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Models of Neutrino Masses & Mixings

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Some recent work by our group

G.A., F. Feruglio, I. Masina, hep-ph/0402155,
G.A., F. Feruglio, hep-ph/0504165,hep-ph/0512103
G.A, R. Franceschini, hep-ph/0512202.

Reviews:

G.A., F. Feruglio, New J.Phys.6:106,2004 [hep-ph/0405048];
G.A., hep-ph/0410101; F. Feruglio, hep-ph/0410131

Lecture 2

Survey of basic ideas on model building
Part 1: "Normal" models

- Degenerate spectrum
  - Anarchy
  - Semianarchy
- Inverse Hierarchy
- Normal Hierarchy
- GUT models
  - $SU(5)xU(1)_F$
  - SO(10)

- Still large space for non maximal 23 mixing

2-σ interval $0.32 < \sin^2\theta_{23} < 0.62$

Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small
  probably accessible to exp.

Very small θ_{13} theoretically hard

"Normal" models: θ_{23} large but not maximal,
θ_{13} not too small (θ_{13} of order λ_C or λ_C^2)

"Exceptional" models: θ_{23} very close to maximal and/or θ_{13} very small
or: a special value for θ_{12} or....

Degenerate ν's

$m^2 >> \Delta m^2$

- Limits on m_{ee} from $0\nu\beta\beta$

$m_{ee} < 0.3\text{-}0.7$ eV (Exp)

$m_{ee} = c^2_{13}\,(m_1 c^2_{12} + m_2 s^2_{12}) + s^2_{13} m_3 \sim m_1 c^2_{12} + m_2 s^2_{12}$

are not very demanding: for $\sin^2\theta \sim 0.3$ $\cos^2\theta - \sin^2\theta \sim 0.4$

and $|m_1| \sim |m_2| \sim |m_3| \sim 1\text{-}2$ eV (with $m_1 = -m_2$) would be perfectly fine

However, WMAP&LSS: $|m| < 0.23$ eV, is very constraining

Only a moderate degeneracy is still allowed:

$m/(\Delta m^2_{atm})^{1/2} < 5$, $m/(\Delta m^2_{sol})^{1/2} < 30$.

If so, constraints from $0\nu\beta\beta$ are satisfied (both $m_1 = \pm m_2$ allowed)

It is difficult to marry degenerate models with see-saw

$$m_\nu \sim m_D^T M^{-1} m_D$$

(needs a sort of conspiracy between M and m_D)

So most degenerate models deny all relation to m_D and directly work in the $L^T L$ Majorana sector

Even if a symmetry guarantees degeneracy at the GUT scale it is difficult to protect it from corrections, e.g. from Renormalisation group running

For degenerate models there can be large ren. group corrections to mixing angles and masses in the running from M_{GUT} dow to m_W

In fact the running rate is inv. prop. to mass differences

For a 2x2 case:

$$U^{Aa} = \begin{pmatrix} c_\vartheta & -s_\vartheta \\ s_\vartheta & c_\vartheta \end{pmatrix} \qquad t = \frac{1}{16\pi^2}\log\frac{m}{m_Z}$$

$$\frac{ds_\vartheta}{dt} = \kappa A_{21}(y_{e_2}^2 - y_{e_1}^2)s_\vartheta c_\vartheta^2 \quad \frac{dc_\vartheta}{dt} = -\kappa A_{21}(y_{e_2}^2 - y_{e_1}^2)s_\vartheta^2 c_\vartheta,$$

with $A_{21} = \frac{m_2 + m_1}{m_2 - m_1}$

k = -3/2 (SM), 1 (MSSM)

$y_e = m_e/v$ (SM), $m_e/v\cos\beta$ (MSSM)

RG corrections are generally negligible and can only be large for degenerate models especially at large $\tan\beta$

The observed mixings and splitting do not fit the typical result from pure evolution.

See, for example, Chankowski, Pokorski '01

In summary: degenerate models are less favoured by now because of:

- No clear physical motivation: after all quark and charged lepton masses are very non degenerate

- Upper bounds on m^2 that limit $m^2/\Delta m^2_{atm}$
At present, no significant amount of hot dark matter is indicated by cosmology
Only a moderate degeneracy is allowed
Can be obtained as a limiting case of hierarchical models.

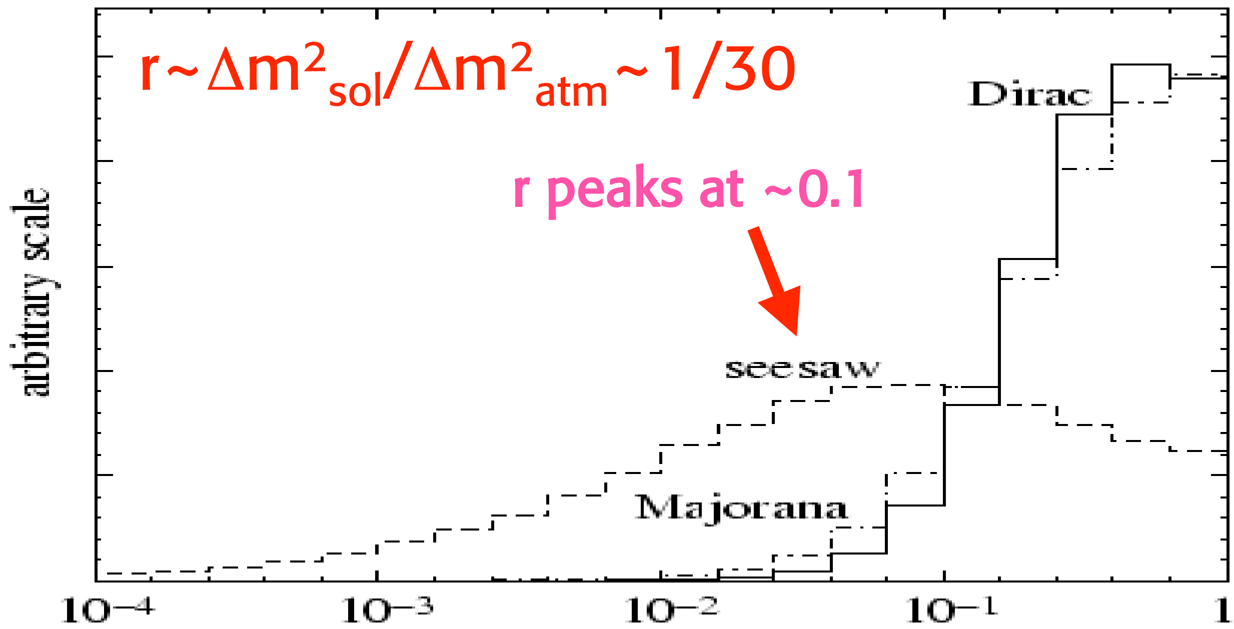
- Possible renormalization group instability
- Disfavoured by see-saw

Anarchy (or accidental hierarchy): No structure in the leptonic sector

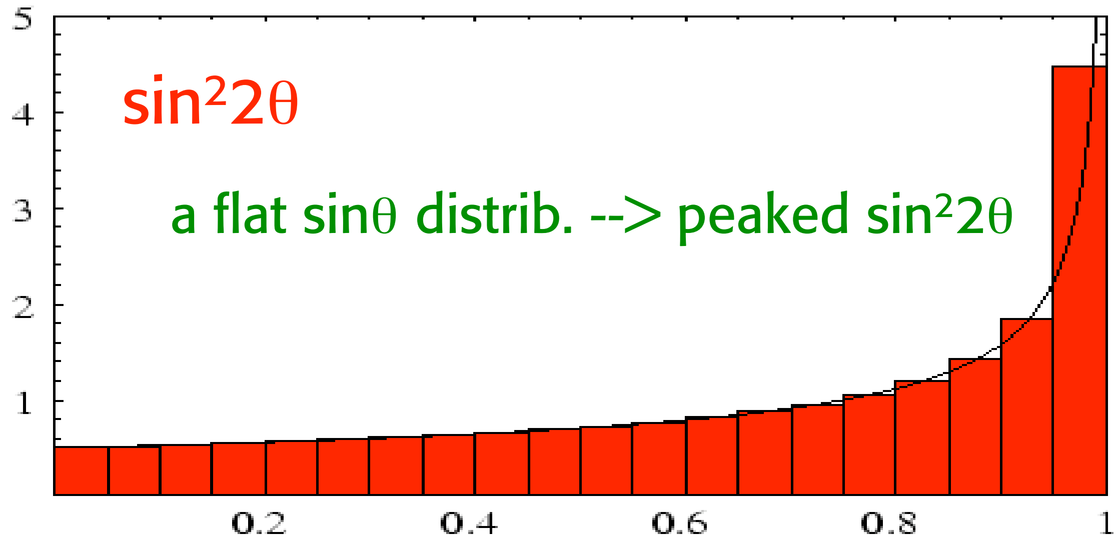
Hall, Murayama, Weiner

See-Saw:
$m_\nu \sim m^2/M$
produces hierarchy from random m,M

could fit the data



But: all mixing angles should be not too large, not too small



Marginal: predicts θ_{13} near bound

Semianarchy: no structure in 23

Consider a matrix like

$$m_\nu \sim \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix}$$

Note: $\theta_{13} \sim \varepsilon$
$\theta_{23} \sim 1$

with coeff.s of o(1) and det23~o(1)
[$\varepsilon \sim 1$ corresponds to anarchy]

After 23 and 13 rotations

$$m_\nu \sim \begin{pmatrix} \varepsilon^2 & \varepsilon & 0 \\ \varepsilon & \eta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Normally two masses are of o(1) or r ~1 and $\theta_{12} \sim \varepsilon$
But if, accidentally, $\eta \sim \varepsilon$, then r is small and θ_{12} is large.

The advantage over anarchy is that θ_{13} is small, but θ_{12} large and the hierarchy $m^2_3 >> m^2_2$ are still accidental

Ramond et al, Buchmuller et al

Hierarchy for masses and mixings via horizontal $U(1)_F$ charges.

Froggatt, Nielsen '79

Principle:

A generic mass term

$\bar{R}_1 m_{12} L_2 H$

is forbidden by U(1)

if $q_1+q_2+q_H$ not 0

q_1, q_2, q_H:
U(1) charges of
$\bar{R}_1, L_2, H$

U(1) broken by vev of "flavon" field θ with U(1) charge $q_\theta = -1$.
If vev $\theta = w$, and $w/M = \lambda$ we get for a generic interaction:

Δ_{charge}

$\bar{R}_1 m_{12} L_2 H\ (\theta/M)^{q1+q2+qH}$ $\qquad m_{12} \to m_{12}\, \lambda^{q1+q2+qH}$

Hierarchy: More Δ_{charge} -> more suppression (λ small)

One can have more flavons (λ, λ', ...)
with different charges (>0 or <0) etc -> many versions

$q(\bar{5}) \sim (2, 0, 0)$ with no see-saw --> no structure in 23

Consider a matrix like $m_\nu \sim L^T L \sim \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix}$ Note: $\theta_{13} \sim \lambda^2$, $\theta_{23} \sim 1$

with coeff.s of o(1) and det23~o(1)
[semianarchy, while $\lambda \sim 1$ corresponds to anarchy]

After 23 and 13 rotations $m_\nu \sim \begin{pmatrix} \lambda^4 & \lambda^2 & 0 \\ \lambda^2 & \eta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Normally two masses are of o(1) or r ~1 and $\theta_{12} \sim \lambda^2$
But if, accidentally, $\eta \sim \lambda^2$, then r is small and θ_{12} is large.

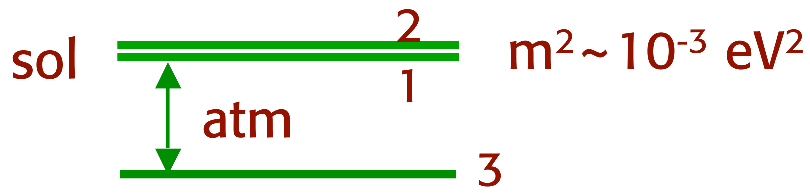
The advantage over anarchy is that θ_{13} is naturally small, but θ_{12} large and the hierarchy $m^2_3 >> m^2_2$ are accidental

Ramond et al, Buchmuller et al

⊕ With see-saw, one can do much better (see later)

Inverted Hierarchy

Zee, Joshipura et al;
Mohapatra et al; Jarlskog et al;
Frampton,Glashow; Barbieri et al
Xing; Giunti, Tanimoto.......



An interesting model:

An exact U(1) L_e-L_μ-L_τ symmetry for m_ν predicts:

(a good 1st approximation)

$$m_\nu = U m_{\nu diag} U^T = m \begin{bmatrix} 0 & 1 & x \\ 1 & 0 & 0 \\ x & 0 & 0 \end{bmatrix} \quad \text{with} \quad m_{\nu diag} = \begin{bmatrix} m' & 0 & 0 \\ 0 & -m' & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- $\theta_{13} = 0$
- $\theta_{12} = \pi/4$
- $\tan^2\theta_{23} = x^2$

θ_{sun} maximal! θ_{atm} generic

Can arise from see-saw or dim-5 $L^T H H^T L$

- 1-2 degeneracy stable under rad. corr.'s

1^{st} approximation

$$m_{\nu diag} = \begin{bmatrix} m' & 0 & 0 \\ 0 & -m' & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad m_\nu = U m_{\nu diag} U^T = m \begin{bmatrix} 0 & 1 & x \\ 1 & 0 & 0 \\ x & 0 & 0 \end{bmatrix}$$

- Data? This texture prefers θ_{sol} closer to maximal than θ_{atm}

In fact: 12-> $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow$ Pseudodirac θ_{12} maximal 23-> $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow \theta_{23} \sim o(1)$

With HO corrections: $\begin{bmatrix} 0 & 1 & x \\ 1 & 0 & 0 \\ x & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} \delta & 1 & 1 \\ 1 & \eta & \eta \\ 1 & \eta & \eta \end{bmatrix}$ (modulo o(1) coeff.s)

one gets $1 - tg^2\,\theta_{12} \sim o(\delta + \eta) \sim (\Delta m^2_{sol}/\Delta m^2_{atm})$

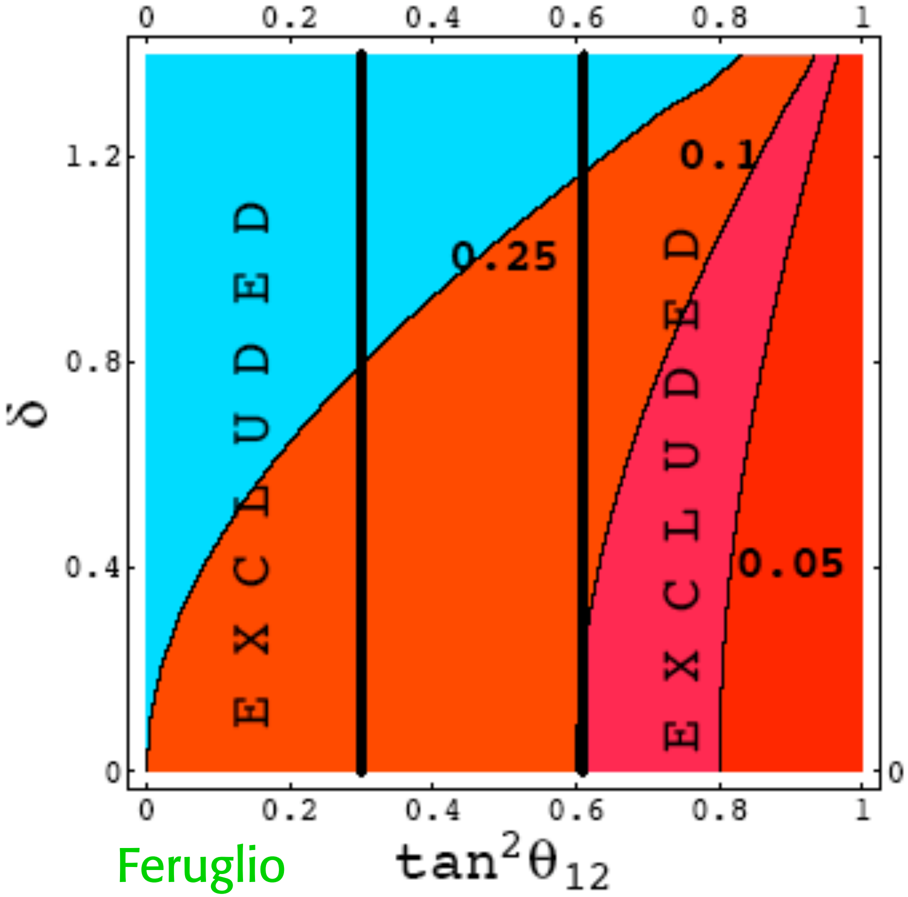
Exp. (3σ): 0.39-0.70 0.018-0.053

- In principle one can use the charged lepton mixing
  to go away from θ_{12} maximal.
  In practice constraints from θ_{13} small ($\delta\theta_{12} \sim \theta_{13}$)

Frampton et al; GA, Feruglio, Masina '04

For the corrections from the charged lepton sector, typically $|\sin\theta_{13}| \sim (1-\tan^2\theta_{12})/4\cos\delta \sim 0.15$

GA, Feruglio, Masina '04



Feruglio

$$\bar{U}_{12} = -\frac{e^{-i(\alpha_1+\alpha_2)}}{\sqrt{2}} + \frac{s^e_{12}e^{-i\alpha_2} + s^e_{13}e^{i\delta_e}}{2}$$

$$\bar{U}_{13} = \frac{s^e_{12}e^{-i\alpha_2} - s^e_{13}e^{i\delta_e}}{\sqrt{2}}$$

$$\bar{U}_{23} = -e^{-i\alpha_2}\frac{1 + s^e_{23}e^{i\alpha_2}}{\sqrt{2}}$$

Corr.'s from $s^e{}_{12}$, $s^e{}_{13}$ to U_{12} and U_{13} are of first order (2nd order to U_{23})

Thus approximate L_e-L_μ-L_τ favours θ_{13} near its bound

There is an intriguing empirical relation:

$$\theta_{12} + \theta_C = (47.0 \pm 1.7)^o \sim \pi/4$$ Raidal

Suggests bimaximal mixing in 1st approximation, corrected by charged lepton diagonalization.

Recall that

$$\lambda_C \approx 0.22 \text{ or } \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$$

While $\theta_{12} + o(\theta_C) \sim \pi/4$ is easy to realize, exactly $\theta_{12} + \theta_C \sim \pi/4$ is more difficult: no compelling model

Minakata, Smirnov

A realistic model (eg $\tan^2 2\theta_{12}$ large) with IH, θ_{13} small can be obtained from see-saw, if L_e-L_μ-L_τ is badly broken in M_{RR}

Grimus, Lavoura; G.A., Franceschini

As ν_R are gauge singlets the large soft breaking in M_{RR} does not invade all other sectors when we do rad. corr's

By adding a small flavon breaking of $U(1)_F$ symmetry with parameter $\lambda \sim m_\mu/m_\tau$ the lepton spectrum is made natural and leads to $\theta_{13} \sim m_\mu/m_\tau \sim 0.05$ or even smaller.

GA, Franceschini; hep-ph/0512202

$U(1)_F$ charges

$$l_i \sim L_e - L_\mu - L_\tau \sim (1,-1,-1)$$
$$l_{Ri} \sim (Q_e, Q_\mu, -1)$$
$$\nu_{Ri} \sim (-Q_R, Q_R, 0)$$

λ	Q_e	Q_μ	Q_τ
$0.25 \sim \xi^{\frac{1}{2}}$	7	-3	-1
$0.15 \sim \xi^{\frac{2}{3}}$	$\frac{11}{2}$	$-\frac{5}{2}$	-1
$0.06 \sim \xi$	4	-2	-1
$4 \times 10^{-3} \sim \xi^2$	$\frac{5}{2}$	$-\frac{3}{2}$	-1
$2 \times 10^{-4} \sim \xi^3$	2	$-\frac{4}{3}$	-1

Charged lepton sector

$$m^l \sim \bar{l} l_R \sim m_\tau \begin{pmatrix} \lambda^{|-1+Q_e|} & \lambda^{|-1+Q_\mu|} & \lambda^2 \\ \lambda^{|1+Q_e|} & \lambda^{|1+Q_\mu|} & 1 \\ \lambda^{|1+Q_e|} & \lambda^{|1+Q_\mu|} & 1 \end{pmatrix} \sim m_\tau \begin{pmatrix} \xi' & \xi\epsilon & \epsilon \\ \xi'\epsilon & \xi & 1 \\ \xi'\epsilon & \xi & 1 \end{pmatrix}$$

$$\epsilon \sim \lambda^2, \quad \xi \sim \lambda^{|1+Q_\mu|} \sim \frac{m_\mu}{m_\tau} \sim 6\ 10^{-2}, \quad \xi' \sim \lambda^{|-1+Q_e|} \sim \frac{m_e}{m_\tau} \sim 3\ 10^{-4}$$

Diagonalisation

$$\begin{pmatrix} \xi' & \xi\epsilon & \epsilon \\ \xi'\epsilon & \xi & 1 \\ \xi'\epsilon & \xi & 1 \end{pmatrix} = U_l \begin{pmatrix} \xi' & 0 & 0 \\ 0 & \xi & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad U_l = R_{23}(\theta_l) R_{13}(\epsilon) R_{12}(\epsilon)$$

θ_l --> large shift to θ_{23}, $O(\lambda^2)$ contrib'ns to θ_{13}, θ_{12}

Neutrino sector

$$\nu_{Ri} \sim (-Q_R, Q_R, 0)$$ Q_R=1

Dirac: $$m_\nu^D \sim \bar{\nu}_R\, l \sim m \begin{pmatrix} y_{11}\lambda^2 & a & b \\ d & y_{22}\lambda^2 & y_{23}\lambda^2 \\ y_{31}\lambda & y_{32}\lambda & y_{33}\lambda \end{pmatrix}$$

a,b,d
W,Z do not
break U(1)

Majorana:

$$m_{RR}^{-1} \sim \frac{1}{M} \begin{pmatrix} x_{11}\lambda^2 & W & x_{13}\lambda \\ W & x_{22}\lambda^2 & x_{23}\lambda \\ x_{13}\lambda & x_{23}\lambda & Z \end{pmatrix}$$

no soft breaking

$$m_{RR}^{-1} \sim \frac{1}{M} \begin{pmatrix} A & W & B \\ W & C & D \\ B & D & Z \end{pmatrix}$$

with soft breaking

$$\begin{aligned} m_\nu &= m_\nu^{DT} m_{RR}^{-1} m_\nu^D \sim m_{\nu 0} + \lambda m_{\nu 1} + \ldots \sim \\ &\sim \frac{m^2}{M} \begin{pmatrix} d^2C & adW & bdW \\ adW & a^2A & abA \\ bdW & abA & b^2A \end{pmatrix} + \\ &+ \lambda \frac{m^2}{M} \begin{pmatrix} 2y_{31}dD & y_{31}aB + y_{32}dD & y_{31}bB + y_{33}dD \\ y_{31}aB + y_{32}dD & 2y_{32}aB & y_{32}bB + y_{33}aB \\ y_{31}bB + y_{33}dD & y_{32}bB + y_{33}aB & 2y_{33}bB \end{pmatrix} + o(\lambda^2) \end{aligned}$$

after see-saw

Various stages:

r=0

exact U(1) $\quad m_1 = -m_2; \quad m_3 = 0; \quad \theta_{13} = 0; \quad \theta_{12} = \frac{\pi}{4}; \quad \tan\theta_{23} = \frac{b}{a}$

$\longrightarrow$ pure L_e-L_μ-L_τ

only soft breaking (λ=0)

$$m_3 = 0; \quad m_1 + m_2 = \bar{C} + \bar{A}; \quad m_1 - m_2 = \sqrt{(\bar{A}-\bar{C})^2 + \bar{W}^2};$$
$$\theta_{13} = 0; \qquad \tan\theta_{23} = \left|\frac{b}{a}\right|; \qquad \tan^2 2\theta_{12} \sim \frac{\bar{W}^2}{(\bar{A}-\bar{C})^2}$$

with

$$\bar{A} = \frac{m^2}{M} A\,(a^2+b^2); \quad \bar{C} = \frac{m^2}{M} C\,d^2; \quad \bar{W}^2 = \frac{m^2}{M} 4W^2(a^2+b^2)d^2$$

$\theta_{12} \longrightarrow \frac{|\bar{A}-\bar{C}|}{|\bar{W}|} \sim 0.40.$

requires large soft breaking

$r \sim 1/30 \longrightarrow |\frac{\bar{A}+\bar{C}}{\bar{A}-\bar{C}}| \cos\delta \sim 0.02$

requires some fine tuning

Summarising: this model with IH

In the limit of exact $U(1)_F$ $\theta_{12}=\pi/4$ and r , $\theta_{13,}$ as well as m_e/m_τ and m_μ/m_τ (for our choice of charges) are all zero.

In general a small symmetry breaking will make them different from zero but small. And θ_{12} will only be sligthly displaced from $\pi/4$ (bad)

A large soft explicit mixing in the M_{RR} sector can decouple θ_{12}, which gets a large shift, from θ_{13}, m_e/m_τ and m_μ/m_τ which remain small.

The only remaining imperfection is that a moderate fine tuning is needed for r.

Normal hierarchy

- A crucial point: in the 2-3 sector we need both
  large m_3-m_2 splitting and large mixing.

  $m_3 \sim (\Delta m^2_{atm})^{1/2} \sim 5\ 10^{-2}$ eV
  $m_2 \sim (\Delta m^2_{sol})^{1/2} \sim 8\ 10^{-3}$ eV

- The "theorem" that large Δm_{32} implies small mixing
  (pert. th.: $\theta_{ij} \sim 1/|E_i - E_j|$)
  is not true in general: all we need is (sub)det[23]~0

- Example: $m_{23} \sim \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$

  Det = 0; Eigenvl's: 0, $1+x^2$
  Mixing: $\sin^2 2\theta = 4x^2/(1+x^2)^2$

So all we need are natural mechanisms for det[23]=0

For x~1
large splitting
and large mixing!

Examples of mechanisms for Det[23]~0

based on see-saw: $m_\nu \sim m^T_D M^{-1} m_D$

1) A ν_R is lightest and coupled to μ and τ

King; Allanach; Barbieri et al......

$$M \sim \begin{bmatrix} \varepsilon & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow M^{-1} \sim \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_\nu \sim \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} \approx 1/\varepsilon \begin{bmatrix} a^2 & ac \\ ac & c^2 \end{bmatrix}$$

2) M generic but m_D "lopsided" $\qquad m_D \sim \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix}$

Albright, Barr; GA, Feruglio, ...

$$m_\nu \sim \begin{bmatrix} 0 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix} = c \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$$

An important property of SU(5)

Left-handed quarks have small mixings (V_{CKM}),
but right-handed quarks can have large mixings (unknown).

In SU(5): LH for d quarks	$\Longleftrightarrow$	RH for l^- leptons

$$\overset{\bar{5}\qquad\quad 10}{m_d \sim \bar{d}_R d_L}$$

$$\overset{10\qquad\quad \bar{5}}{m_e \sim \bar{e}_R e_L}$$

$$\bar{5} : (\underbrace{\bar{d},\bar{d},\bar{d},}_{R}\ \underbrace{\nu, e^-}_{L})$$

$$\boxed{m_d = m_e^T}$$

cannot be exact, but approx.

Most "lopsided" models are based on this fact. In these models large atmospheric mixing arises (at least in part) from the charged lepton sector.

- The correct pattern of masses and mixings, also including ν's, is obtained in simple models based on

  $SU(5)xU(1)_{flavour}$

  Ramond et al; GA, Feruglio+Masina; Buchmuller et al; King et al; Yanagida et al, Berezhiani et al; Lola et al.......

Offers a simple description of hierarchies, but it is not very predictive (large number of undetermined o(1) parameters

- **SO(10)** models could be more predictive, as are non abelian flavour symmetries, eg $O(3)_F$, $SU(3)_F$

  Albright, Barr; Babu et al; Bajic et al; Barbieri et al; Buccella et al; King et al; Mohapatra et al; Raby et al; G. Ross et al

SU(5)xU(1)

1st fam. 2nd 3rd

$$\left\{\begin{array}{l}\Psi_{10}: (5, 3, 0)\\ \Psi_5: (2, 0, 0)\\ \Psi_1: (1,-1, 0)\end{array}\right.$$

Equal 2,3 ch. for lopsided

Recall: $m_u \sim 10\ 10$
$m_d = m_e^T \sim \bar{5}\ 10$
$m_{\nu D} \sim \bar{5}\ 1$; $M_{RR} \sim 1\ 1$

No structure for leptons

No automatic det23 = 0

Automatic det23 = 0

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Anarchical (A)	(3,2,0)	(0,0,0)	(0,0,0)	(0,0)
Semi-Anarchical (SA)	(2,1,0)	(1,0,0)	(2,1,0)	(0,0)
Hierarchical (H_I)	(6,4,0)	(2,0,0)	(1,-1,0)	(0,0)
Hierarchical (H_{II})	(5,3,0)	(2,0,0)	(1,-1,0)	(0,0)
Inversely Hierarchical (IH_I)	(3,2,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)
Inversely Hierarchical (IH_{II})	(6,4,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)

all charges positive

not all charges positive

With suitable charge assignments all relevant patterns can be obtained

All entries are a given power of λ times a free o(1) coefficient

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix}$$

In a statistical approach we generate these coeff.s as random complex numbers $\rho e^{i\phi}$ with ϕ =[0,2π] and ρ= [0.5,2] (default) or [0.8,1.2], or [0.95,1.05] or [0,1] (real numbers also considered for comparison)

For each model we evaluate the success rate (over many trials) for falling in the exp. allowed window:

(boundaries ~3σ limits)

Maltoni et al, hep-ph/0309130

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm}$

$0.018 < r < 0.053$
$|U_{e3}| < 0.23$
$0.30 < \tan^2\theta_{12} < 0.64$
$0.45 < \tan^2\theta_{23} < 2.57$

for each model the λ,λ' values are optimised

The optimised values of λ are of the order of λ_C or a bit larger (moderate hierarchy)

model	$\lambda(=\lambda')$
A_{SS}	0.2
SA_{SS}	0.25
$H_{(SS,II)}$	0.35
$H_{(SS,I)}$	0.45
$IH_{(SS,II)}$	0.45
$IH_{(SS,I)}$	0.25

Example: Normal Hierarchy

G.A., Feruglio, Masina

1st fam. 2nd 3rd

$q(10)$: (5, 3, 0)
$q(\bar{5})$: (2, 0, 0)
$q(1)$: (1,-1, 0)

Note: not all charges positive
--> det23 suppression

$q(H) = 0$, $q(\bar{H}) = 0$
$q(\theta) = -1$, $q(\theta') = +1$

In first approx., with $\langle\theta\rangle/M \sim \lambda \sim \lambda' \sim 0.35 \sim o(\lambda_C)$

$10_i 10_j$

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix},$$

$10_i \bar{5}_j$

$$m_d = m_e^T \sim v_d \begin{bmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

"lopsided"

$\bar{5}_i 1_j$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{bmatrix},$$

$1_i 1_j$

$$M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}$$

Note: coeffs. 0(1) omitted, only orders of magnitude predicted

$$\bar{5}_i 1_j \qquad\qquad\qquad\qquad 1_i 1_j$$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda & 1 \\ \lambda & \lambda & 1 \end{bmatrix}, \qquad M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{bmatrix}$$

see-saw $\quad m_\nu \sim m_{\nu D}{}^T M_{RR}{}^{-1} m_{\nu D}$

$$m_\nu \sim v_u{}^2/M \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & \boxed{\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}} \\ \lambda^2 & & \end{bmatrix},$$

$$\det_{23} \sim \lambda^2$$

The 23 subdeterminant is automatically suppressed,
$\theta_{13} \sim \lambda^2$, θ_{12} , $\theta_{23} \sim 1$

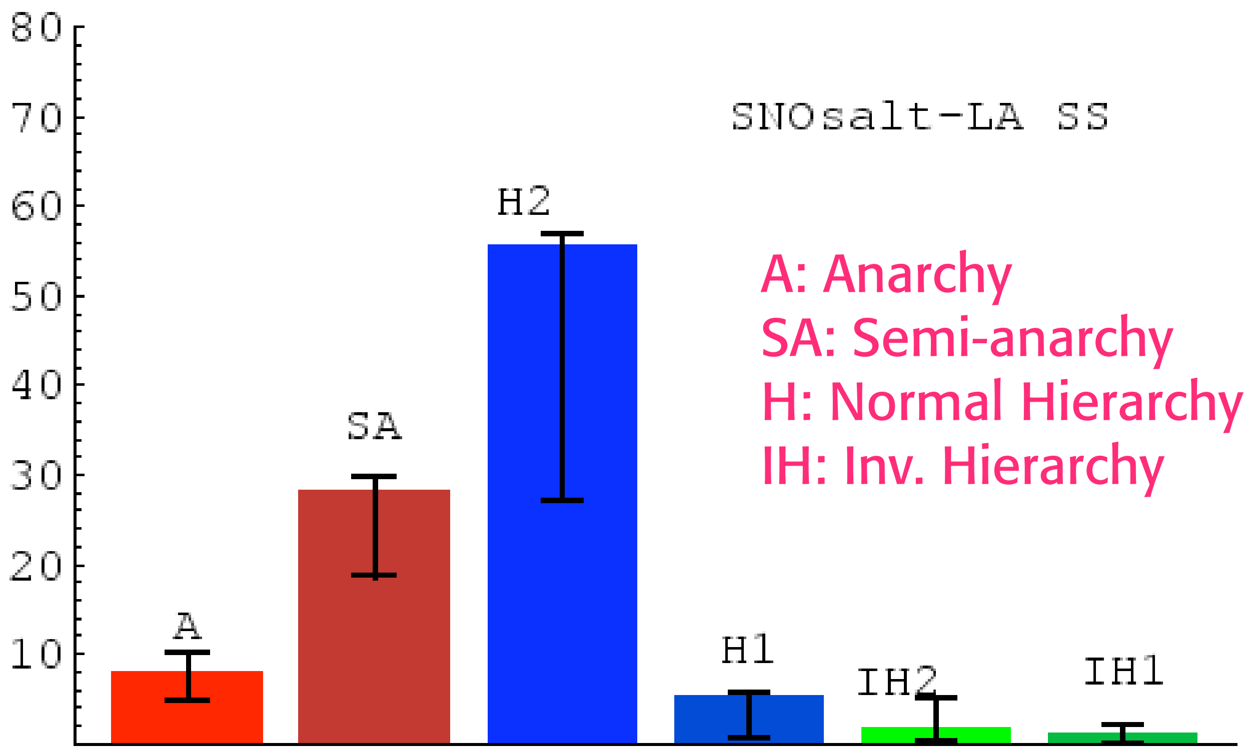
This model works, in the sense that all small parameters are naturally due to various degrees of suppression.
But too many free parameters!!

Results with see-saw dominance (updated in Nov. '03):

Scale: Σrates=100

1 or 2 refer to models with 1 or 2 flavons of opposite ch.

With charges of both signs and 1 flavon some entries are zero



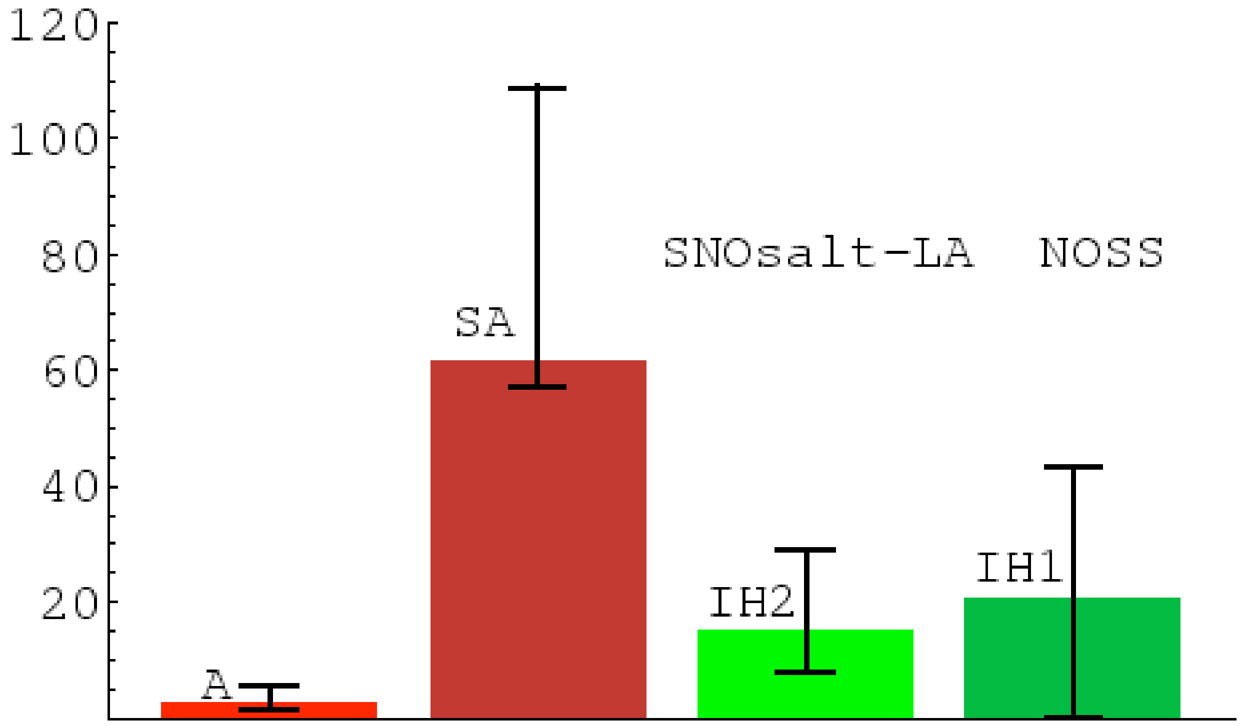
A: Anarchy
SA: Semi-anarchy
H: Normal Hierarchy
IH: Inv. Hierarchy

Errors are linear comb. of stat. and syst. errors (varying the extraction procedure: interval of ρ, real or complex)

H2 is better than SA, better than A, better than IH

With no see-saw (m_ν generated directly from $L^T m_\nu L \sim \overline{5}\,\overline{5}$) IH is better than A

[With no-see-saw H coincide with SA]



Note: we always include the effect of diagonalising charged leptons

SO(10) in principle has several advantages vs SU(5). More predictive but less flexible.

$16_{SO(10)} = (10 + 5\mathrm{bar} + 1)_{SU(5)}$

1 is ν_R: important for see-saw

The Majorana term $M\nu_R{}^T\nu_R$ is SU(5) but not SO(10) invariant:
M could be larger than the scale where SU(5) is broken, while, in SO(10), M should be of order of the scale where B-L is broken [SO(10) contains B-L]

Masses in SO(10) models

$16\mathrm{x}16 = 10 + 126 + 120$

If no non-ren mass terms are allowed a simplest model needs a 10 and a 126:

Bajc, Senjanovic, Vissani '02
Goh, Mohapatra, Ng '03

$$\mathcal{L}_Y = 10_H 16 y_{10} 16 + 126_H 16 y_{126} 16,$$

leading to

$$m_d = \alpha y_{10} + \beta y_{126}, \qquad m_e = \alpha y_{10} - 3\beta y_{126},$$

and $m_\nu \propto m_d - m_e \propto 126$

In the 23 sector, both m_d and m_e can be obtained (by $U(1)_F$) as:

$$m_{d,e} \sim \begin{pmatrix} \lambda^2 & \lambda^2 \\ \lambda^2 & 1 \end{pmatrix}$$

Then b-τ unification forces a cancellation $1 \to \lambda^2$, which in turn makes a large 23 neutrino mixing.

Also predicts θ_{12} large, $r \sim \lambda^2$, θ_{13} near the bound

Problems: Doublet-Triplet splitting worse, some fine tuning

In other SO(10) models one avoids large Higgs represent'ns (120, 126) by relying on non ren. operators like
$16_i\ 16_H\ 16_j\ 16'_H$ or $16_i\ 16_j\ 10_H\ 45_H$

In the F-symmetry limit, the lowest dimension mass terms $16_3 16_3 10_H$ is only allowed for the 3rd family.

In particular, both lopsided and L-R symmetric models can be obtained in this way

Babu, Pati, Wilczek
Albright, Barr
Ji, Li, Mohapatra
....

I do not know a GUT model which is exempt from some ad hoc ansatz or fine tuning.
On the other hand the goal is very ambitious.

Large neutrino mixings can induce observable $\tau \to \mu\gamma$ and $\mu \to e\gamma$ transitions

In fact, in SUSY models large lepton mixings induce large s-lepton mixings via RG effects (boosted by the large Yukawas of the 3rd family)

Detailed predictions depend on the model structure and the SUSY parameters.
Lopsided models tend to lead to the largest rates.

Typical values: $B(\mu \to e\gamma) \sim 10^{-11} - 10^{-14}$ (now: $\sim 10^{-11}$)
$B(\tau \to \mu\gamma) < \sim 10^{-7}$ (now: $\sim 10^{-7}$)

See, e.g., ••••• Lavignac, Masina, Savoy'02
Masiero, Vempati, Vives'03; Babu, Dutta, Mohapatra'03;
Babu, Pati, Rastogi'04; Blazek, King '03; Petcov et al '04;
Barr '04 ••••••

Conclusion

"Normal" models are not too difficult to build

In fact there are quite a number of different examples

Some of them require θ_{13} near the bound
All of them prefer $\theta_{13} \gtrsim \lambda^2_C$

Good chances for next generation of experiments!