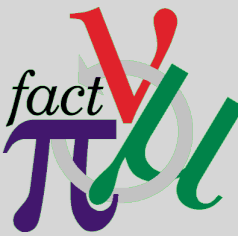
$$\frac{d^2 N_{\bar{\nu}_e, \nu_e}}{dy d\Omega} = \frac{24 n_\mu}{\pi L^2 m_\mu^6} E_\mu^4 y^2 (1 - \beta \cos \varphi) \times \left[\left\{ m_\mu^2 - 2E_\mu^2 y (1 - \beta \cos \varphi) \right\} \mp P_\mu \left\{ m_\mu^2 - 2E_\mu^2 y (1 - \beta \cos \varphi) \right\} \right]$$

$$y = \frac{E_\nu}{E_\mu}; \beta = \sqrt{1 - m_\mu^2 / E_\mu^2}; n_\mu = \text{\# of muons};$$

φ = angle between beam and detector; L = distance

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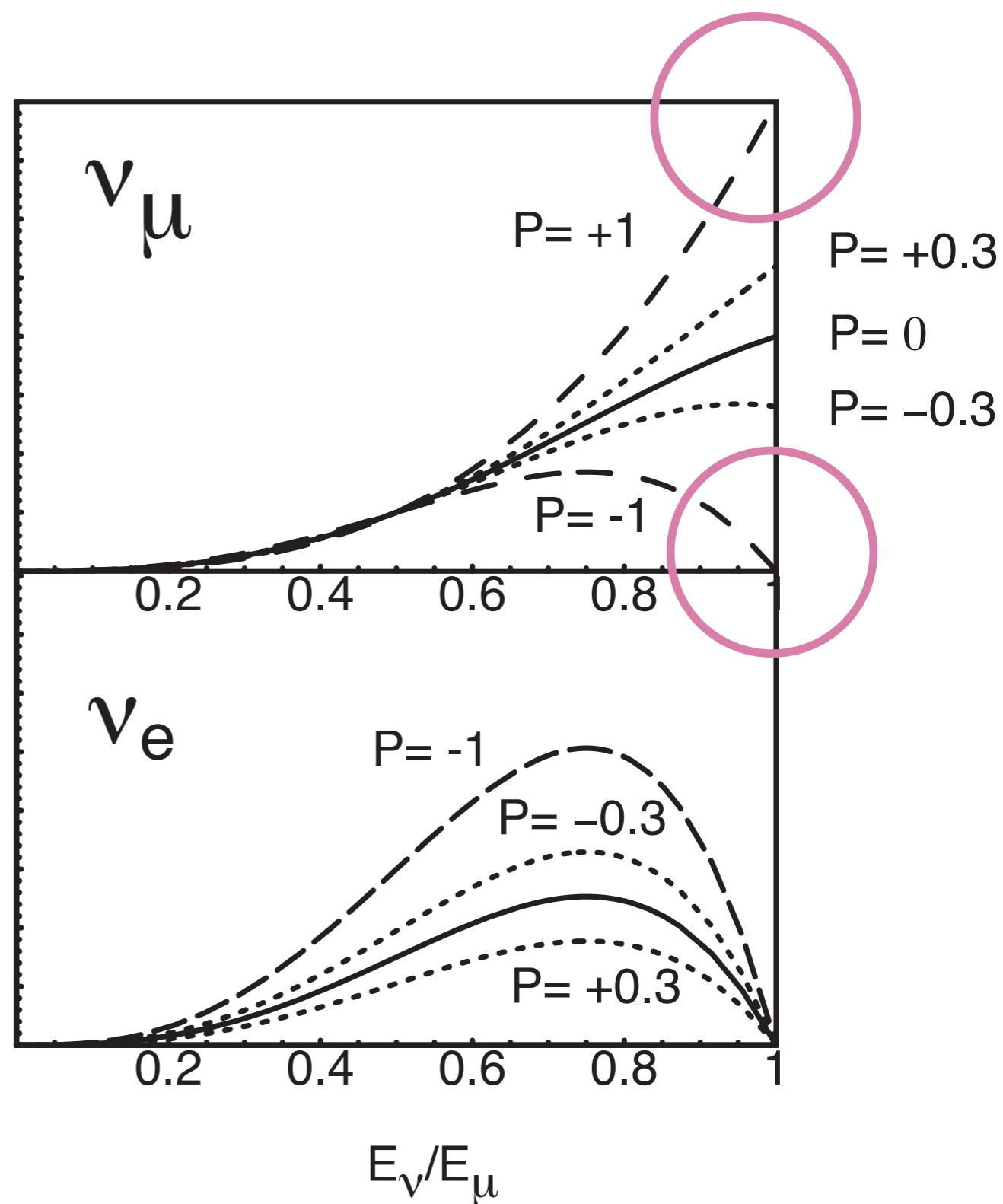
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φ = angle between beam and detector; L = distance

Exercise 1

- If a muon is polarized, neutrino spectra changes. The right figures show the energy spectra of muon neutrino and electron neutrino (not shown whether they are neutrino or anti-neutrino). Here, $P=+1$ ($P=-1$) implies that the direction of neutrino and the direction of the muon polarization is the same (opposite), Show whether this is the case of a positive muon or a negative muon ?



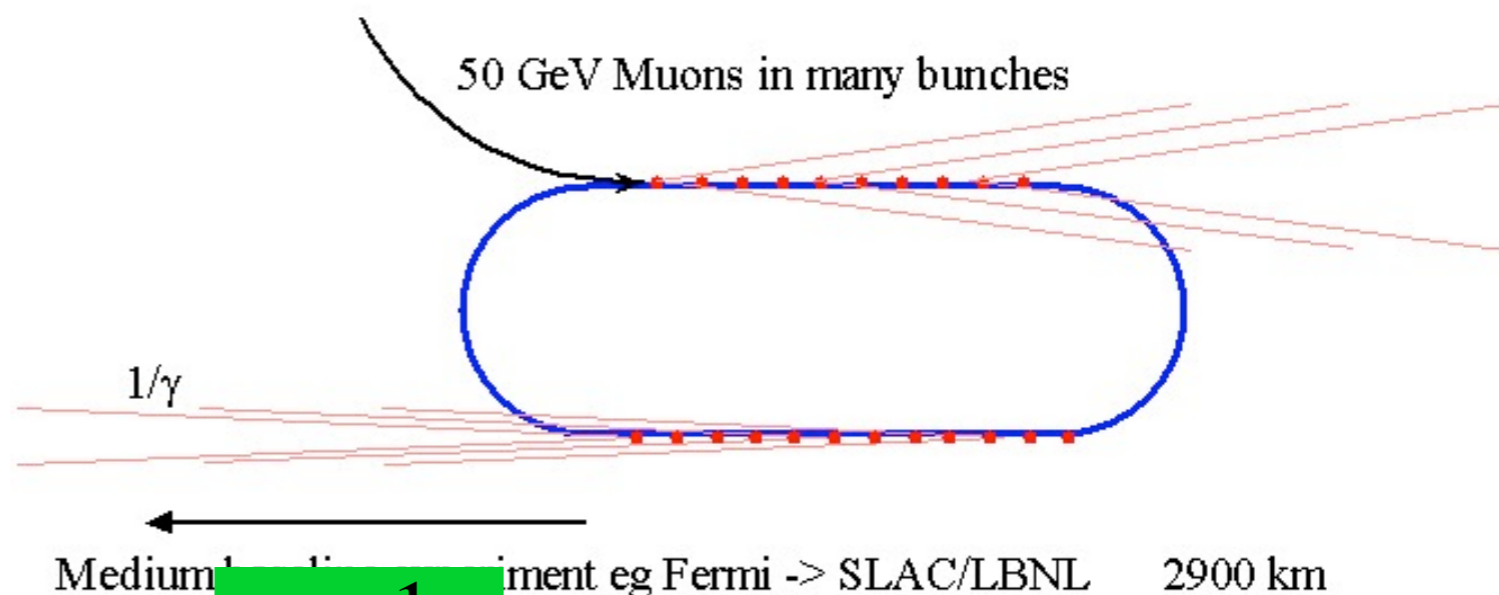
Storage Ring is Needed !

- Muons accelerated at high energy do not decay quickly !
 - at 10 GeV, muon lifetime is about 200 microseconds.
- A storage ring is needed with long straight sections.
 - Two straight sections give automatically two experiments (with different baselines) at a time.

Parameters for the Muon Storage Ring		
Energy	GeV	50
decay ratio	%	>40
Designed for inv. Emittance	m*rad	0.0032
Cooling designed for inv. Emitt.	m*rad	0.0016
β in straight	m	160
N_{μ} /pulse	10^{12}	6
typical decay angle of $\mu = 1/\gamma$	mrad	2.0
Beam angle $(\sqrt{\epsilon/\beta_0}) = (\sqrt{\epsilon} \gamma)$	mrad	0.2
Lifetime $c*\gamma*\tau$	m	3×10^5

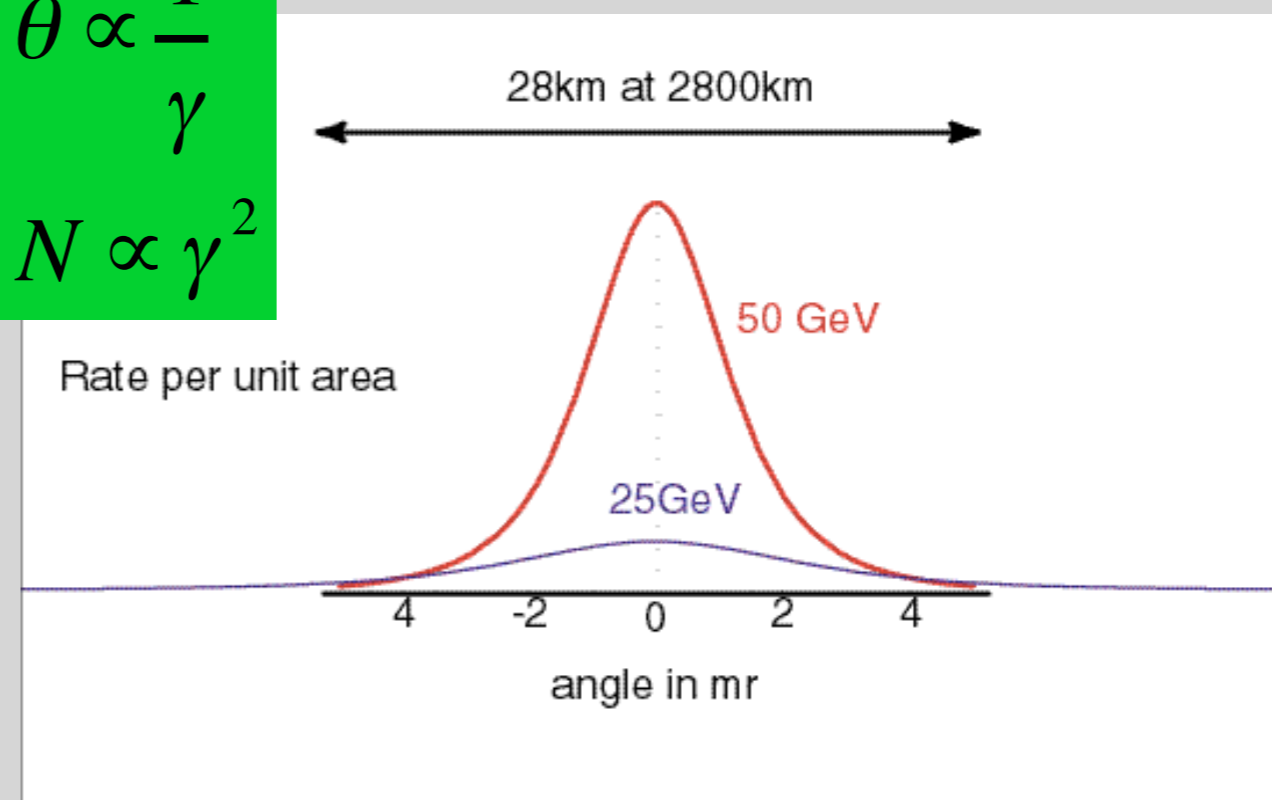
$\gamma = (1-\alpha^2)/\beta$

Muon Storage Ring as a Neutrino Source



$\theta \propto \frac{1}{\gamma}$

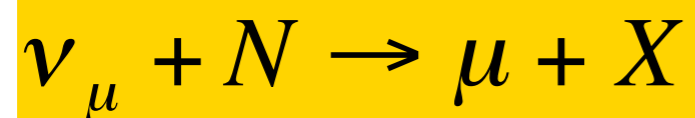
$N \propto \gamma^2$



At 50 GeV, $\gamma=500$ and beam spread is 2 mrad. (At 100m, +-20cm beam size.)

Neutrino Cross Sections

- Deep Inelastic Scattering Processes at High Energy.

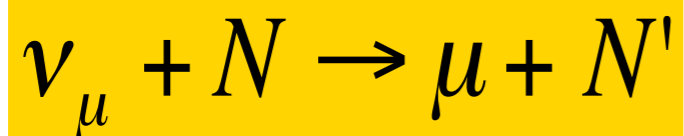


$$\sigma(\nu) \approx 0.67 \times 10^{-38} \text{ cm}^2 \times E_{\nu} (\text{GeV})$$

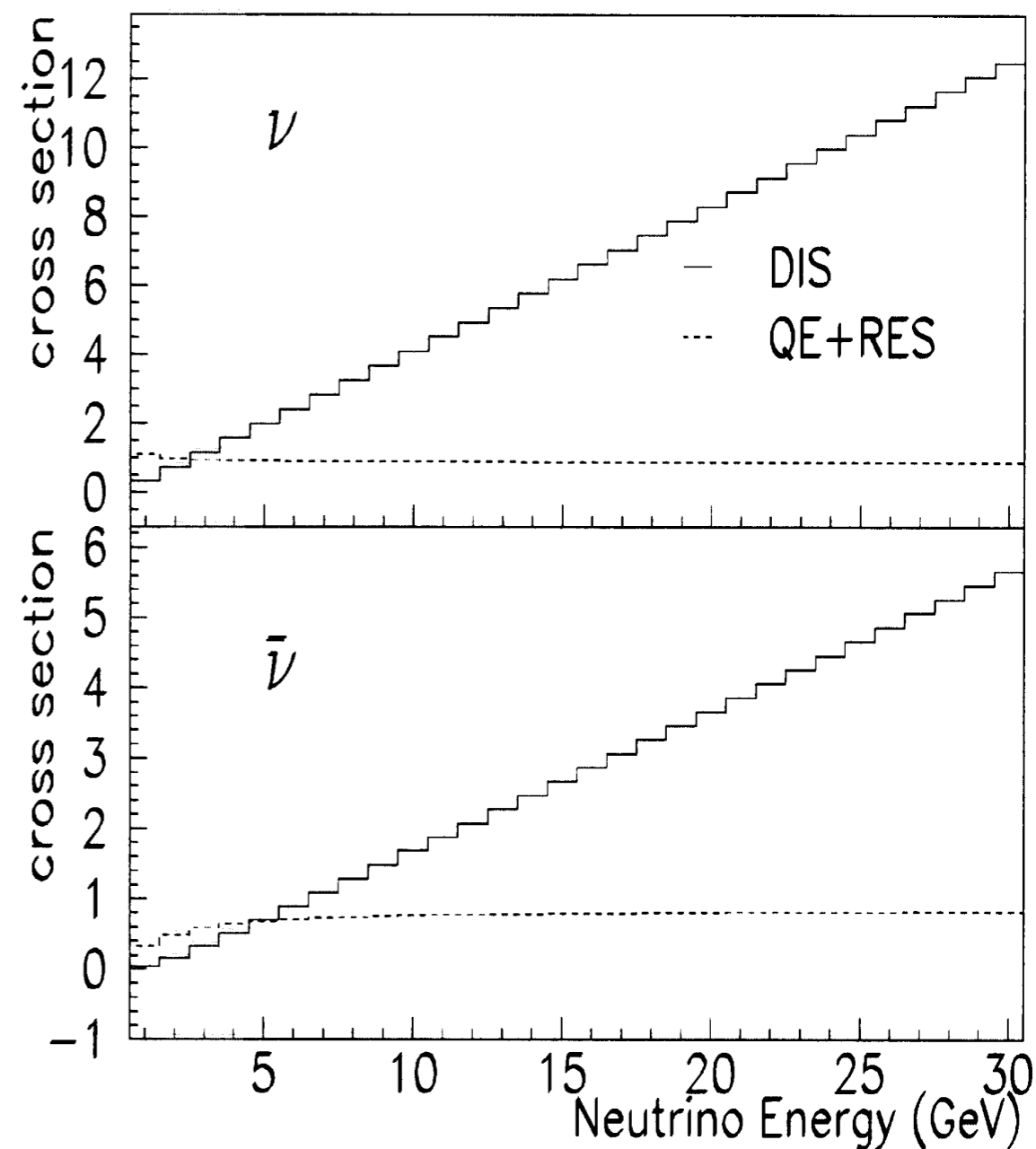
$$\sigma(\bar{\nu}) \approx 0.34 \times 10^{-38} \text{ cm}^2 \times E_{\nu} (\text{GeV})$$

$$\sigma(\bar{\nu}) / \sigma(\nu) \approx 0.5$$

- Quasi Elastic Scattering Processes at 1 GeV

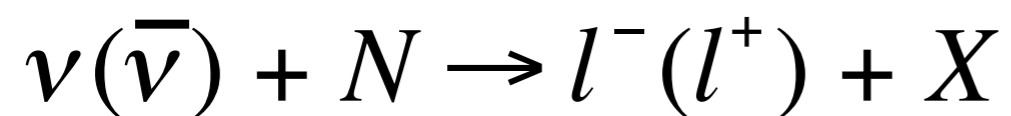


$$\sigma(\bar{\nu}) / \sigma(\nu) \approx 1$$



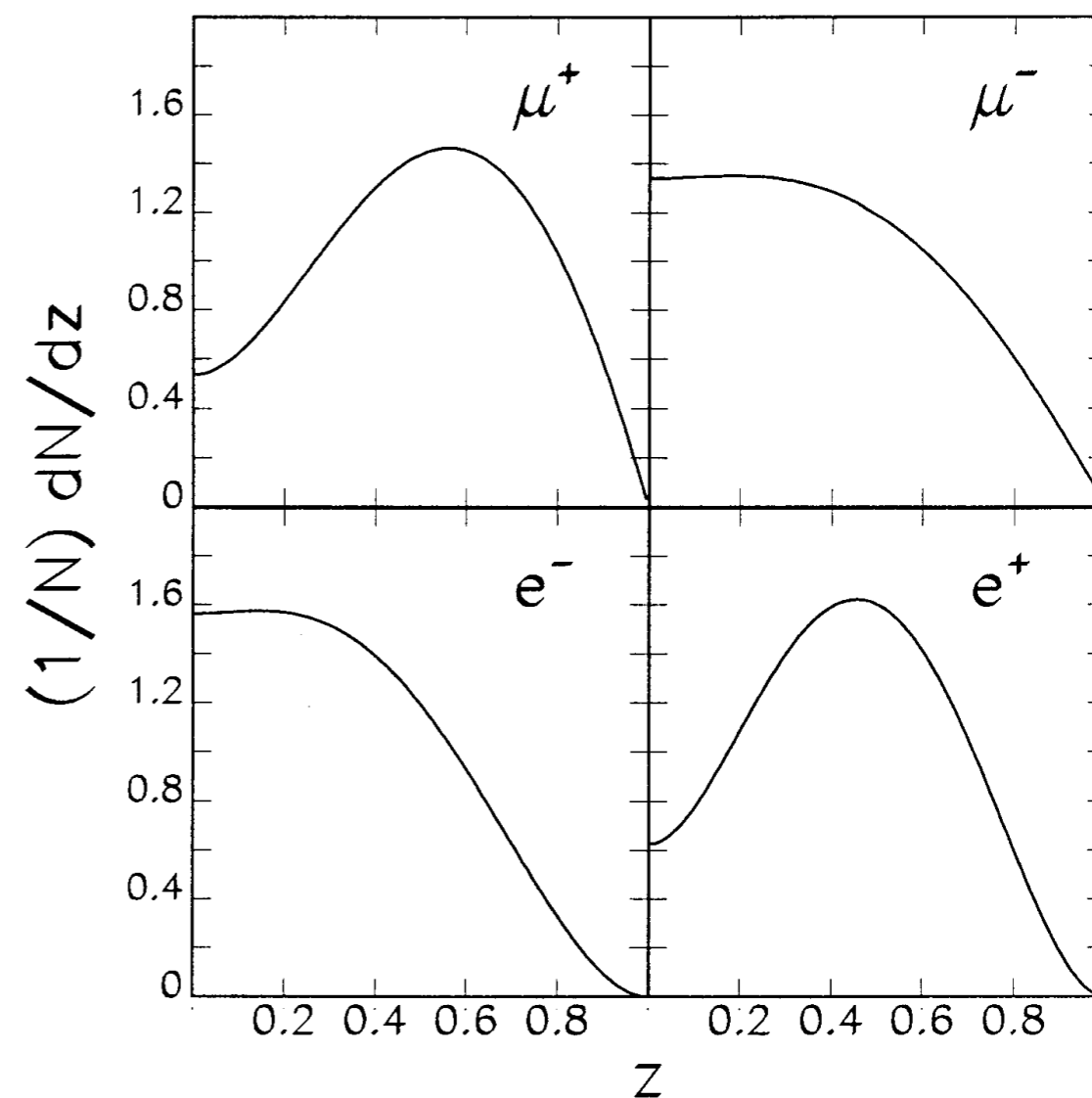
Lepton Spectra from CC events

- neutrino CC events



- different for neutrinos and antineutrinos

- low energy region is important for neutrino events (not antineutrino events.)



Advantages of Neutrino Factory

- **Very highly intense neutrino source**
 - a few orders of magnitude higher at a few 10 GeV energy range.
- **Both muon (anti-)neutrinos and electron (anti-)neutrinos are available.**
 - Many variety of oscillation modes can be studied.
- **Extremely low backgrounds**
 - for wrong signed muon detection, a background level would be less than 10^{-4} .
- **Precise Knowledge on Neutrino Flux**
 - Neutrino flux normalization can be done at the level of 0.1%.

12 Oscillation Processes in a Neutrino Factory

12 Oscillation Processes from (simultaneous) beams of positive and negative muons in a neutrino Factory.

$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$	$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$	
$\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$	$\nu_\mu \rightarrow \nu_\mu$	disappearance
$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$	$\nu_\mu \rightarrow \nu_e$	appearance (challenging)
$\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$	$\nu_\mu \rightarrow \nu_\tau$	appearance (atm. oscillation) platinum
$\nu_e \rightarrow \nu_e$	$\bar{\nu}_e \rightarrow \bar{\nu}_e$	disappearance
$\nu_e \rightarrow \nu_\mu$	$\bar{\nu}_e \rightarrow \bar{\nu}_\mu$	appearance: “golden” channel golden
$\nu_e \rightarrow \nu_\tau$	$\bar{\nu}_e \rightarrow \bar{\nu}_\tau$	appearance: “silver” channel silver

Event Rates

- Charged Current (CC) Event Rates

$$N_{CC}(\nu_\ell \rightarrow \ell) \propto N_\nu \cdot \sigma$$

$$\propto \frac{E^2}{L^2} \cdot E = \frac{E^3}{L^2}$$

- example

- 10^{21} muons decay /year with a 10 kton detector

	L=1000 km	L=1500 km
$E_\mu=20$ GeV	3.2×10^5	1.4×10^5
$E_\mu=30$ GeV	1.1×10^6	4.8×10^5

MINOS (low energy 3GeV, 732 km) : 5000 CC events/10 kton/year

- Oscillation Event Rates

$$N_{osc}(\nu_\ell \rightarrow \ell')$$

$$\propto N_\nu \cdot \sigma \cdot P(\nu_\ell \rightarrow \nu_{\ell'})$$

$$\propto \frac{E^3}{L^2} \cdot \frac{L^2}{E^2} = E$$

Neutrino Oscillation Signature at NuFact

- The signature of neutrino oscillation is wrong-signed leptons.
- Charge identification of the lepton(s) is needed.
 - Muons are easy.
 - Electrons are difficult.

$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$$

oscillation



$$\bar{\nu}_\mu$$

$$\begin{matrix} \mu^- \\ \mu^+ \end{matrix}$$

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$

oscillation



$$\nu_\mu$$

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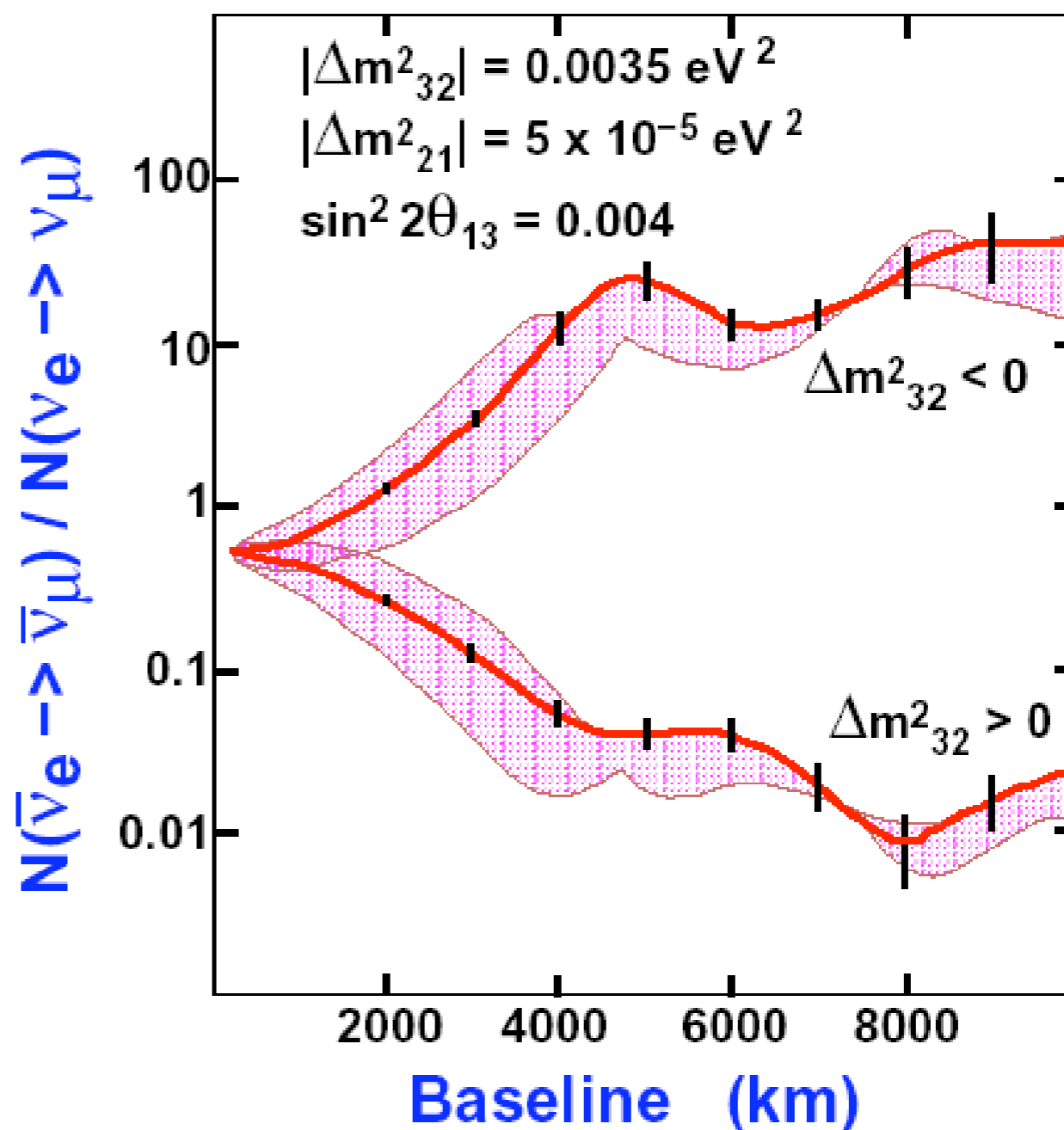
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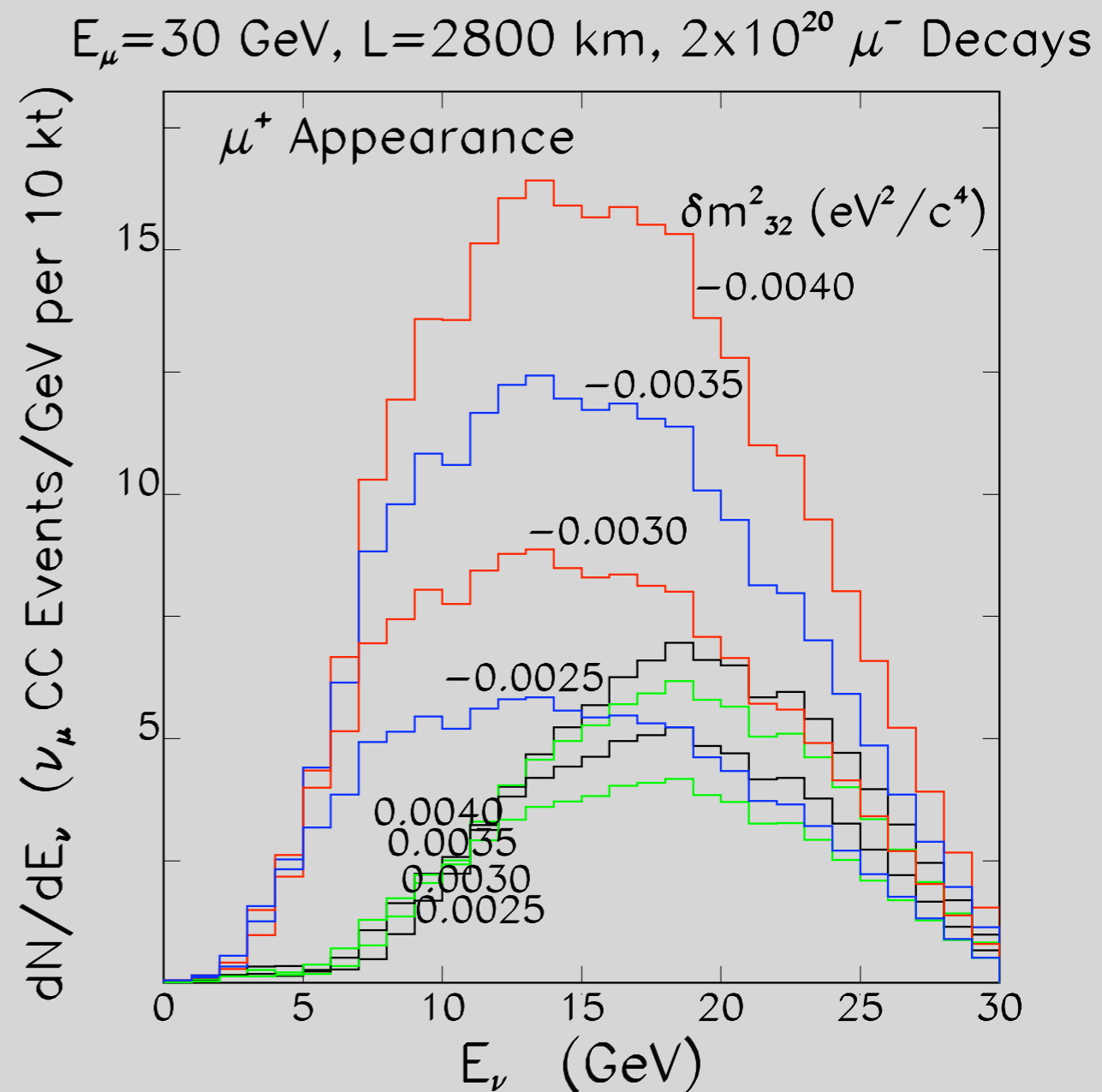
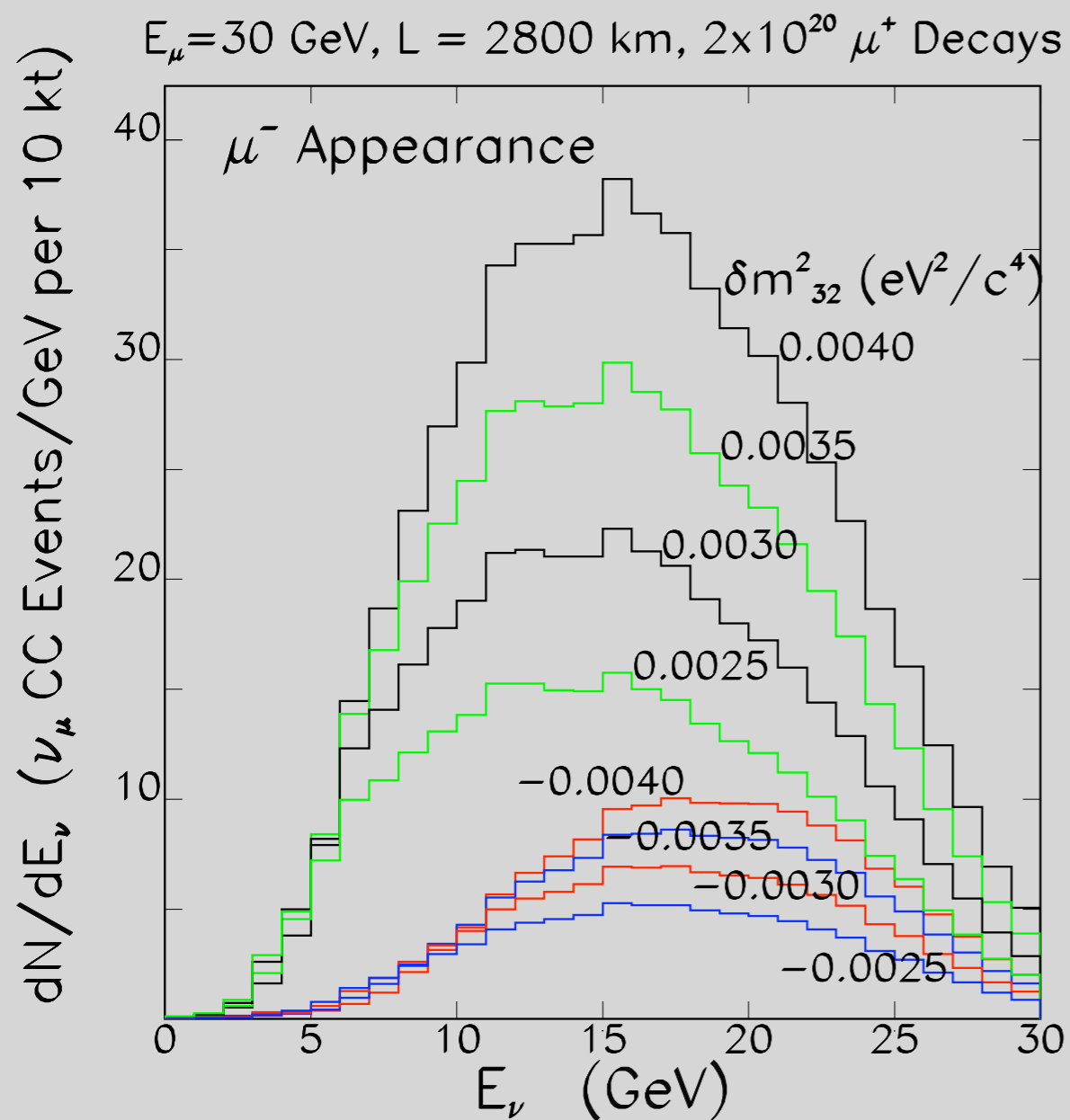
Look for wrong signed Muons.

Ratio of Wrong Sign Muon Events

- Wrong sign muons are clean signals.
- Background level for wrong sign muons would be 10^{-4} .
- The matter effect enhance anti-neutrino events if $\Delta m_{32}^2 < 0$, and it enhance neutrino events if $\Delta m_{32}^2 > 0$.
- The band shows CP violation effect where the phase changes.



20 GeV Neutrino Factory, 4 MeV threshold. Two lines are for two mass hierarchy. The statistical error represents the sample of 1021 muon decays with a 50 kton detector.



Wrong Sign Muon Event
Spectra

2E20 decays, $E_\mu = 30 \text{ GeV}$,
 $L = 2800 \text{ km}$

Goals of Neutrino Oscillation Physics (at Neutrino Factory and Superbeams)

Search for θ_{13}

Mass Hierarchy

Discovery of Leptonic
CP Violation δ

$$\delta \mp Y_{\pm}^s \sin \delta) \sin 2\theta_{13} + Z$$

known parameters

These effects
are strongly
correlated.

$$\left(\frac{AL}{2}\right) \sin\left(\frac{\tilde{B}_{\mp}}{2}\right)$$

$$\left(\frac{AL}{2}\right) \sin\left(\frac{\tilde{B}_{\mp}}{2}\right)$$

$$\frac{AL}{2}$$

the m



Daruma

Degeneracies



Golden Appearance Channel in Neutrino Factory

$\nu_e \rightarrow \nu_\mu$ ($\bar{\nu}_e \rightarrow \bar{\nu}_\mu$) Oscillation + for neutrino, - for antineutrino

$$P_{\nu_e \nu_\mu}^\pm(\theta_{13}, \delta) \approx X_\pm \sin^2 2\theta_{13} + \left(Y_\pm^c \cos \delta \mp Y_\pm^s \sin \delta \right) \sin 2\theta_{13} + Z$$

with X_\pm, Y_\pm^c, Y_\pm^s and Z functions of the known parameters:

$$\left\{ \begin{array}{l} X_\pm = \boxed{\sin^2 \theta_{23}} \left(\frac{\Delta_{23}}{\tilde{B}_\mp} \right)^2 \sin^2 \left(\frac{\tilde{B}_\mp L}{2} \right) \\ Y_\pm^c = \boxed{\sin 2\theta_{23}} \sin 2\theta_{12} \frac{\Delta_{12}}{A} \frac{\Delta_{23}}{\tilde{B}_\mp} \sin \left(\frac{AL}{2} \right) \sin \left(\frac{\tilde{B}_\mp L}{2} \right) \boxed{\cos \left(\frac{\Delta_{23}L}{2} \right)} \\ Y_\pm^s = \boxed{\sin 2\theta_{23}} \sin 2\theta_{12} \frac{\Delta_{12}}{A} \frac{\Delta_{23}}{\tilde{B}_\mp} \sin \left(\frac{AL}{2} \right) \sin \left(\frac{\tilde{B}_\mp L}{2} \right) \boxed{\sin \left(\frac{\Delta_{23}L}{2} \right)} \\ Z = \boxed{\cos^2 \theta_{23}} \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{A} \right)^2 \sin^2 \left(\frac{AL}{2} \right) \end{array} \right.$$

where $\Delta_{ij} = \Delta m_{ij}^2 / 2E$, $B_\mp = |A \mp \Delta_{23}|$ and A is the matter parameter.

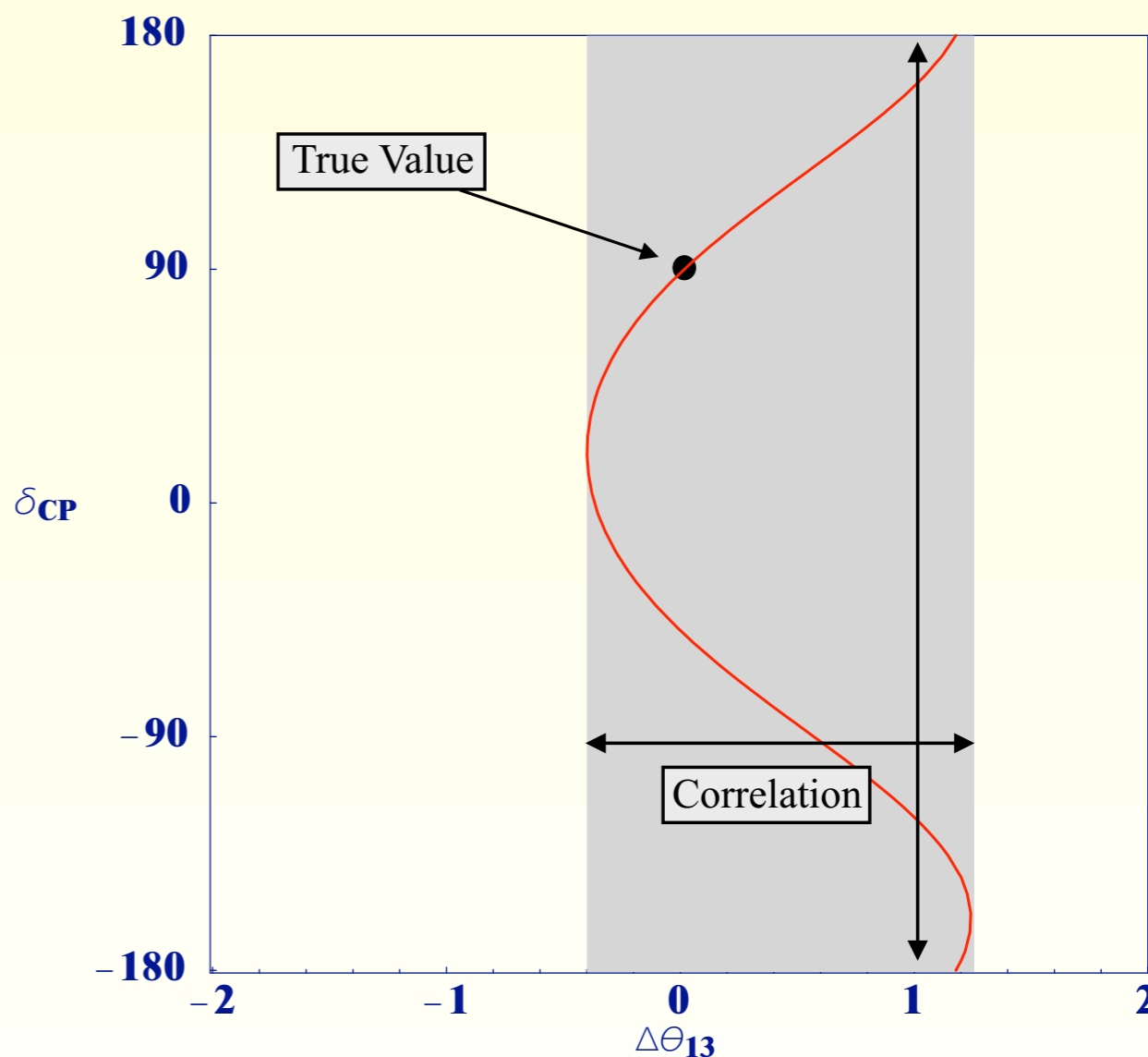
Degeneracies of δ and θ_{13} (1)

- Just One Counting Measurement

- With neutrino oscillation of given L/E , for the true value set of $(\bar{\delta}, \bar{\theta}_{13})$, another set of (δ, θ_{13}) would give the same oscillation probability.
- no sensitivity to δ
- large uncertainty in θ_{13}

Intrinsic
Degeneracy

$$P_+(\bar{\theta}_{13}, \bar{\delta}) = P_+(\theta_{13}, \delta)$$



Appearance Oscillation Channels

$\nu_e \rightarrow \nu_\mu$ ($\bar{\nu}_e \rightarrow \bar{\nu}_\mu$) Oscillation + for neutrino, - for antineutrino

$$P_{\nu_e \nu_\mu}^\pm(\theta_{13}, \delta) \approx X_\pm \sin^2 2\theta_{13} + \left(Y_\pm^c \cos \delta \mp Y_\pm^s \sin \delta \right) \sin 2\theta_{13} + Z$$

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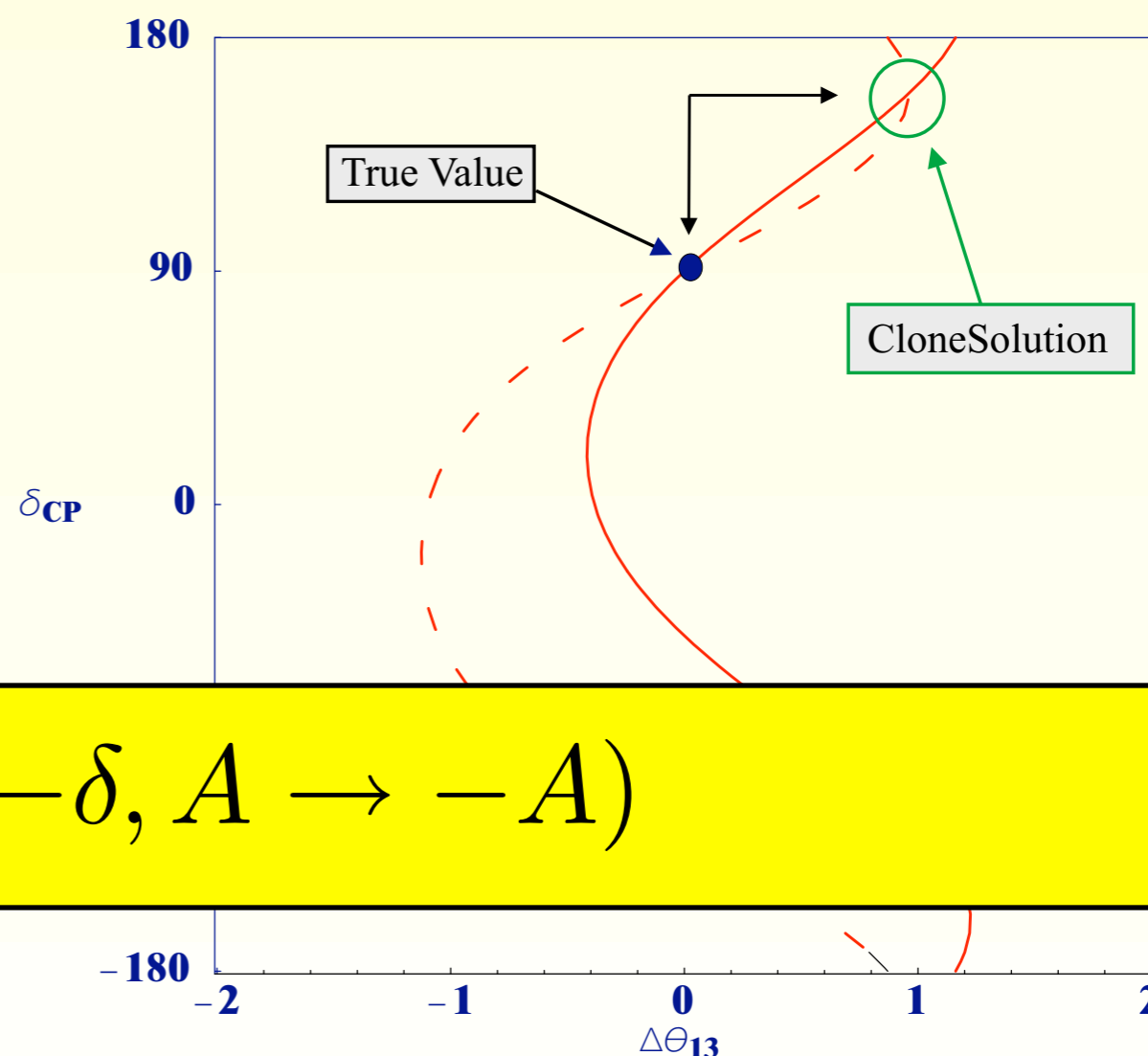
Degeneracies of δ and θ_{13} (2)

- Compare Neutrinos and Anti-Neutrinos

- We have the same measurements for neutrino (P_+) and anti-neutrinos (P_-).
- Two solutions to give the same oscillation probabilities.
 - one is the true, and the other is clone.

$$P_+(\bar{\theta}_{13}, \bar{\delta}) = P_+(\theta_{13}, \delta)$$

$$P_-(\bar{\theta}_{13}, \bar{\delta}) = P_-(\theta_{13}, \delta)$$

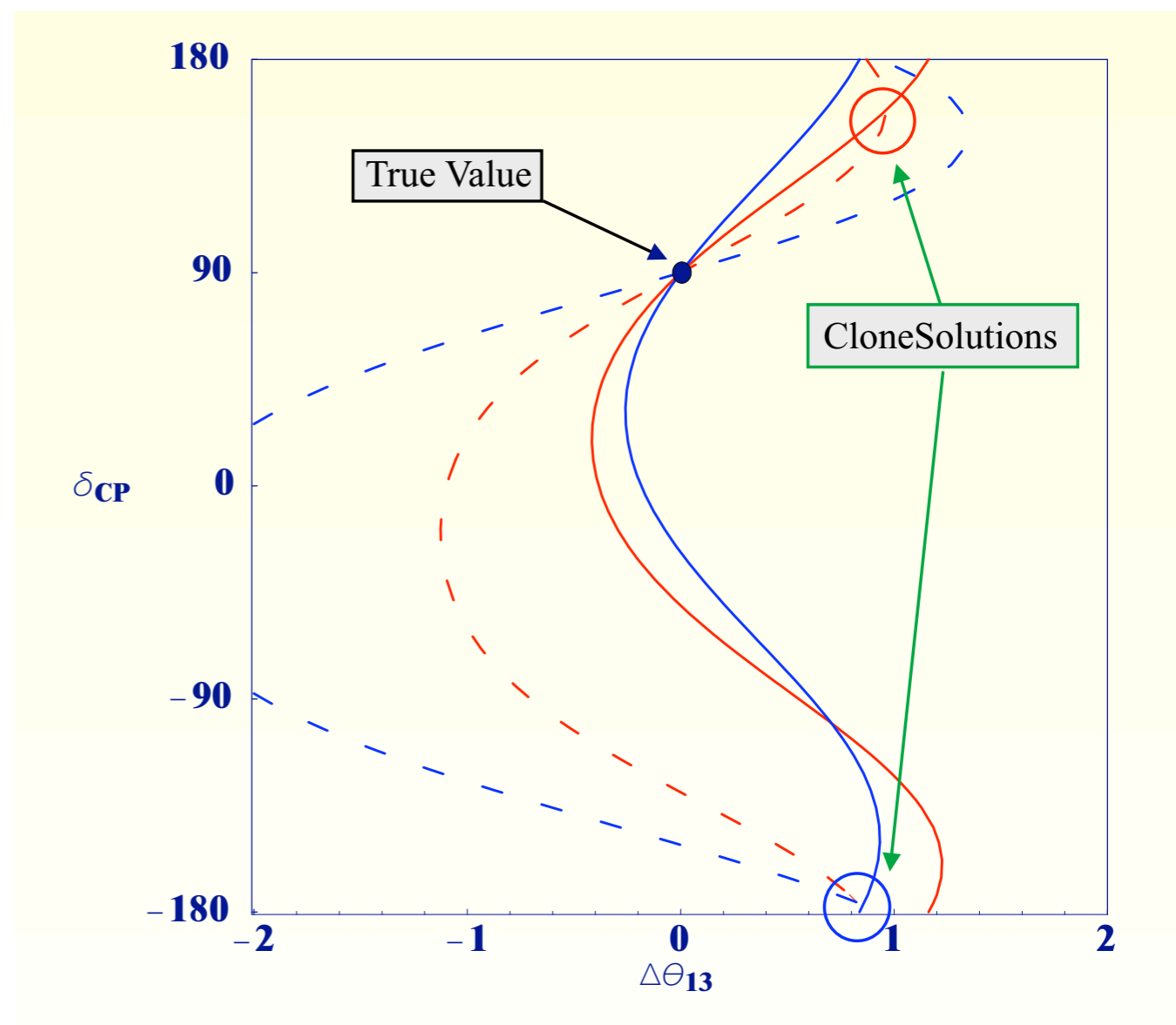


$$P_{\bar{\nu}_e \bar{\nu}_\mu} = P_{\nu_e \nu_\mu} (\delta \rightarrow -\delta, A \rightarrow -A)$$

Degeneracies of δ and θ_{13} (3)

- Compare Different L/E

- We have the same measurements for neutrino (P_+) and anti-neutrinos (P_-) for either
 - two counting experiments at two different L/E values, or
 - binning the energy spectra (where each energy bin corresponds to different experiments).



More Degeneracies - Eight-fold !

- Besides δ and θ_{13} , the following values are not known.

– The SIGN of the ATM mass difference

$$s_{atm} = \text{sign}(\Delta m_{23}^2)$$

– The OCTANT of the ATM angle

$$s_{oct} = \text{sign}(\tan 2\theta_{23})$$

$$\Delta m_{23}^2 \rightarrow -\Delta m_{23}^2$$

$$\theta_{23} \rightarrow \frac{\pi}{2} - \theta_{23}$$

Eight-fold Degeneracy

Eight-fold Degeneracy

intrinsic degeneracy (Burguet01)

$$N_i^\pm(\bar{\theta}_{13}, \bar{\delta}; \bar{s}_{atm}, \bar{s}_{oct}) = N_i^\pm(\theta_{13}, \delta; s_{atm} = \bar{s}_{atm}, s_{oct} = \bar{s}_{oct})$$

sign degeneracy (Minakata01)

$$N_i^\pm(\bar{\theta}_{13}, \bar{\delta}; \bar{s}_{atm}, \bar{s}_{oct}) = N_i^\pm(\theta_{13}, \delta; s_{atm} = -\bar{s}_{atm}, s_{oct} = \bar{s}_{oct})$$

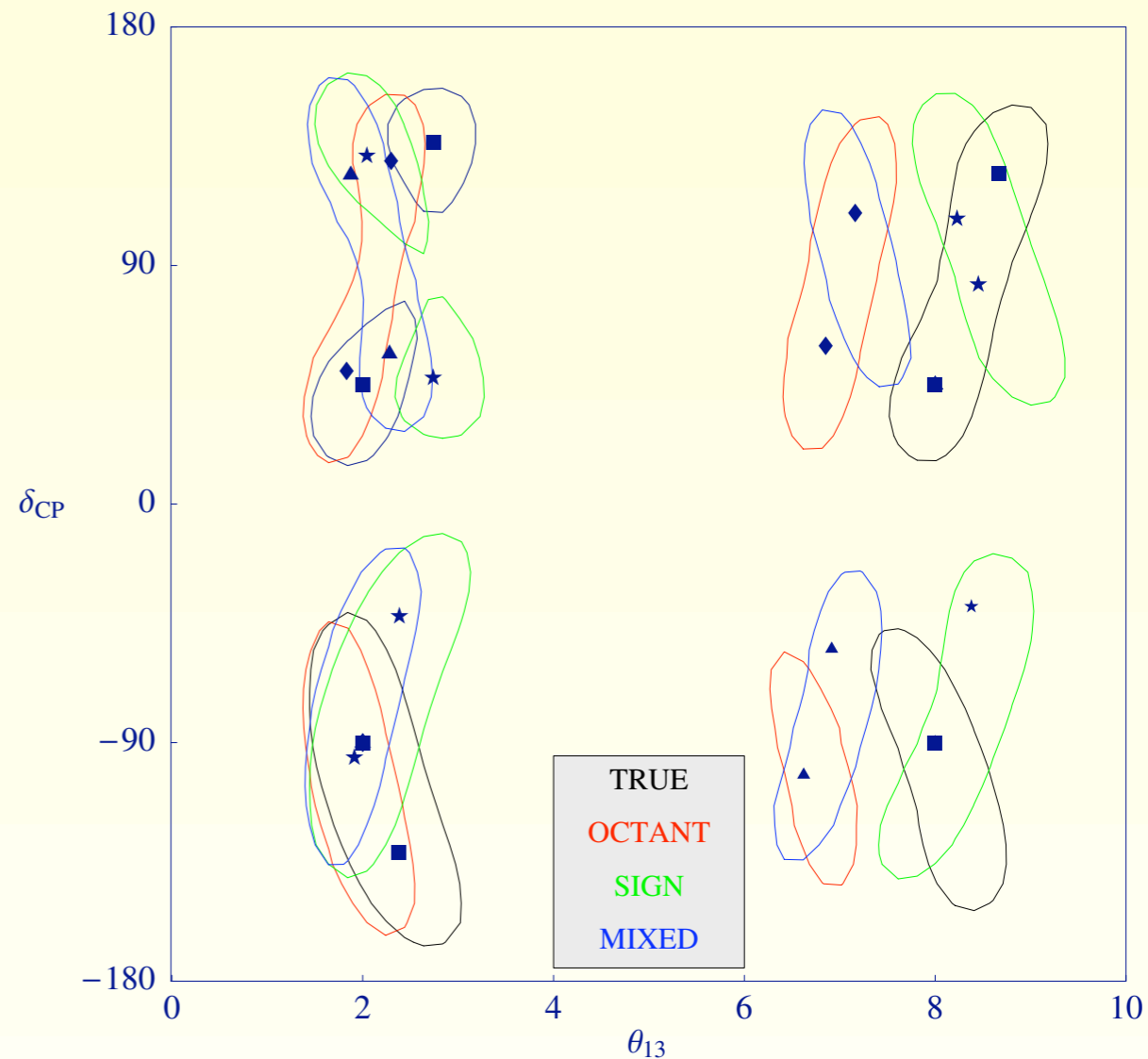
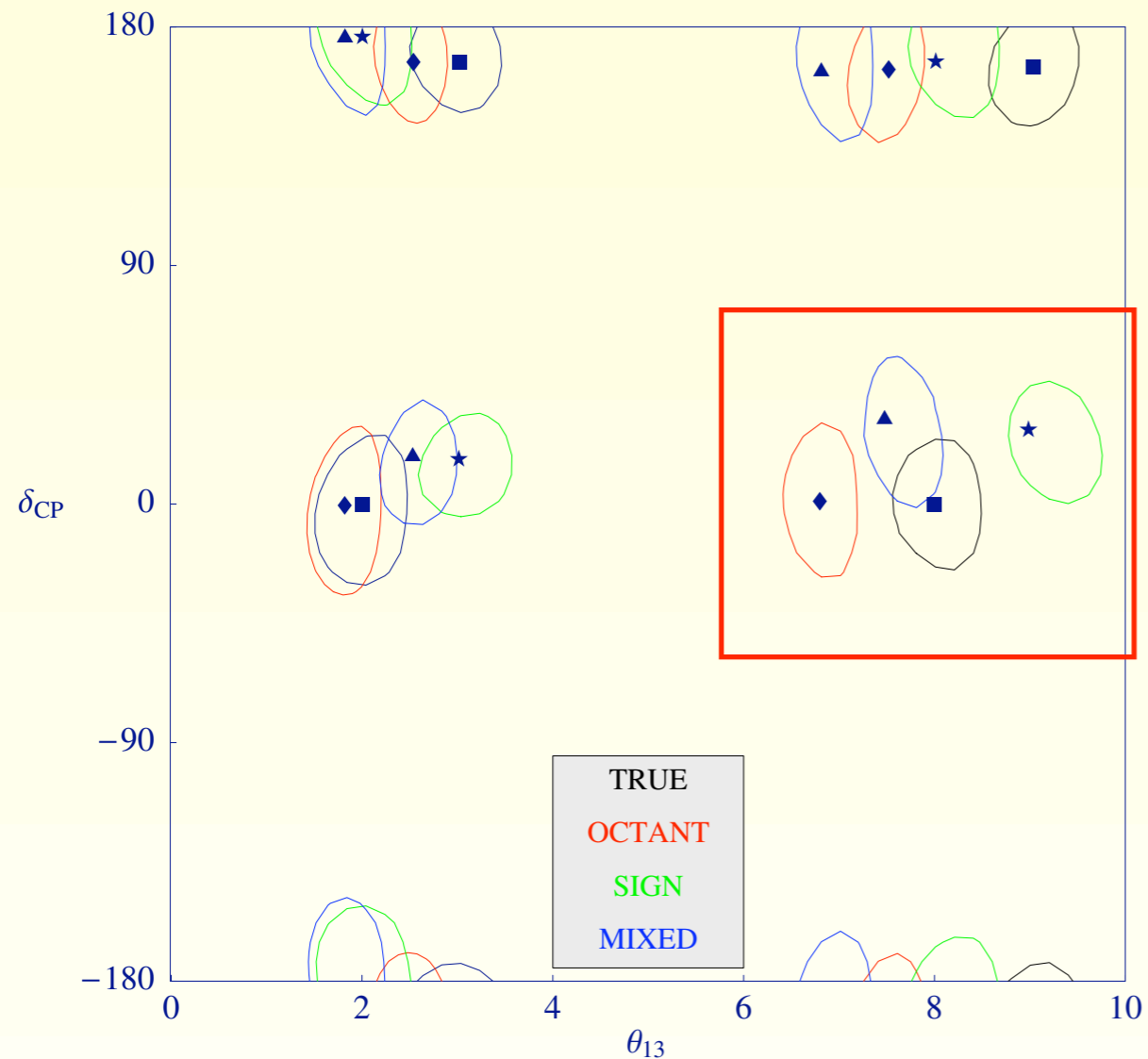
octant degeneracy (Fogli96, Barger01)

$$N_i^\pm(\bar{\theta}_{13}, \bar{\delta}; \bar{s}_{atm}, \bar{s}_{oct}) = N_i^\pm(\theta_{13}, \delta; s_{atm} = \bar{s}_{atm}, s_{oct} = -\bar{s}_{oct})$$

mixed degeneracy (Barger01)

$$N_i^\pm(\bar{\theta}_{13}, \bar{\delta}; \bar{s}_{atm}, \bar{s}_{oct}) = N_i^\pm(\theta_{13}, \delta; s_{atm} = -\bar{s}_{atm}, s_{oct} = -\bar{s}_{oct})$$

Sensitivity of θ_{13} and δ



(plots from A. Donini et al, hep-ph/0406132)

Example

Eight-fold degeneracy

Silver Appearance Channel in Neutrino Factory

$\nu_e \rightarrow \nu_\tau$ ($\bar{\nu}_e \rightarrow \bar{\nu}_\tau$) Oscillation

$$P_{\nu_e \nu_\tau}^\pm(\theta_{13}, \delta) \approx X_\pm^\tau \sin^2 2\theta_{13} + \left(Y_\pm^{\tau,c} \cos \delta \mp Y_\pm^{\tau,s} \sin \delta \right) \sin 2\theta_{13} + Z^\tau$$

with X_\pm^τ , $Y_\pm^{\tau,c}$, $Y_\pm^{\tau,s}$ and Z^τ functions of the known parameters:

$$\left\{ \begin{array}{l} X_\pm^\tau = \cos^2 \theta_{23} \left(\frac{\Delta_{23}}{\tilde{B}_\mp} \right)^2 \sin^2 \left(\frac{\tilde{B}_\mp L}{2} \right) \\ Y_\pm^{\tau,c} = - Y_\pm^c \\ Y_\pm^{\tau,s} = - Y_\pm^s \\ Z^\tau = \sin^2 \theta_{23} \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{\Delta} \right)^2 \sin^2 \left(\frac{AL}{2} \right) \end{array} \right.$$

$$P_{\nu_e \nu_\tau} = P_{\nu_e \nu_\mu} (s_{23}^2 \rightarrow c_{23}^2, \sin^2 2\theta_{23} \rightarrow -\sin^2 2\theta_{23})$$

Platinum Appearance Channel in Neutrino Factory

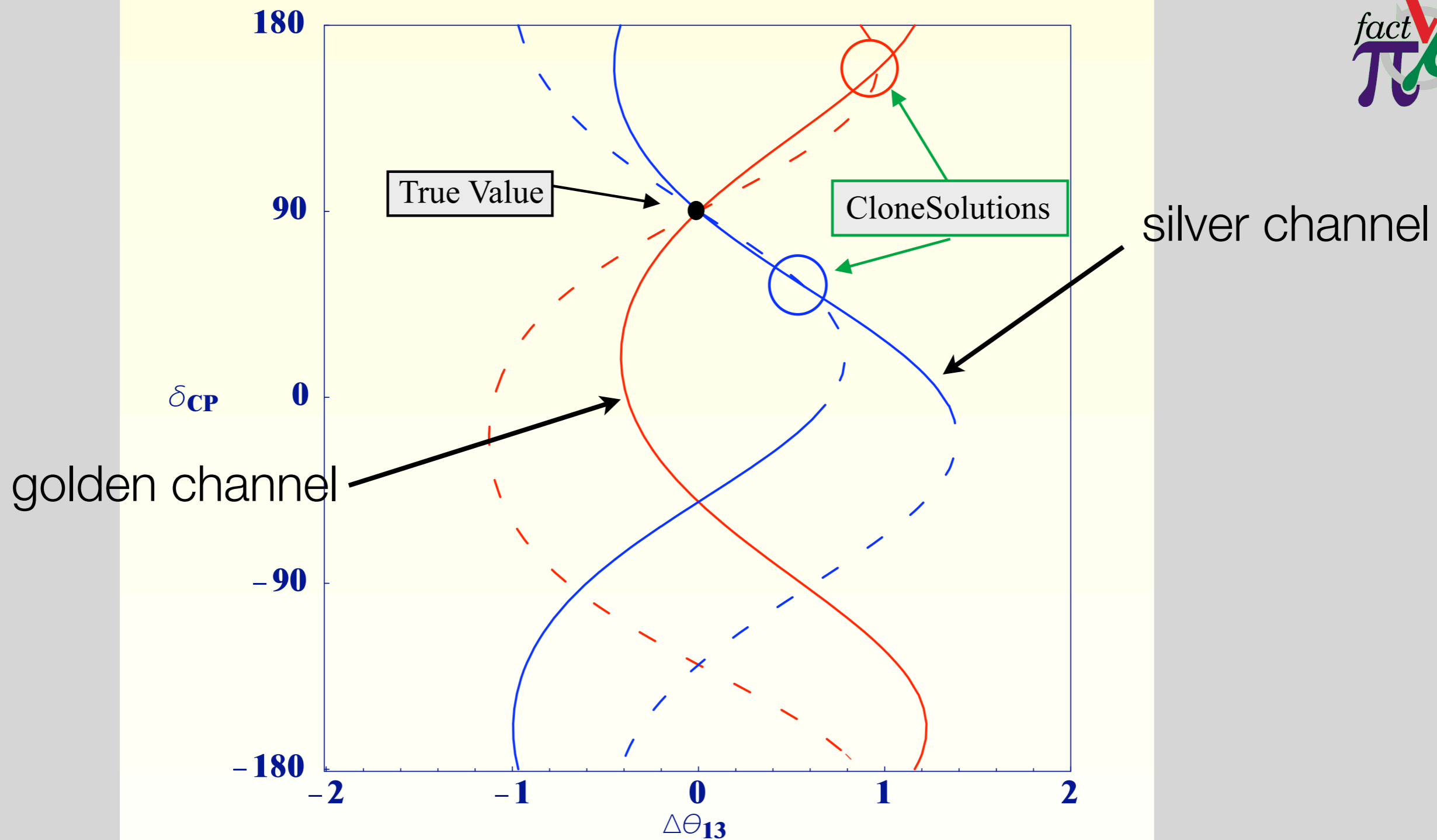
$\nu_\mu \rightarrow \nu_e$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_e$) Oscillation

- Exercise

The oscillation probability of the silver channel is given from that of golden channel by the following transformation.

$$P_{\nu_e \nu_\tau} = P_{\nu_e \nu_\mu} (s_{23}^2 \rightarrow c_{23}^2, \sin^2 2\theta_{23} \rightarrow -\sin^2 2\theta_{23})$$

Show what transformation would give the oscillation probability of the platinum channel from that of the golden channel ?



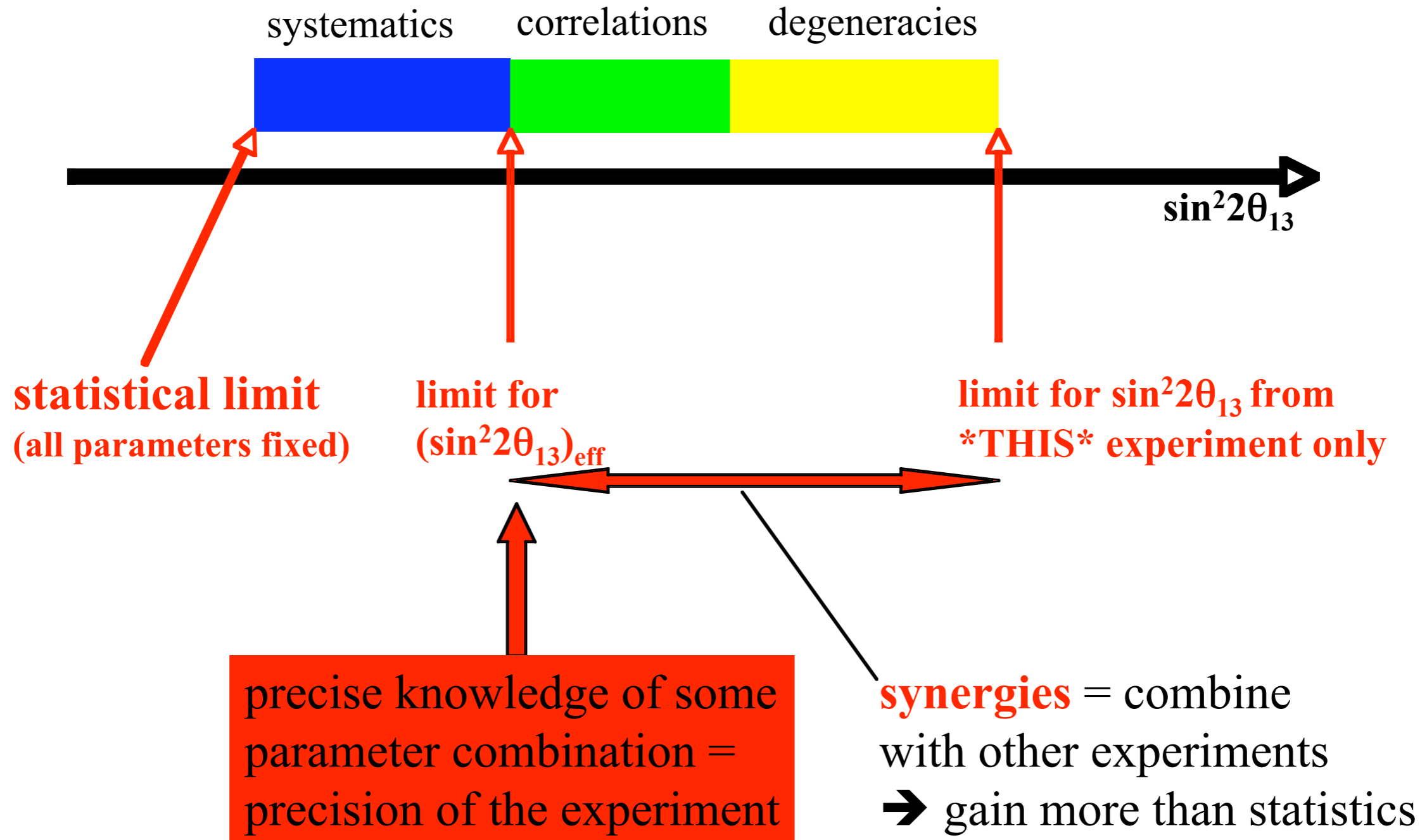
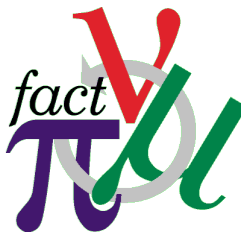
Combine Golden and Platinum Channels

Use different channels to solve the degeneracy

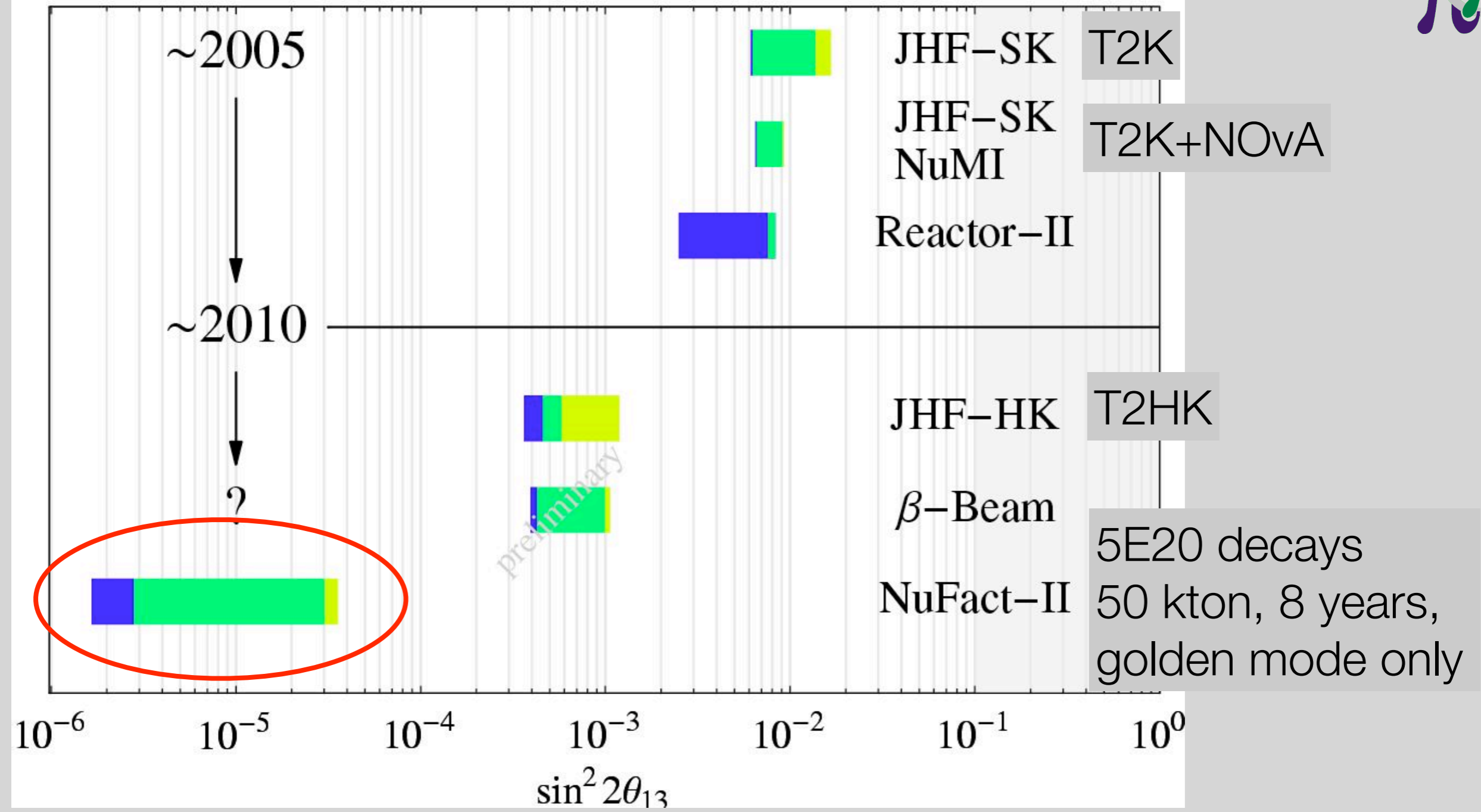
Neutrino Factory Sensitivities And Optimization



Comments : Definitions of Sensitivity Plots with Systematics, Correlations, degeneracies

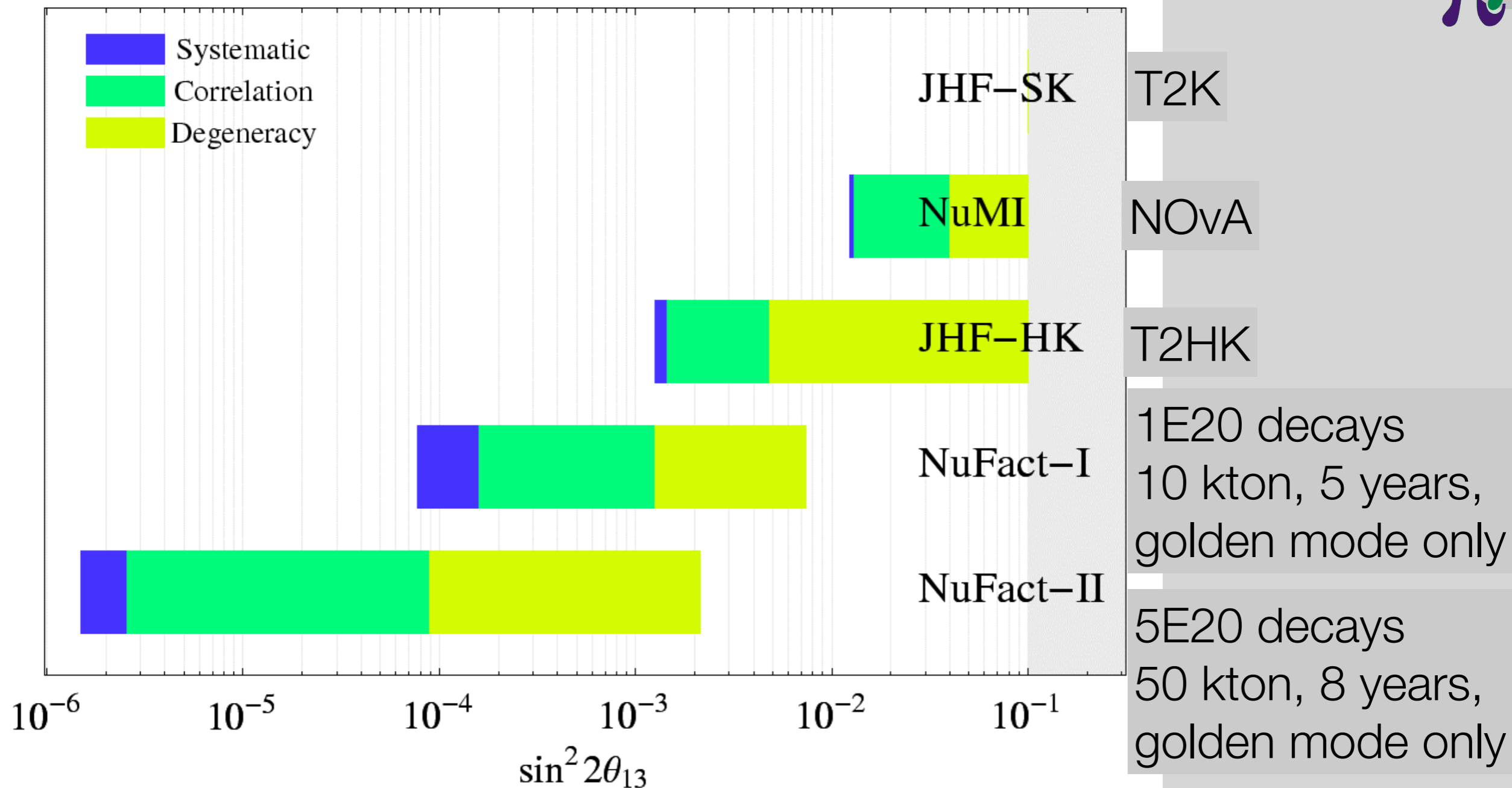


Sensitivity to $\sin^2 2\theta_{13}$ at 90% cl



Exclusion Sensitivity to $\sin^2 2\theta_{13}$

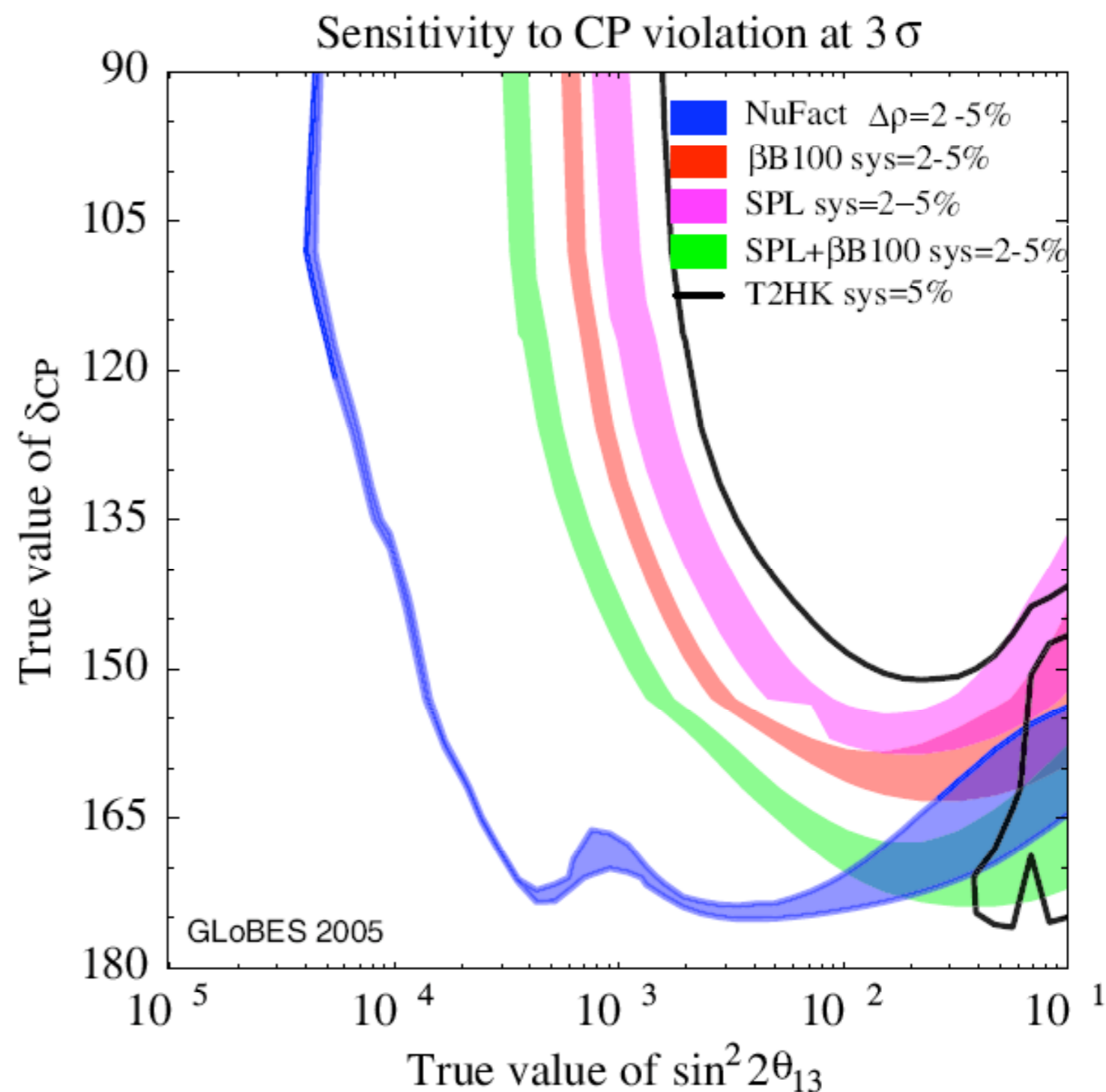
Sensitivity to the sign of Δm_{31}^2



Exclusion Sensitivity to Mass Hierarchy

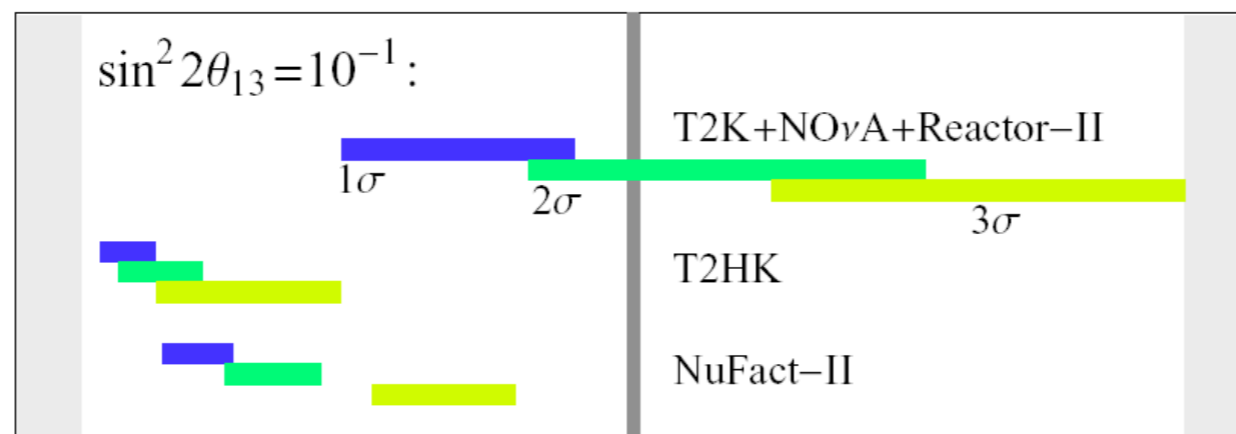
CP Coverage

- Neutrino Factory overperforms for most of the cases, except for large $\sin^2 2\theta_{13}$.
- For large $\sin^2 2\theta_{13}$, systematics dominates. In particular, uncertainty of matter effects is important.
- Need to study matter density or others.



CP (anti-) Coverage for Different $\sin^2 2\theta_{13}$

CP coverage range of possible values

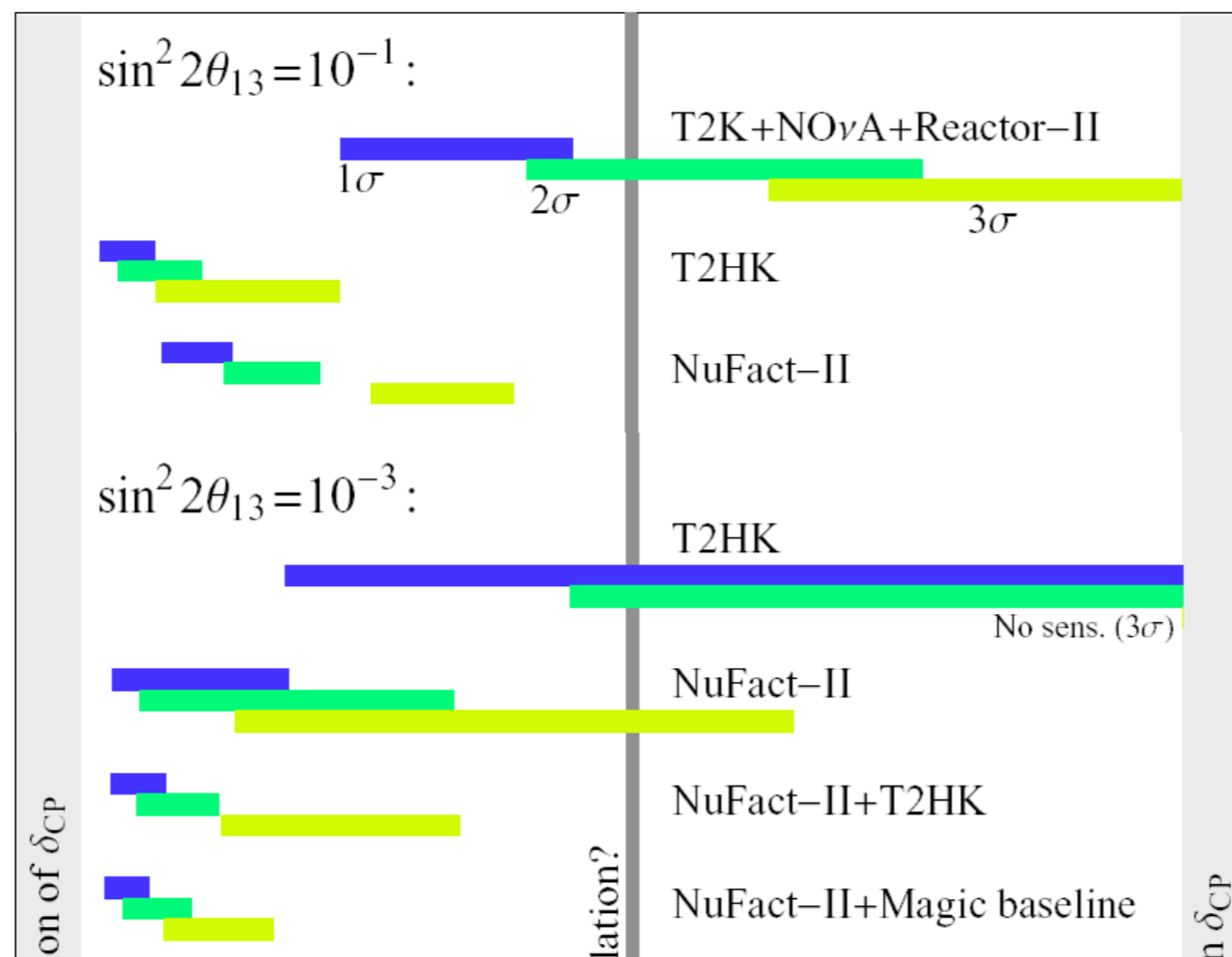


$\sin^2 2\theta_{13} = 10^{-1}$

T2HK and NF
Comparable

CP (anti-) Coverage for Different $\sin^2 2\theta_{13}$

CP coverage range of possible values



$\sin^2 2\theta_{13} = 10^{-1}$

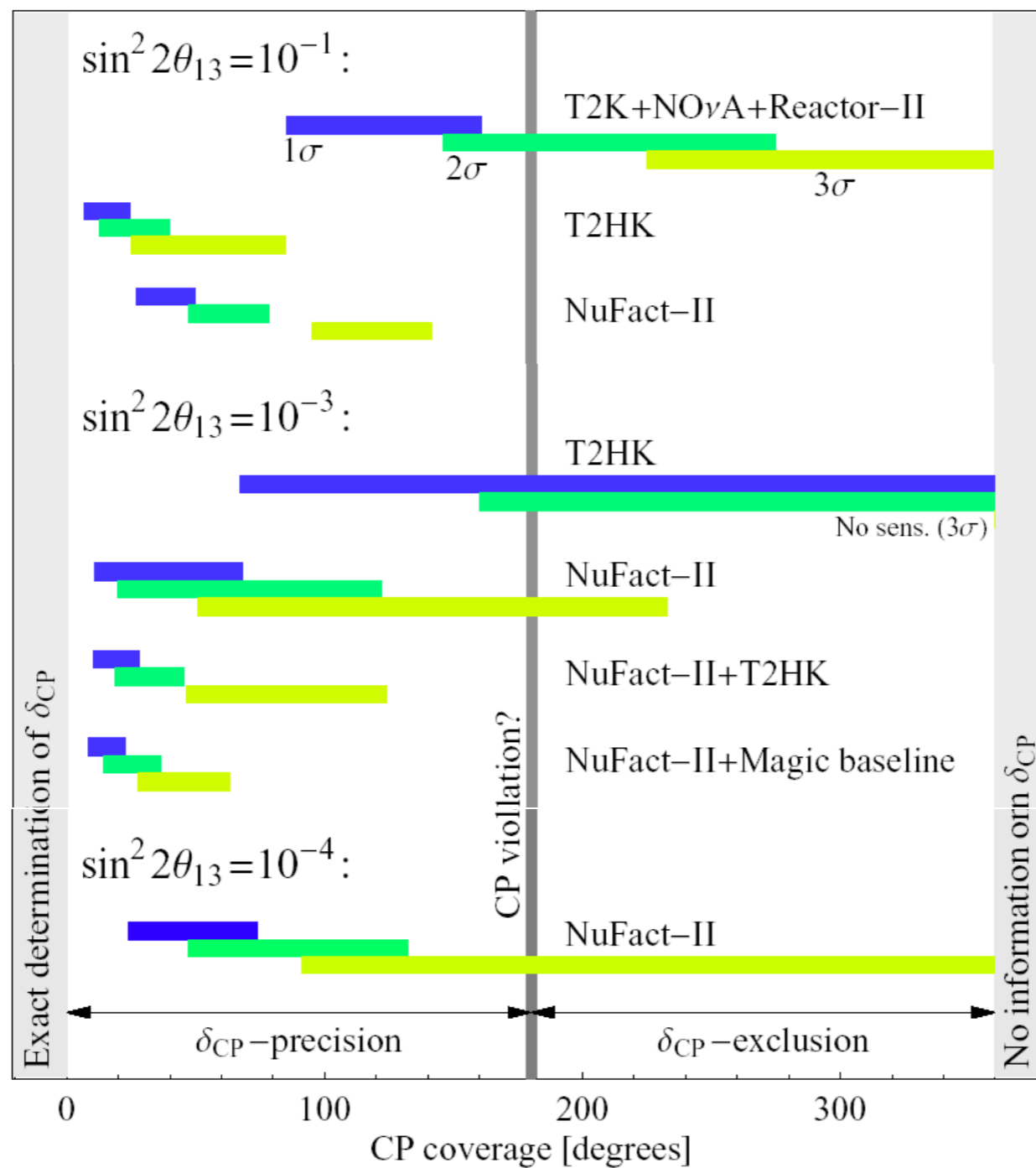
T2HK and NF
Comparable

$\sin^2 2\theta_{13} = 10^{-3}$

Synergy between
T2HK and NF

CP (anti-) Coverage for Different $\sin^2 2\theta_{13}$

CP coverage range of possible values



$\sin^2 2\theta_{13} = 10^{-1}$

T2HK and NF
Comparable

$\sin^2 2\theta_{13} = 10^{-3}$

Synergy between
T2HK and NF

$\sin^2 2\theta_{13} = 10^{-4}$

NF outperforms

NuFACT Strategy

Optimization to Resolve Degeneracies

- Combine with anti-neutrinos
- Combine with “Silver Channel” $\nu_e \rightarrow \nu_\mu$
 Donini, Meloni, Migliozzi, 2002; Autiero et al, 2004.
- Combine with “Platinum Channel” $\nu_\mu \rightarrow \nu_e$
- Use better detectors with high energy resolutions and low threshold.
- Locate the second detector at the magic baseline.
 Lipari, 2002 ; Burguet-Gastell et al. 2001; Barger,
 Mafatia, Whisnant, 2002; Huber, Winter, 2003; others

Magic Baseline (7300 - 7600 km)

$$\sin\left(\frac{AL}{2}\right) = 0 \rightarrow \sqrt{2}G_F n_e L = 2\pi \rightarrow L \sim 7300 - 7600\text{km}$$

$$P_{\nu_e \nu_\mu}^\pm(\theta_{13}, \delta) \approx X_\pm \sin^2 2\theta_{13} + \left(Y_\pm^c \cos \delta \mp Y_\pm^s \sin \delta\right) \sin 2\theta_{13} + Z$$

with X_\pm, Y_\pm^c, Y_\pm^s and Z functions of the known parameters:

$$\left\{ \begin{array}{l} X_\pm = \sin^2 \theta_{23} \left(\frac{\Delta_{23}}{\tilde{B}_\mp}\right)^2 \sin^2 \left(\frac{\tilde{B}_\mp L}{2}\right) \\ Y_\pm^c = \sin 2\theta_{23} \sin 2\theta_{12} \frac{\Delta_{12}}{A} \frac{\Delta_{23}}{\tilde{B}_\mp} \sin\left(\frac{AL}{2}\right) \sin\left(\frac{\tilde{B}_\mp L}{2}\right) \cos\left(\frac{\Delta_{23}L}{2}\right) \\ Y_\pm^s = \sin 2\theta_{23} \sin 2\theta_{12} \frac{\Delta_{12}}{A} \frac{\Delta_{23}}{\tilde{B}_\mp} \sin\left(\frac{AL}{2}\right) \sin\left(\frac{\tilde{B}_\mp L}{2}\right) \sin\left(\frac{\Delta_{23}L}{2}\right) \\ Z = \cos^2 \theta_{23} \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{A}\right)^2 \sin^2 \left(\frac{AL}{2}\right) \end{array} \right.$$

where $\Delta_{ij} = \Delta m_{ij}^2 / 2E$, $B_\mp = |A \mp \Delta_{23}|$ and A is the matter parameter.

Magic Baseline (7300 - 7600 km)

$$\sin\left(\frac{AL}{2}\right) = 0 \rightarrow \sqrt{2}G_F n_e L = 2\pi \rightarrow L \sim 7300 - 7600\text{km}$$

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Magic Baseline (7300 - 7600 km)

$$\sin\left(\frac{AL}{2}\right) = 0 \rightarrow \sqrt{2}G_F n_e L = 2\pi \rightarrow L \sim 7300 - 7600\text{km}$$

$P_{\nu_e \nu_\mu}^\pm(\theta)$ with a clean determination of $\sin^2 2\theta_{13}$ $+ Z$

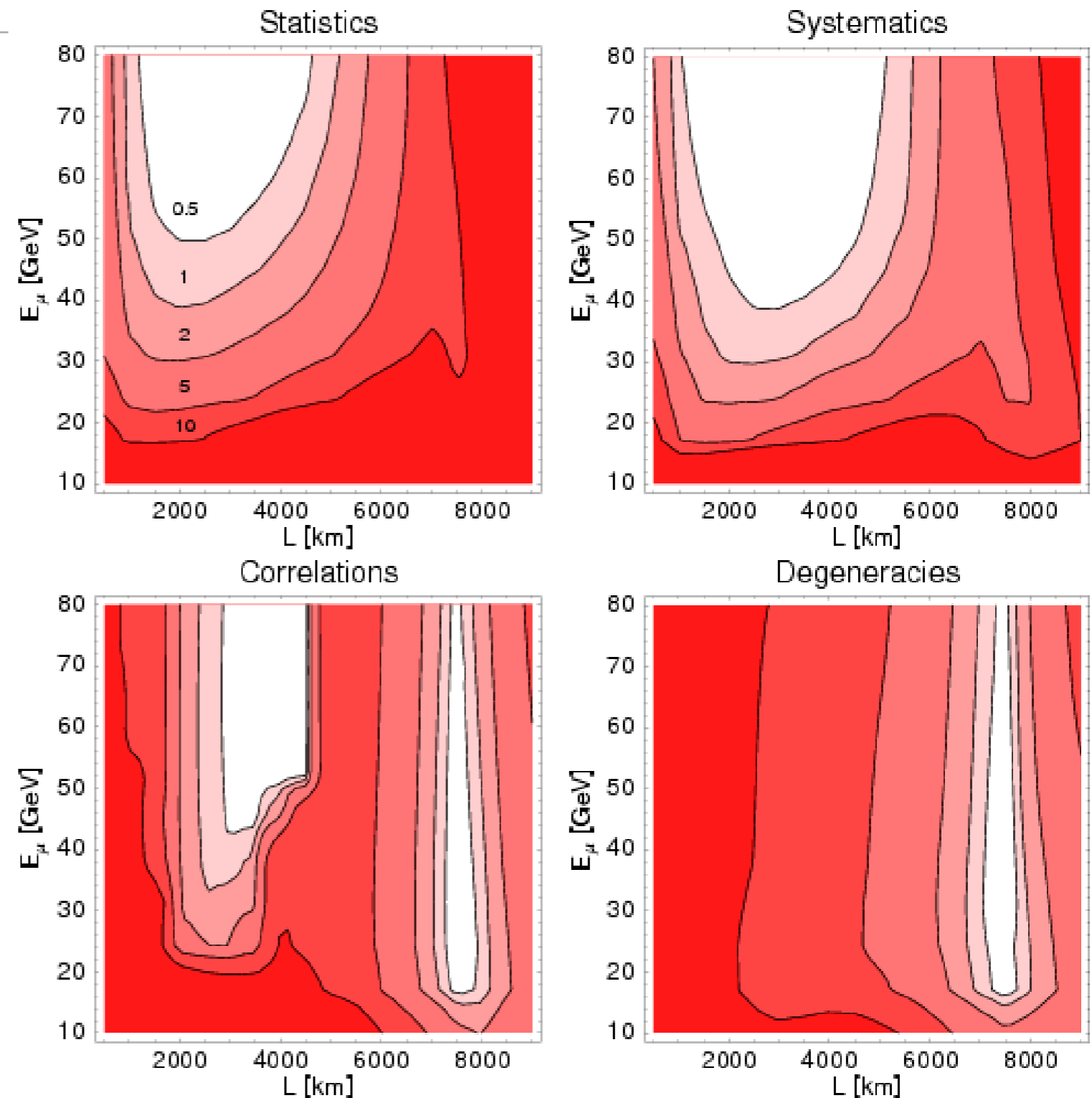
$$\left\{ \begin{array}{l} X_\pm = \boxed{\sin^2 \theta_{23}} \left(\frac{\Delta_{23}}{\tilde{B}_\mp}\right)^2 \sin^2\left(\frac{\tilde{B}_\mp L}{2}\right) \text{ only this term !} \\ Y_\pm^c = \boxed{\sin 2\theta_{23}} \sin 2\theta_{12} \frac{\Delta_{12}}{A} \frac{\Delta_{23}}{\tilde{B}_\mp} \cancel{\sin\left(\frac{AL}{2}\right)} \sin\left(\frac{\tilde{B}_\mp L}{2}\right) \boxed{\cos\left(\frac{\Delta_{23}L}{2}\right)} \\ Y_\pm^s = \boxed{\sin 2\theta_{23}} \sin 2\theta_{12} \frac{\Delta_{12}}{A} \frac{\Delta_{23}}{\tilde{B}_\mp} \cancel{\sin\left(\frac{AL}{2}\right)} \sin\left(\frac{\tilde{B}_\mp L}{2}\right) \boxed{\sin\left(\frac{\Delta_{23}L}{2}\right)} \\ Z = \boxed{\cos^2 \theta_{23}} \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{A}\right)^2 \cancel{\sin^2\left(\frac{AL}{2}\right)} \end{array} \right.$$

where $\Delta_{ij} = \Delta m_{ij}^2 / 2E$, $B_\mp = |A \mp \Delta_{23}|$ and A is the matter parameter.

$$\sin^2 2\theta_{13}$$

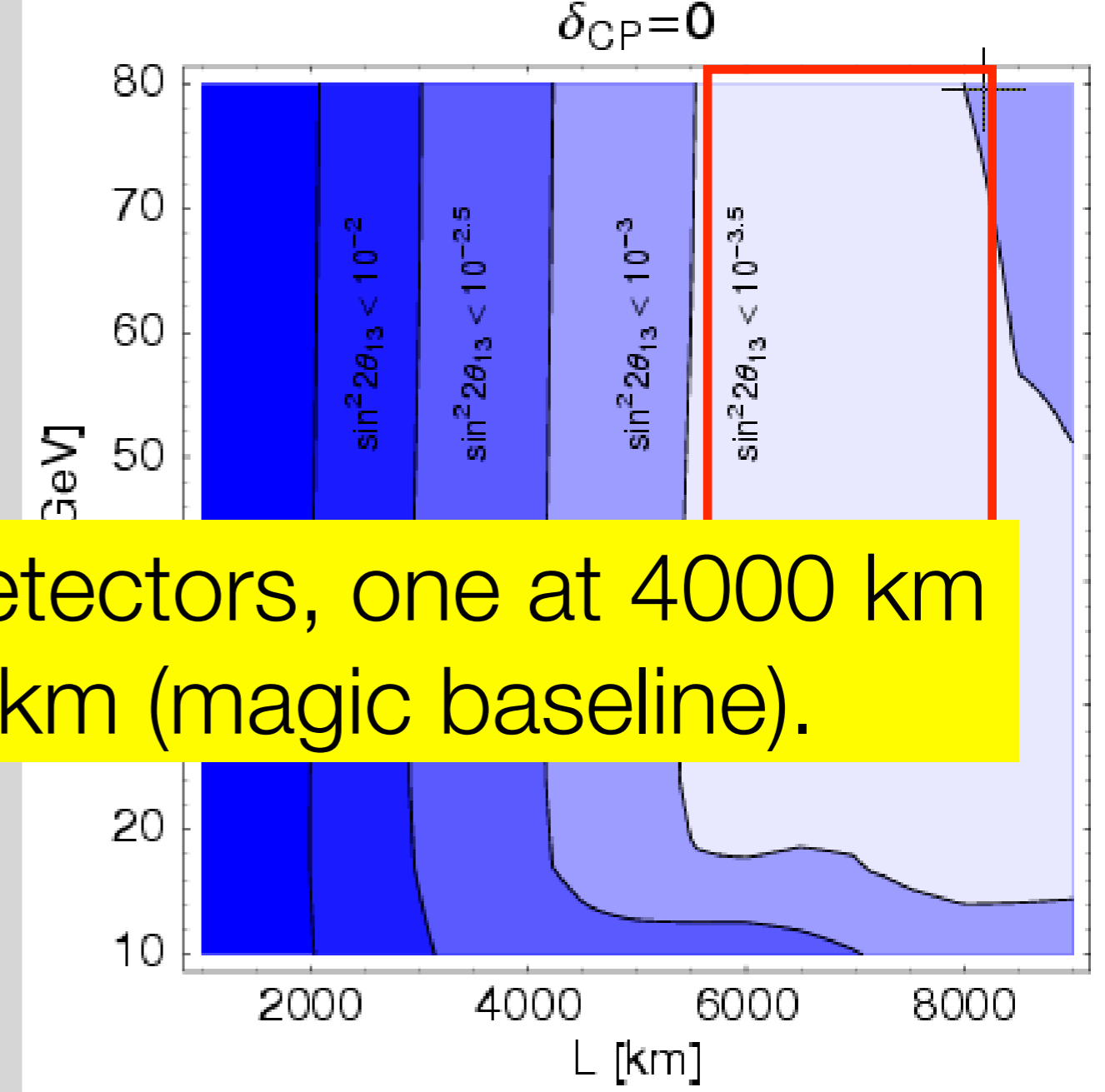
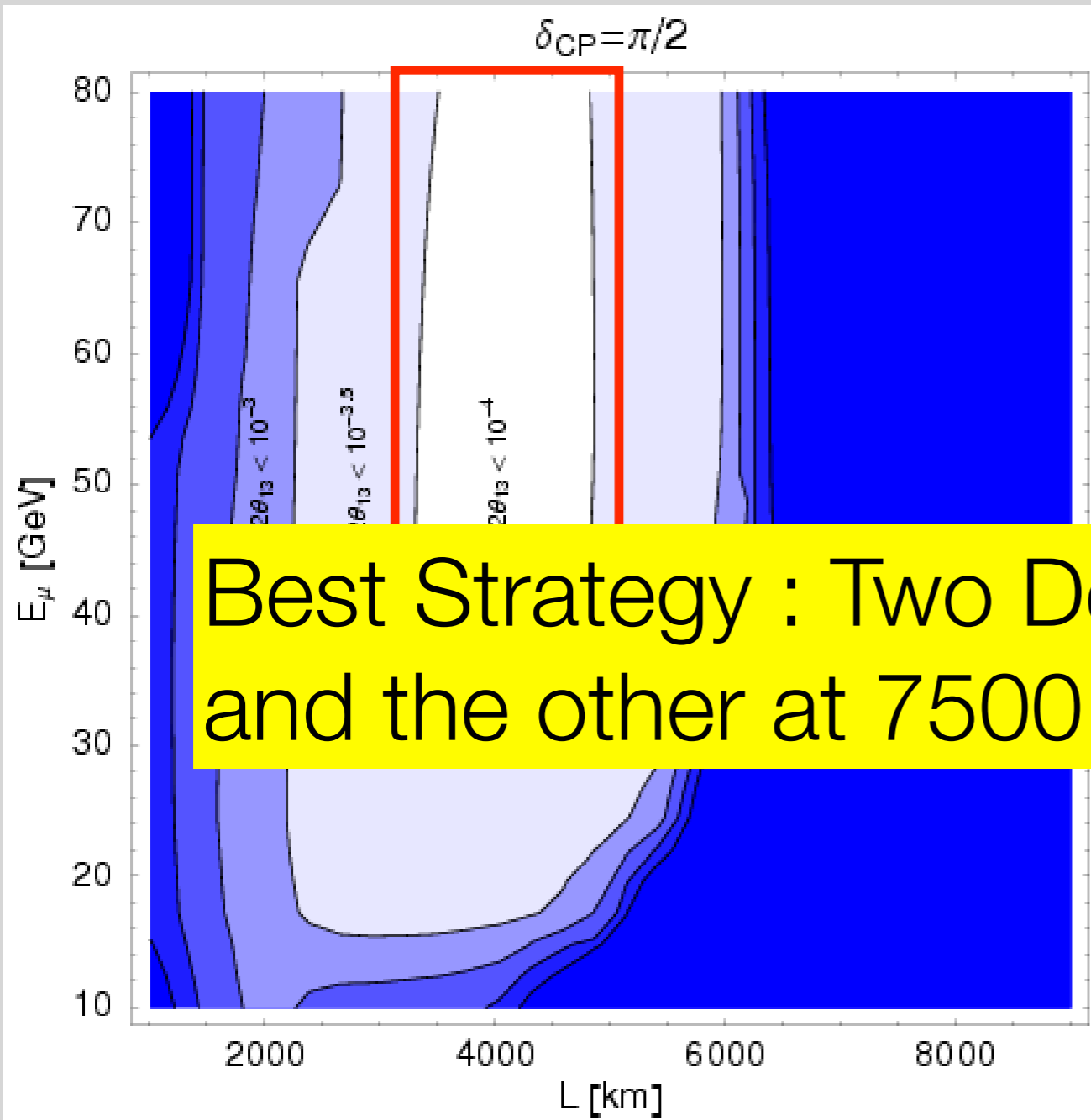
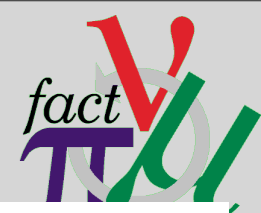
Optimization of L/E

- Magic baseline is useful to resolve degeneracy.
- L=2000 - 4000 km is good for statistics



CP Violation

Mass Hierarchy



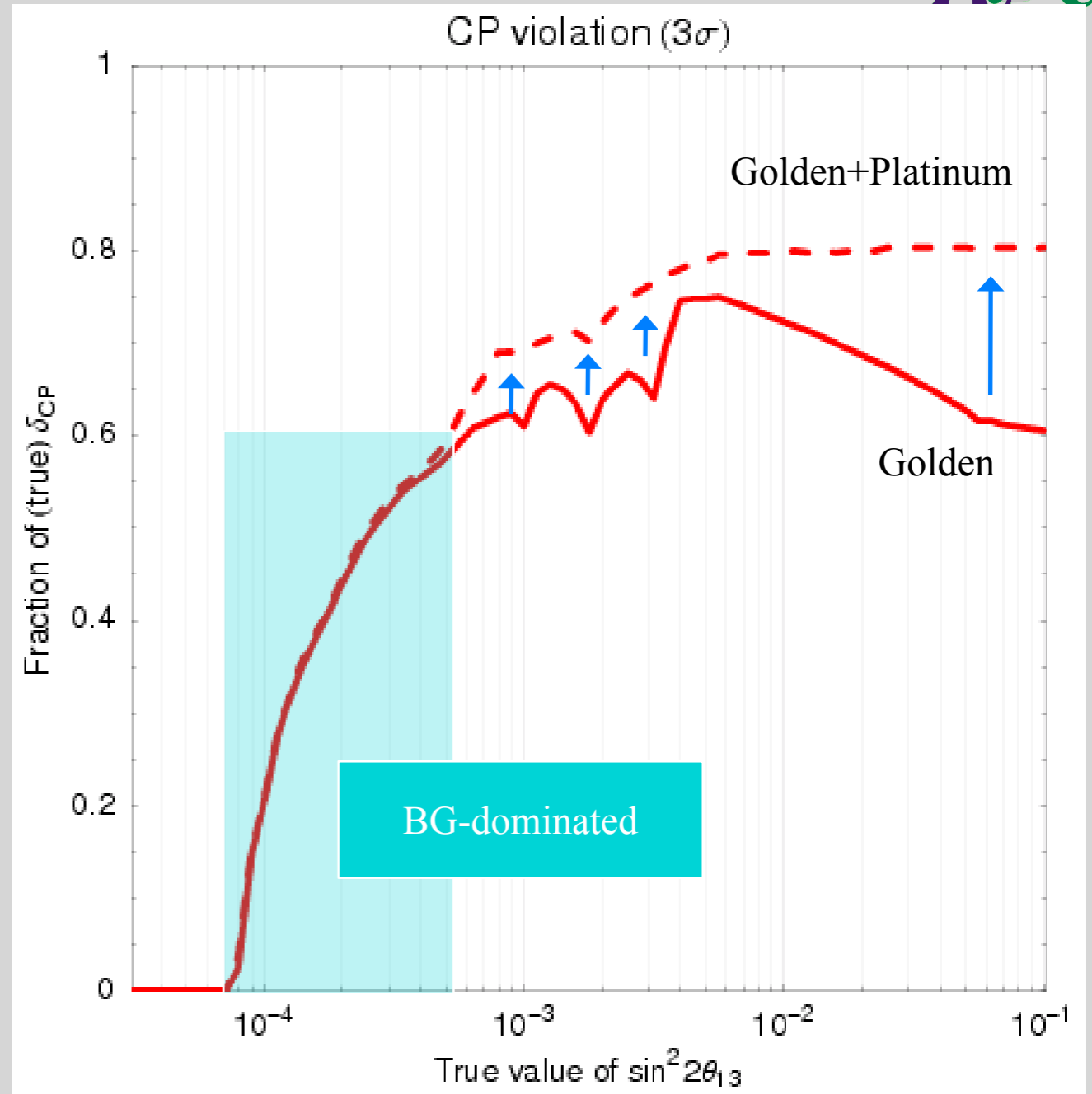
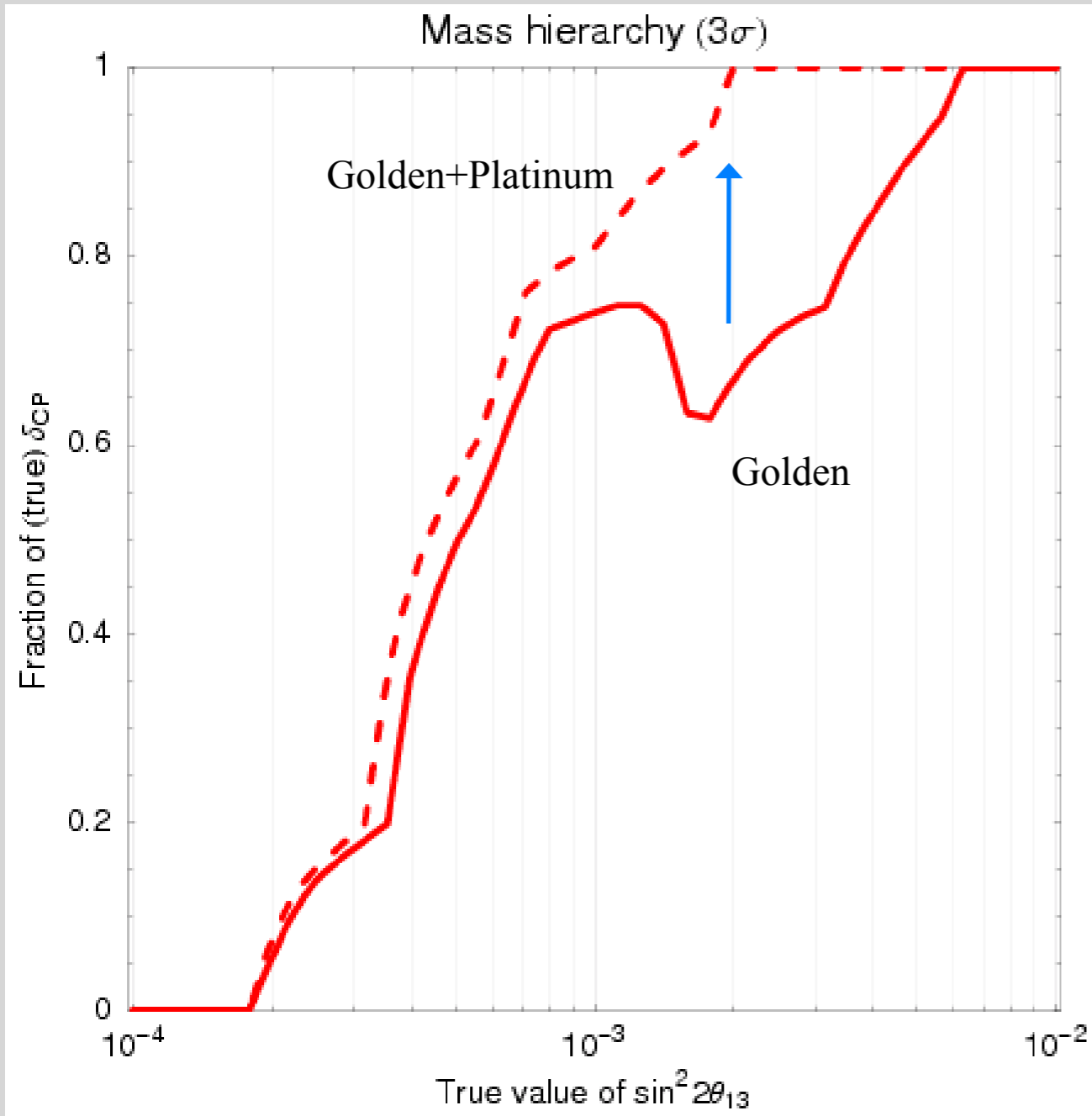
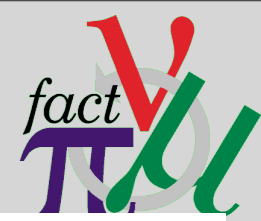
Best Strategy : Two Detectors, one at 4000 km and the other at 7500 km (magic baseline).

Optimization of L/E

L=3000 - 5000 km for CP
L>6000 km for mass hierarchy.

Mass Hierarchy

CP Violation

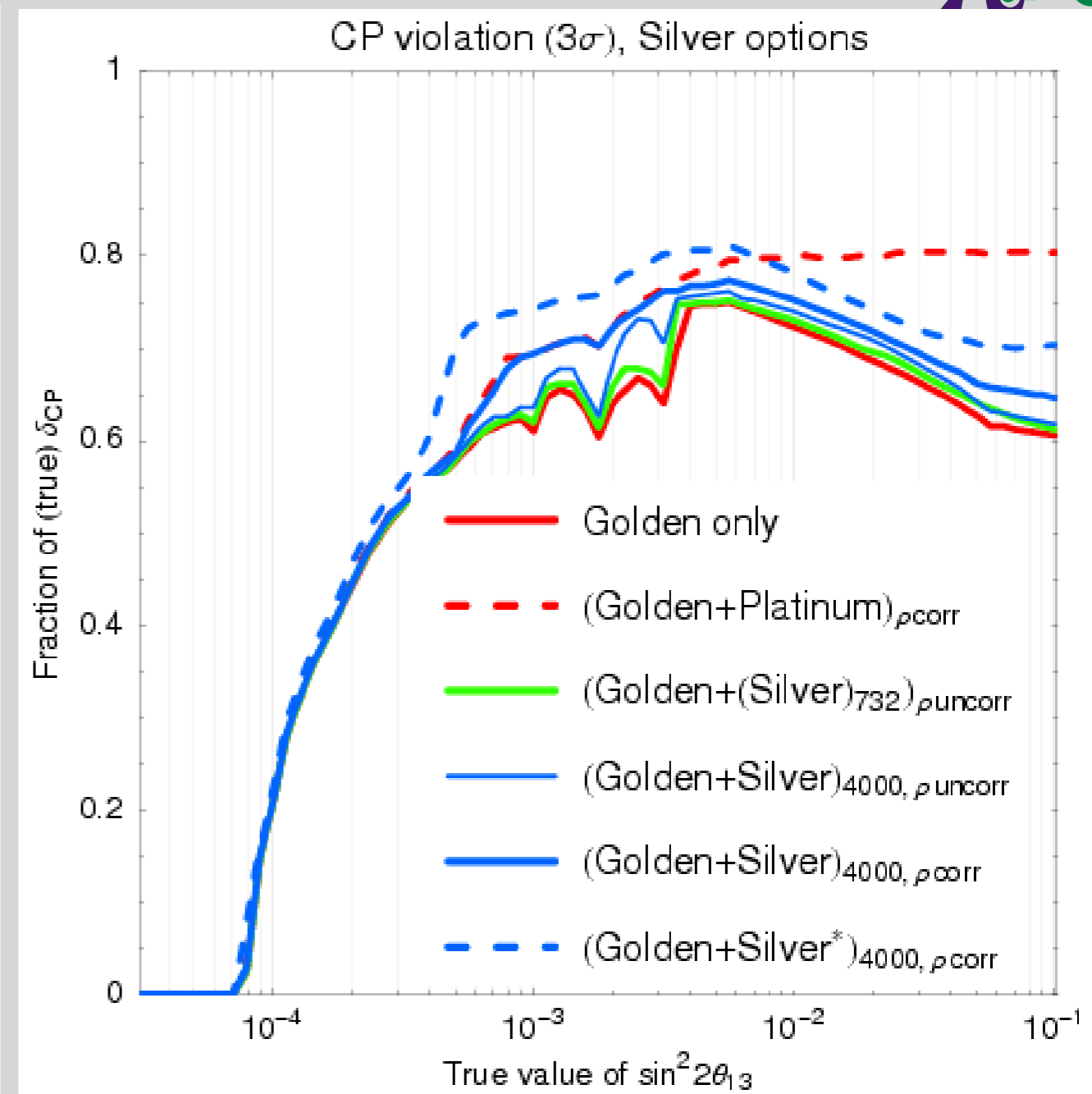
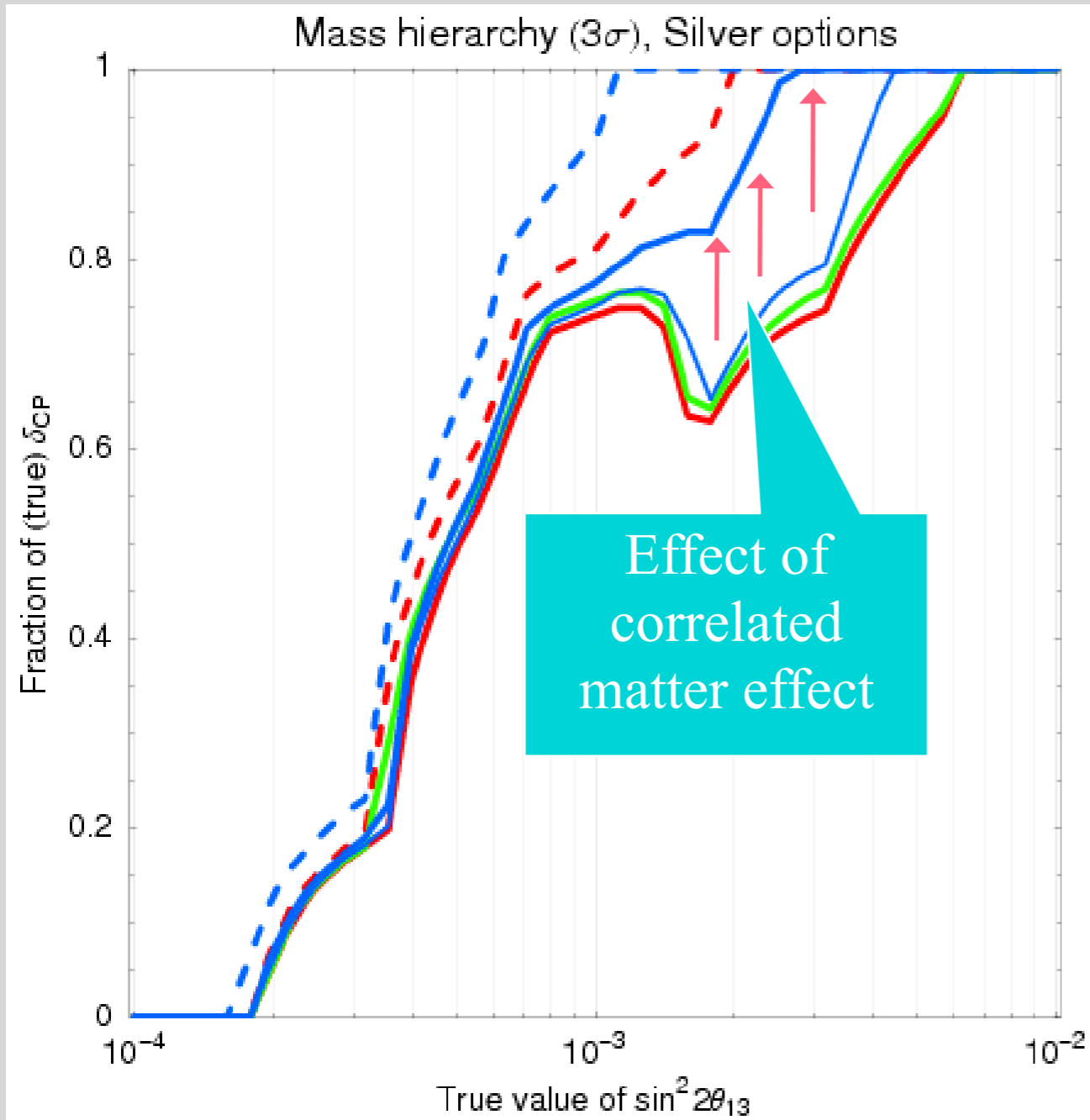
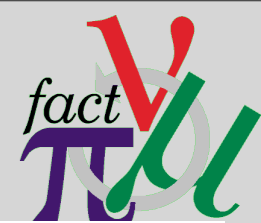


Add Platinum Channel

degeneracy can be resolved for large θ_{13}

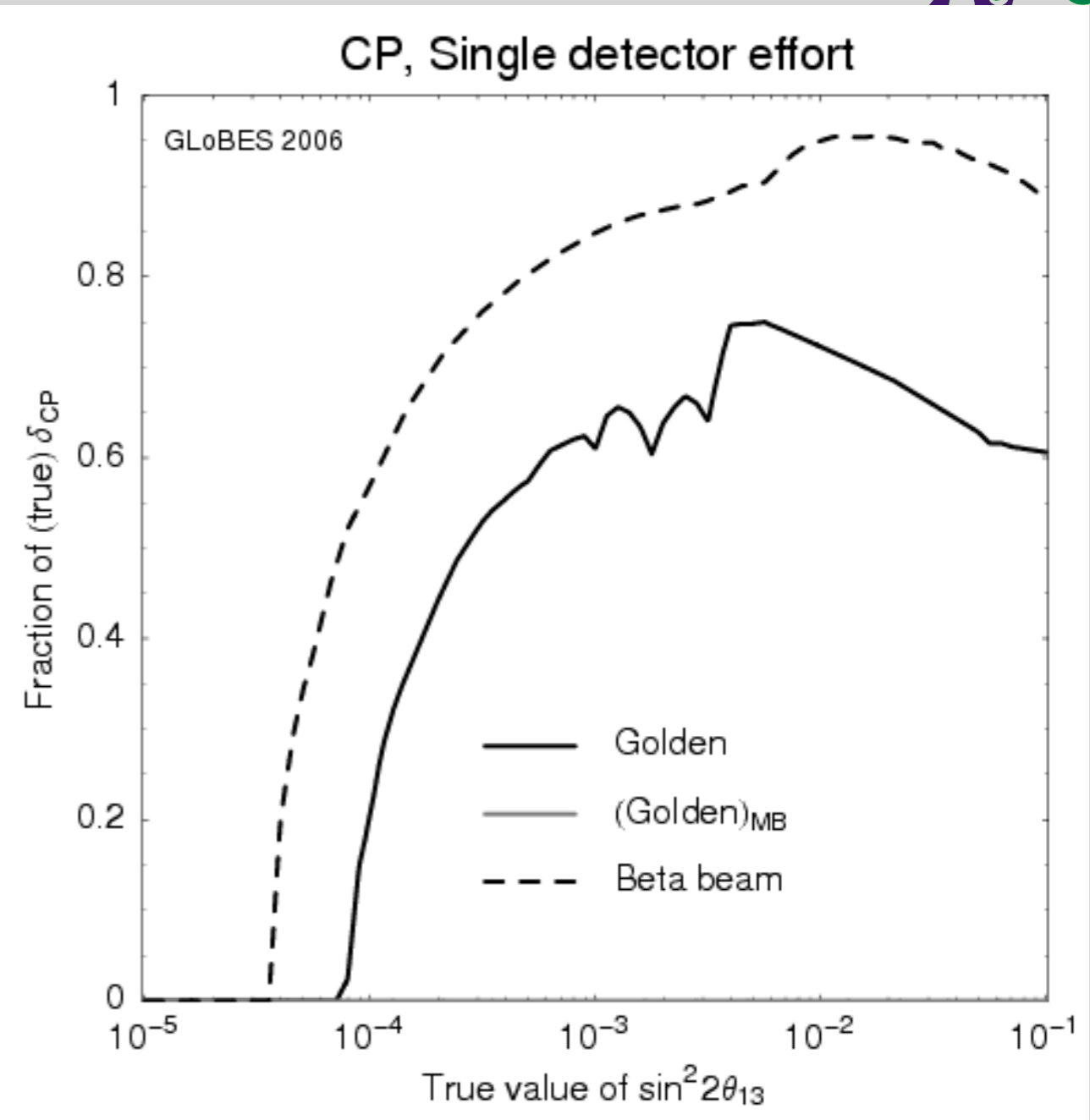
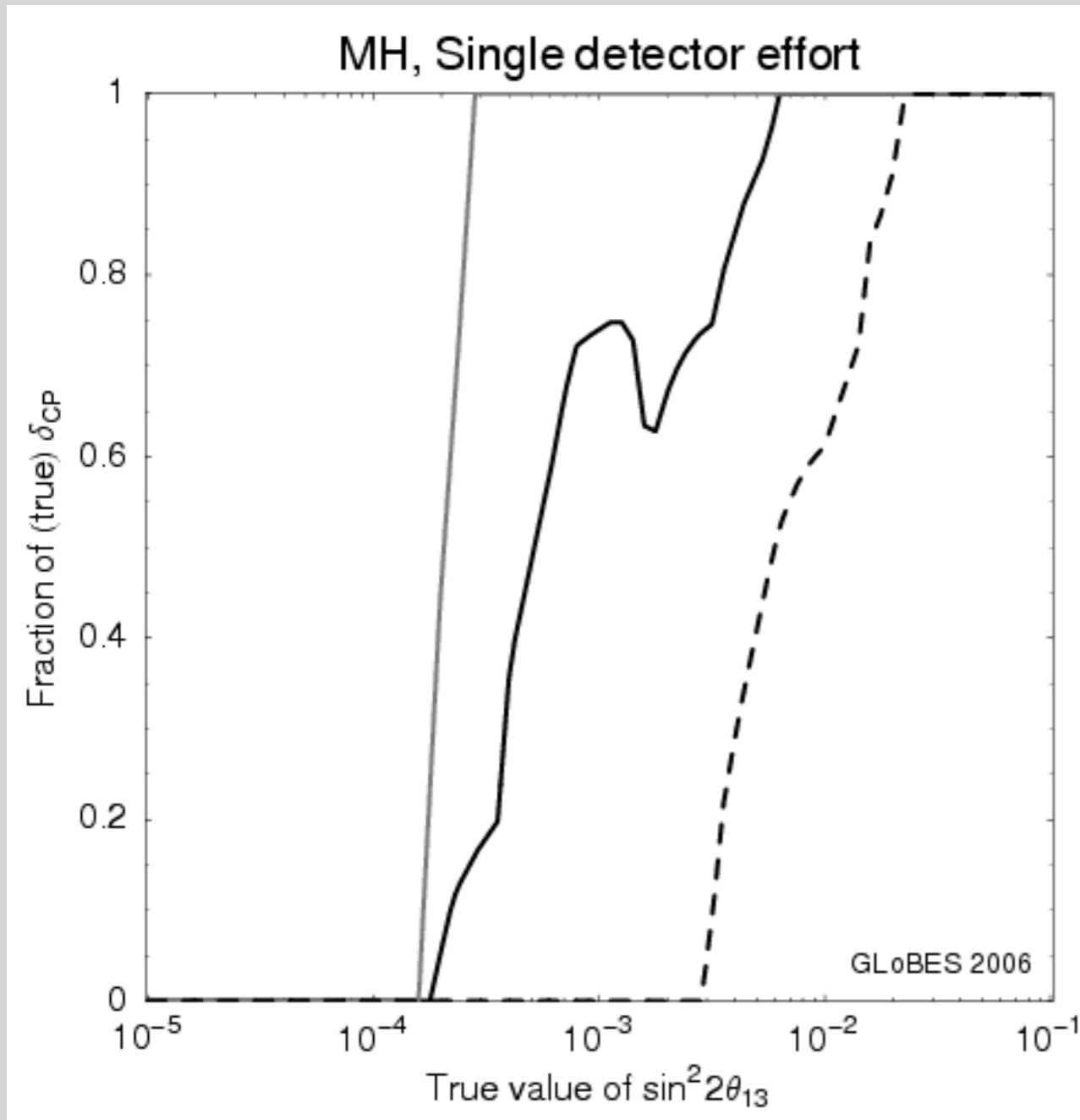
Mass Hierarchy

CP Violation



Add Silver Channels

Matter density correlation better not competitive to platinum with detector upgrade



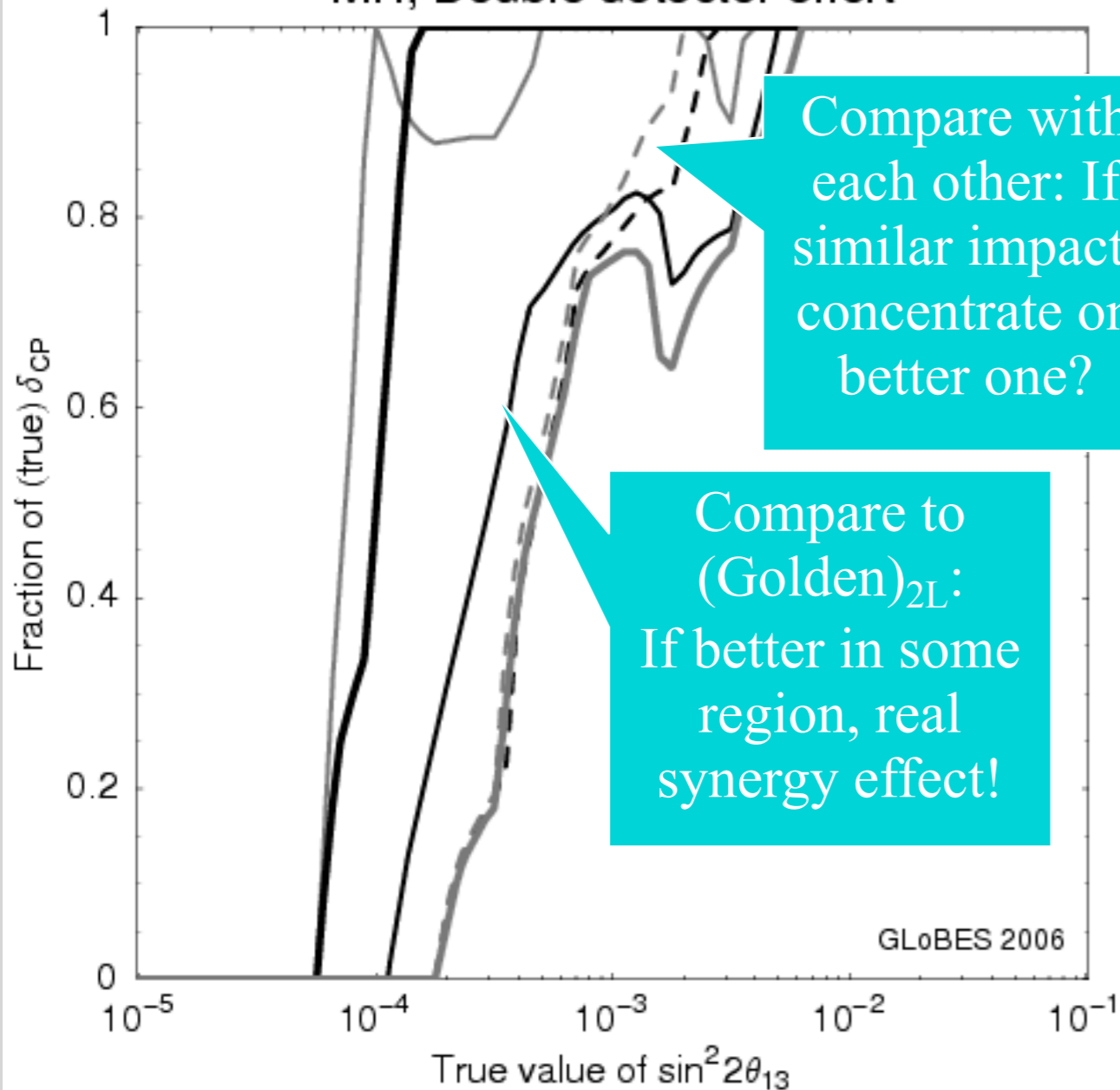
Single Detector Effort

Mass Hierarchy

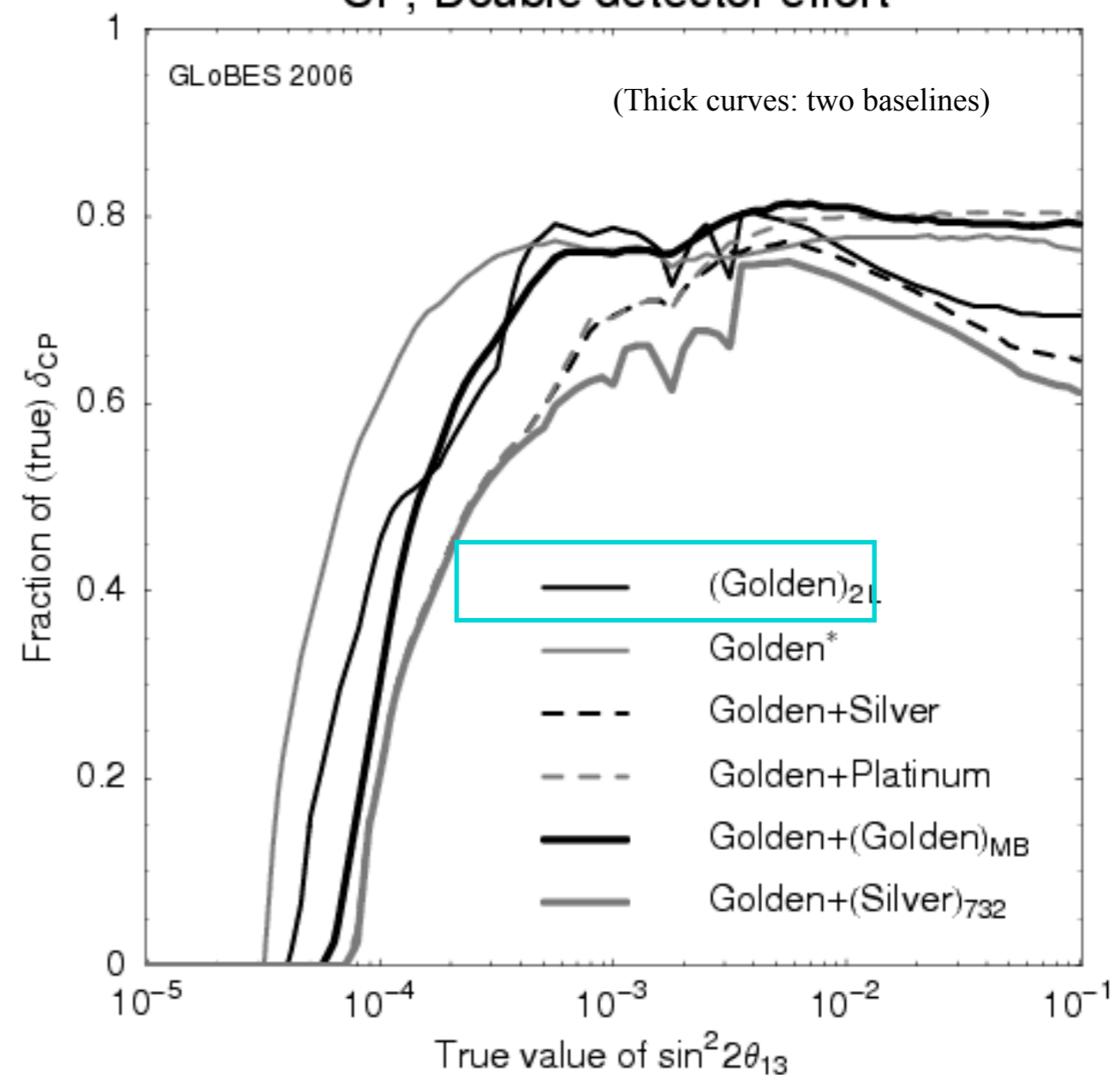
CP Violation



MH, Double detector effort



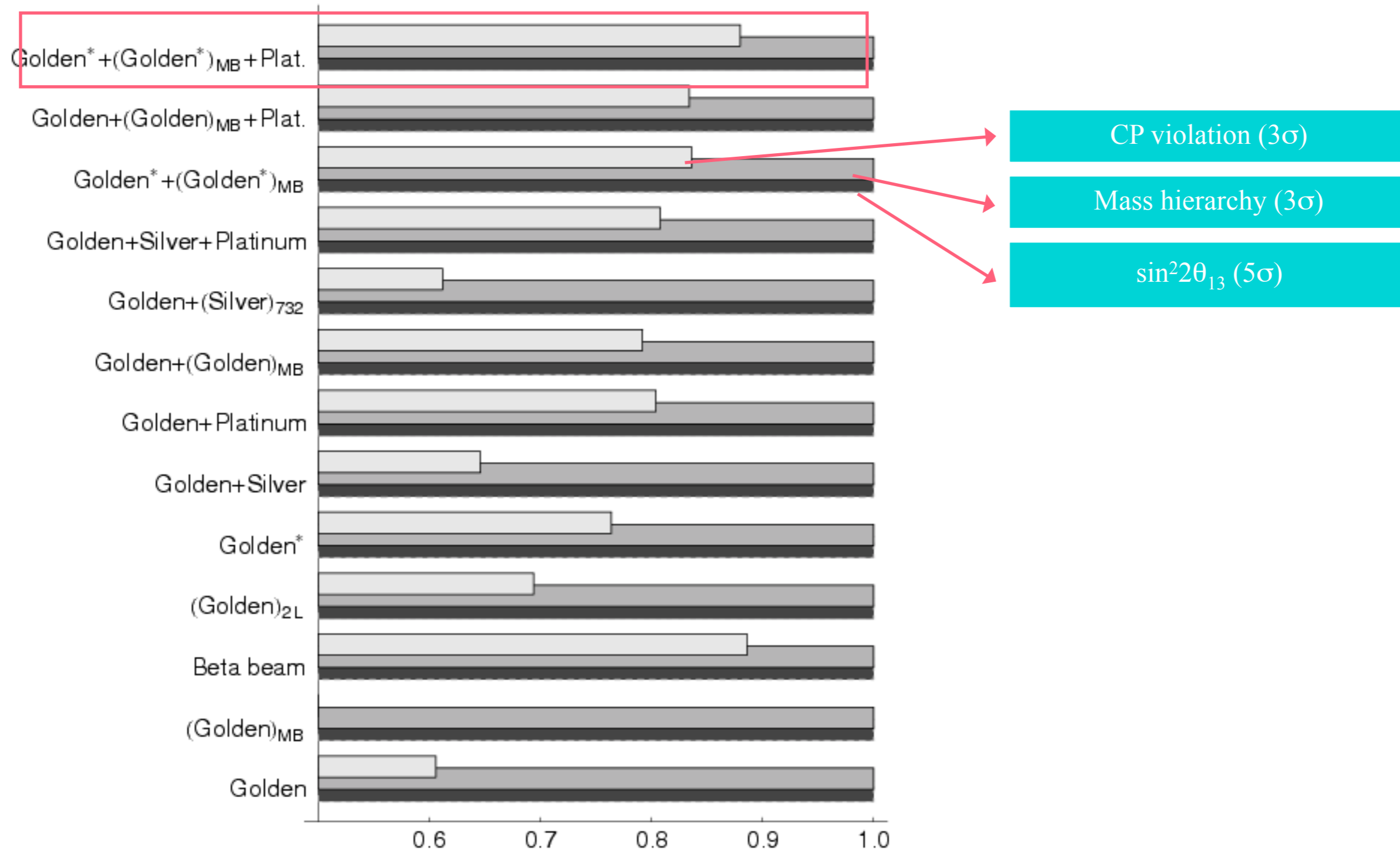
CP, Double detector effort



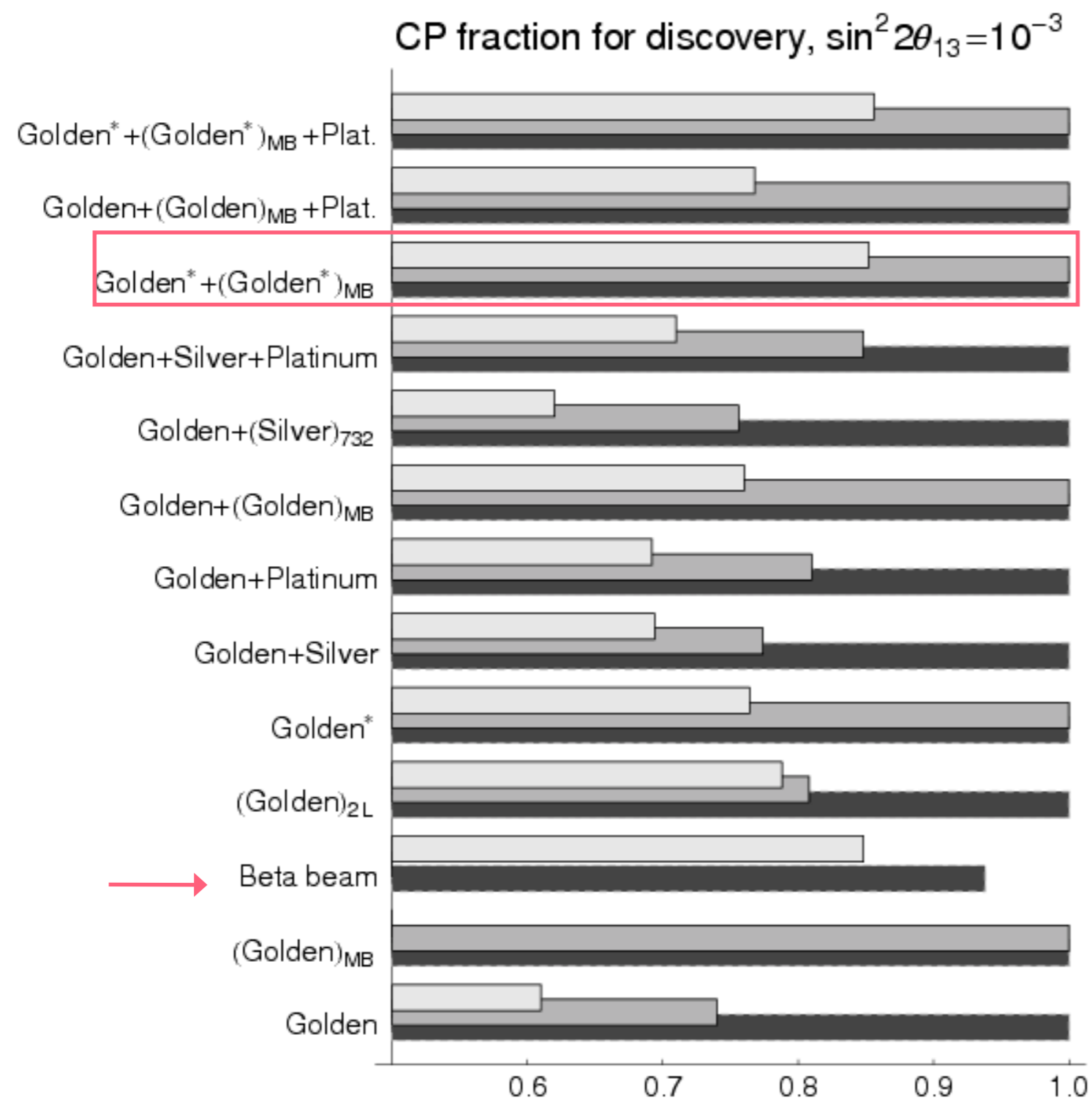
Double Detector Effort

Physics Case : Large $\sin^2 2\theta_{13}$

CP fraction for discovery, $\sin^2 2\theta_{13}=0.1$



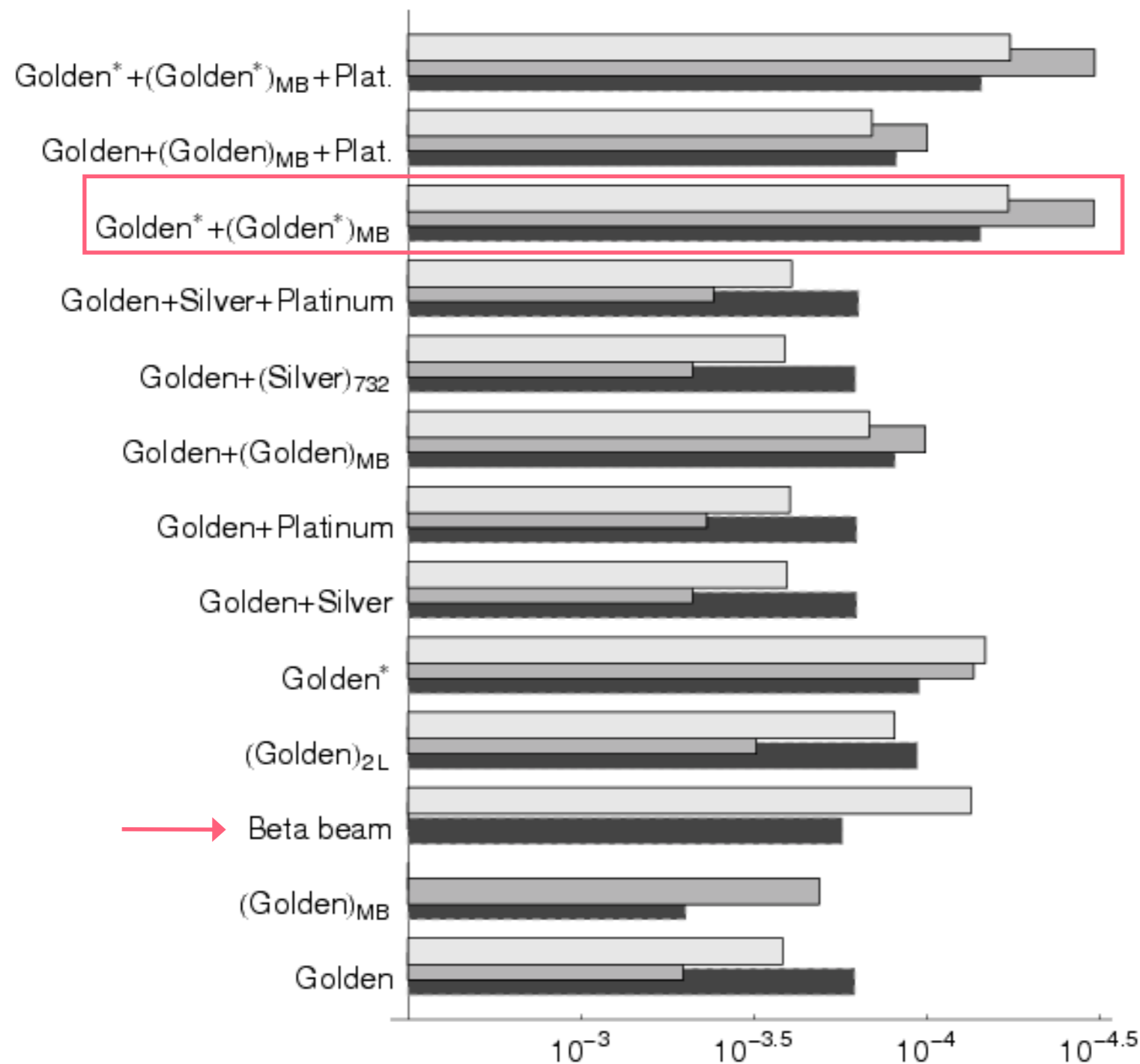
Physics Case : Intermediate $\sin^2 2\theta_{13}$



- Typical physics case for a neutrino factory.
- Improved detector and magic baseline is sufficient to make physics case.

Physics Case : Small $\sin^2 2\theta_{13}$

Discovery reaches in $\sin^2 2\theta_{13}$ for typical δ_{CP}



- Clear Physics Case for neutrino factory with eon with moderate improvement.
- Optimal reach for improved detector and magic baseline.

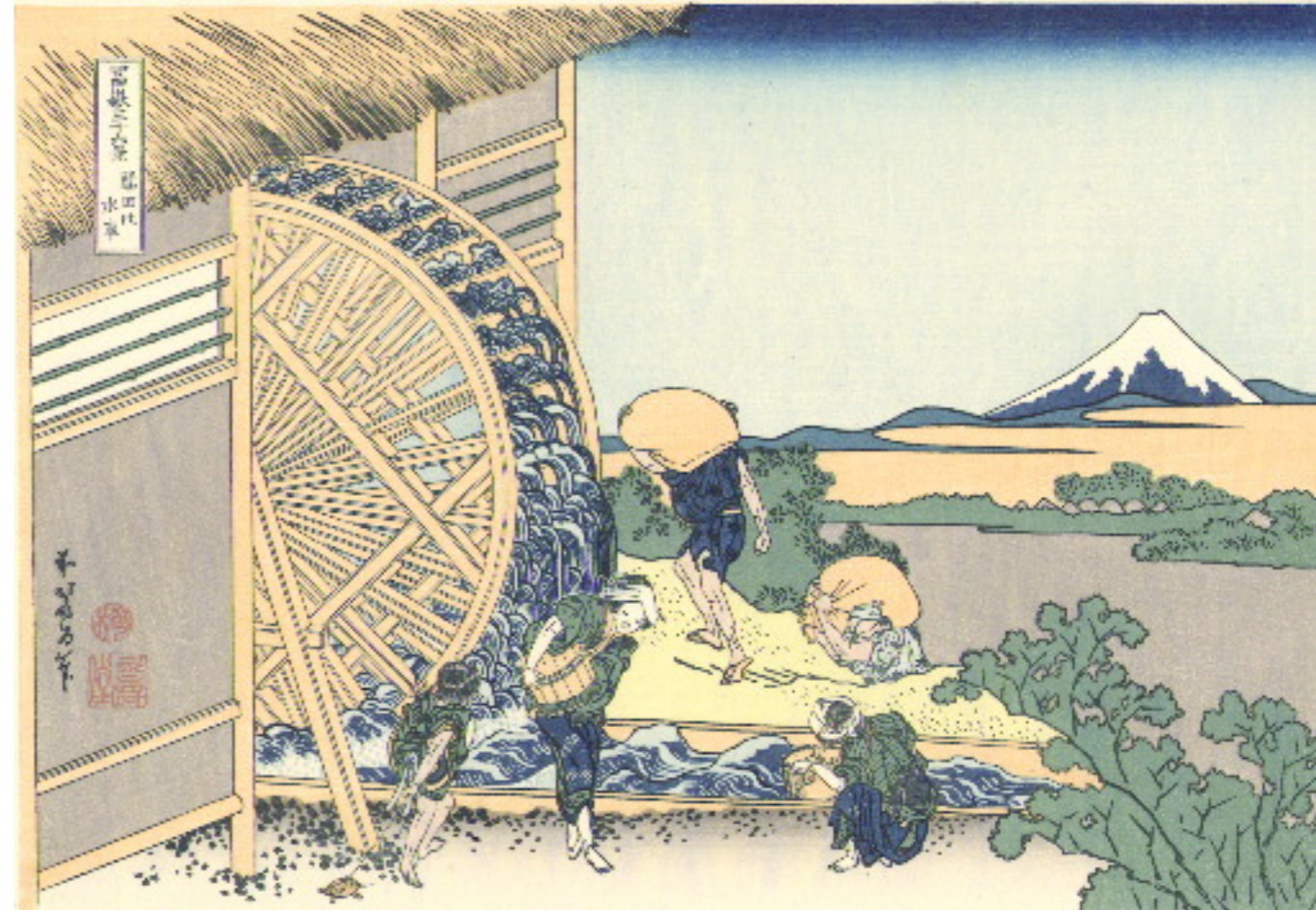
Summary of Neutrino Factory Optimization

- A lot of works have been done and more works are being undertaken.
- For $\sin^2 2\theta_{13} < 0.01$ there is a strong case for a neutrino factory, which gives the best sensitivity of CP violation.
- For $\sin^2 2\theta_{13} > 0.01$, T2HK and a neutrino factory are comparable. For a neutrino factory, systematic uncertainty, in particular from matter density, is important and should be reduced. (The study is going.)

Summary of the Second Lecture

- A neutrino factory is a next-generation highly intense neutrino facility.

Beta Beam



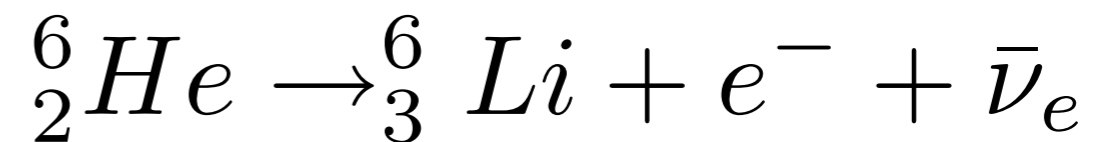
What Is a Beta Beam ?

- The “Beta beam” is a future neutrino facility which produce pure and intense (anti) electron neutrino beams, by accelerating radioactive ions and storing them in a decay ring.
- Proposed by Piero Zucchelli
 - Phys. Lett. B532 (2002) 166 - 172.
-

Ion Choice for Beta Beam

- Considerations
 - need to produce reasonable amounts of ions.
 - not too short lifetime to get reasonable intensities.
 - not too long lifetime otherwise no decays at high energy.
 -

Electron Anti-neutrinos



average energy = 1.94 MeV

lifetime = 1.94 MeV

Electron Neutrinos



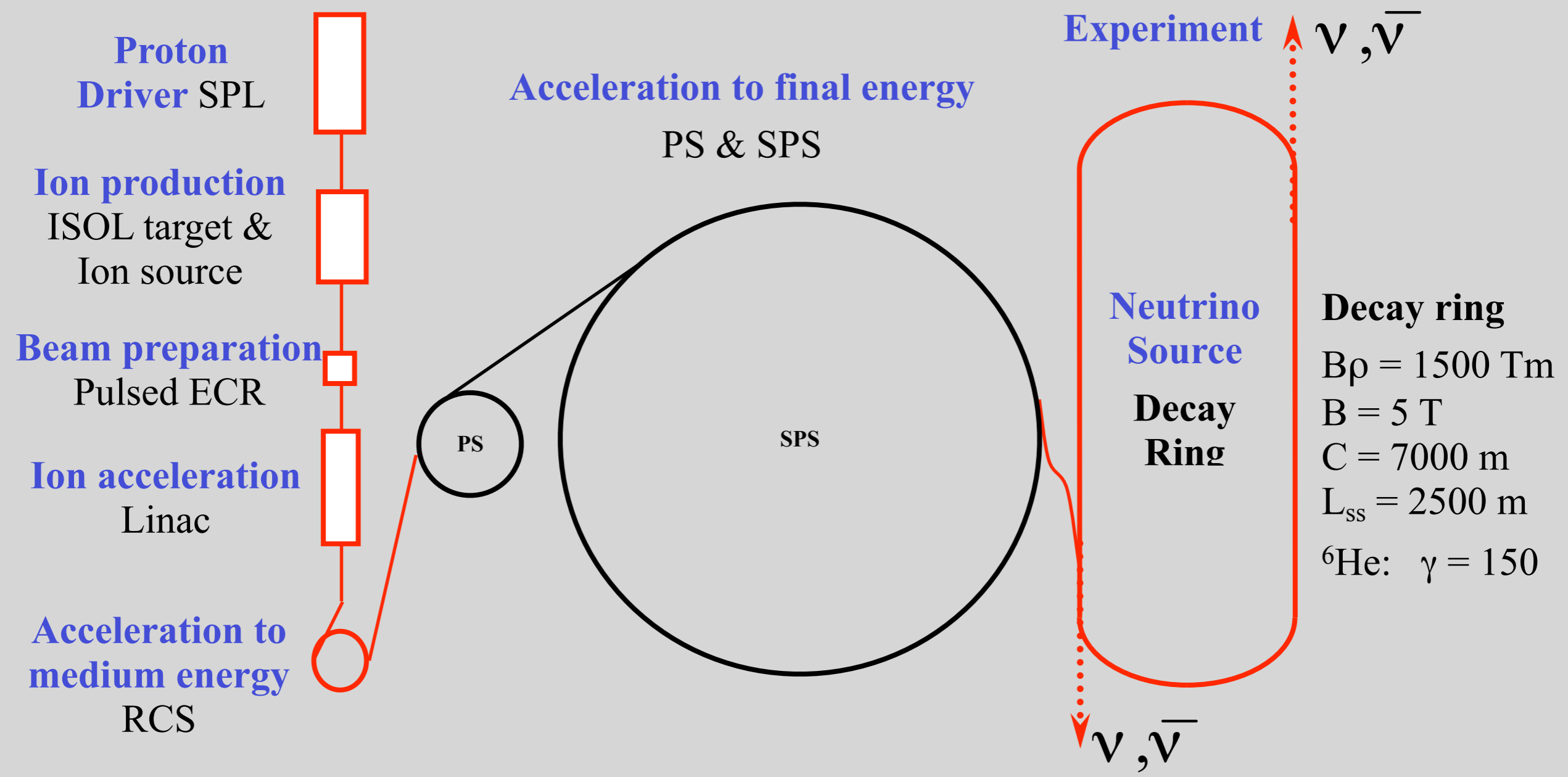
average energy = 1.86 MeV

lifetime = 1.86 MeV

Ion production

Acceleration

Neutrino source



Beta Beam Concept

Monochromatic Neutrino Beam (Electron Capture)

Decay	$T_{1/2}$	BR_ν	EC/ ν	B(GT)	E_{GR}	Γ_{GR}	Q_{EC}	E_ν	ΔE_ν
$^{148}\text{Dy} \rightarrow ^{148}\text{Tb}^*$	3.1 m	1	0.96	0.96	0.46	620	2682	2062	
$^{150}\text{Dy} \rightarrow ^{150}\text{Tb}^*$	7.2 m	0.64	1	1	0.32	397	1794	1397	
$^{152}\text{Tm}2^- \rightarrow ^{152}\text{Er}^*$	8.0 s	1	0.45	0.50	0.48	4300	520	4400	520
$^{150}\text{Ho}2^- \rightarrow ^{150}\text{Dy}^*$	72 s	1	0.77	0.56	0.25	4400	400	3000	400

