Machine learning quantification of cluster structure

Oliver Lomax

Matthew Bates & Anthony Whitworth

6th September 2018

The Wonders of Star Formation

Outline

Outline

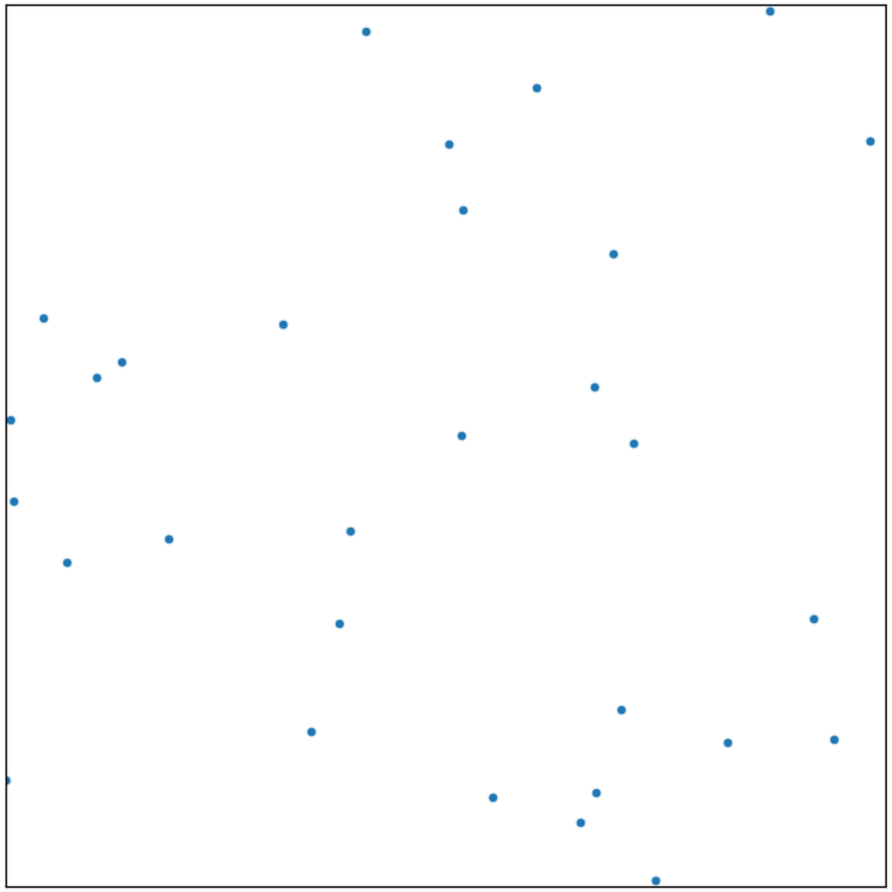
- Q parameter, used to measure substructure in star clusters, is unreliable.

Outline

- Q parameter, used to measure substructure in star clusters, is unreliable.
- We have tools available to do better!

Outline

- Q parameter, used to measure substructure in star clusters, is unreliable.
- We have tools available to do better!
- Disclaimer: I am neither an observer or an N-body numericist!



Q parameter

$$Q = \frac{\bar{m}}{\bar{s}}$$

$$\bar{m} = \frac{\sum_{i=1}^{N_m} m_i}{(N_m + 1)^{\frac{E-1}{E}} R}$$

$$\bar{s} = \frac{\sum_{i=1}^{N_s} s_i}{N_s R}$$

Cartwright and Whitworth, 2004, MNRAS, 348, 589

Q parameter

Minimum Spanning Tree

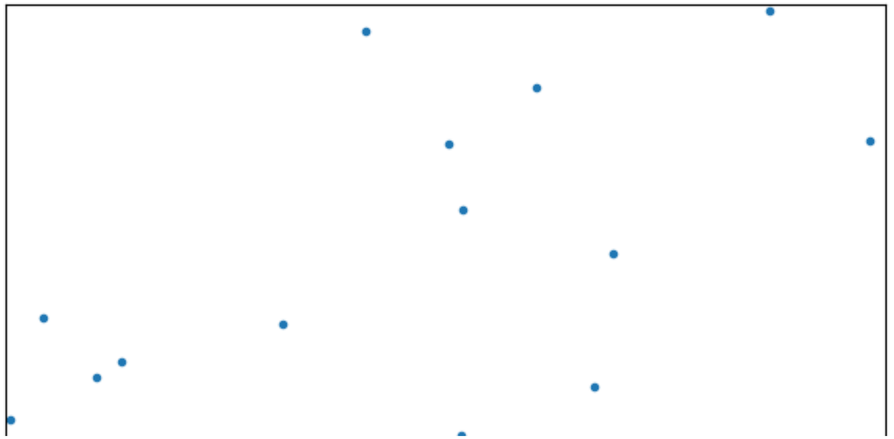
$$Q = \frac{\bar{m}}{\bar{s}}$$

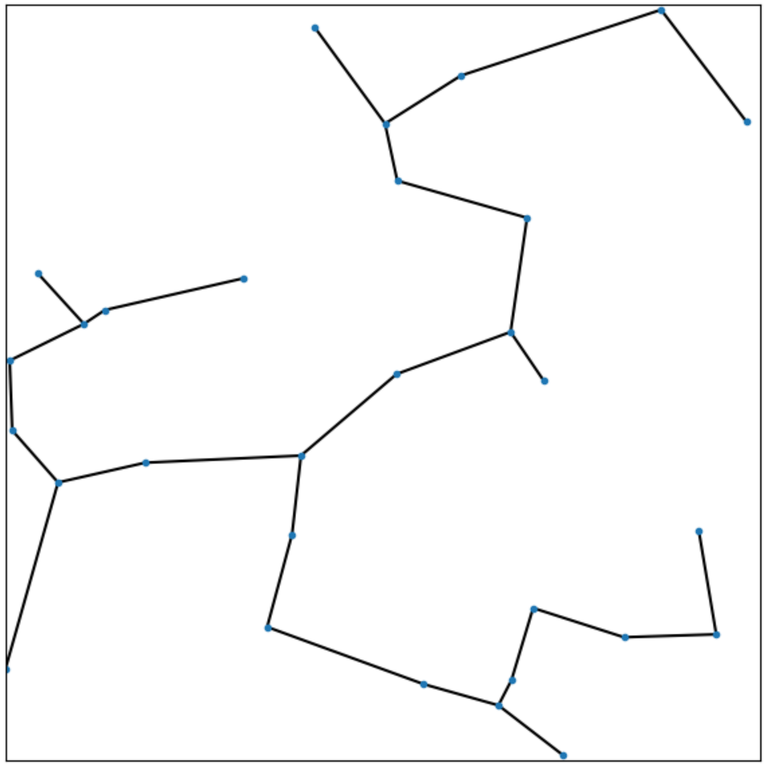
$$\bar{m} = \frac{\sum_{i=1}^{N_m} m_i}{(N_m + 1)^{\frac{E-1}{E}} R}$$

$$\bar{s} = \frac{\sum_{i=1}^{N_s} s_i}{N_s R}$$

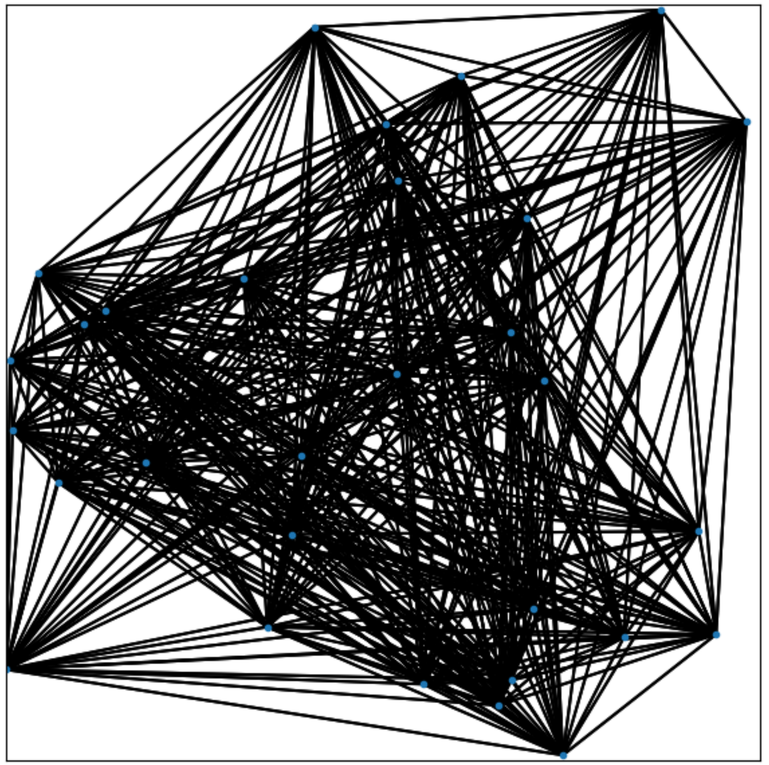
Cartwright and Whitworth, 2004, MNRAS, 348, 589

Q parameter





—



$$Q = \frac{\bar{m}}{\bar{s}}$$

$$\bar{m} = \frac{\sum_{i=1}^{N_m} m_i}{(N_m + 1)^{\frac{E-1}{E}} R}$$

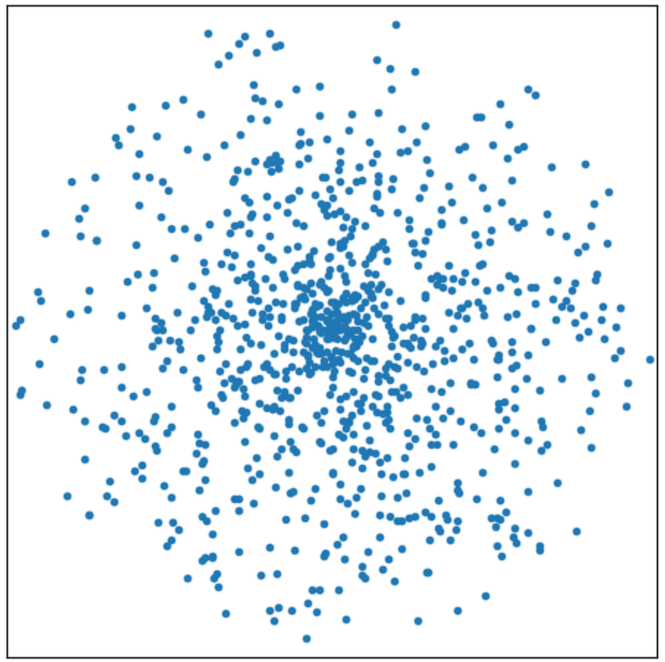
$$\bar{s} = \frac{\sum_{i=1}^{N_s} s_i}{N_s R}$$

Minimum Spanning Tree

Complete Graph

Cartwright and Whitworth, 2004, MNRAS, 348, 589

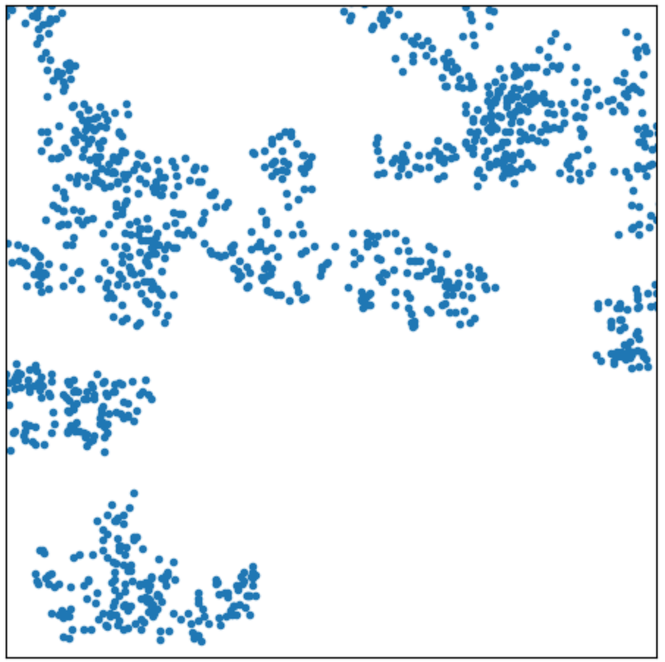
Q parameter



Radial Profile

$$\rho(r) \propto r^{-\alpha}$$

$$Q \gtrsim 0.8$$



Box Fractal

$$P(\text{sub-cube}) = \frac{2^D}{2^E}$$

$$Q \lesssim 0.8$$

$$Q = \frac{\bar{m}}{\bar{s}}$$

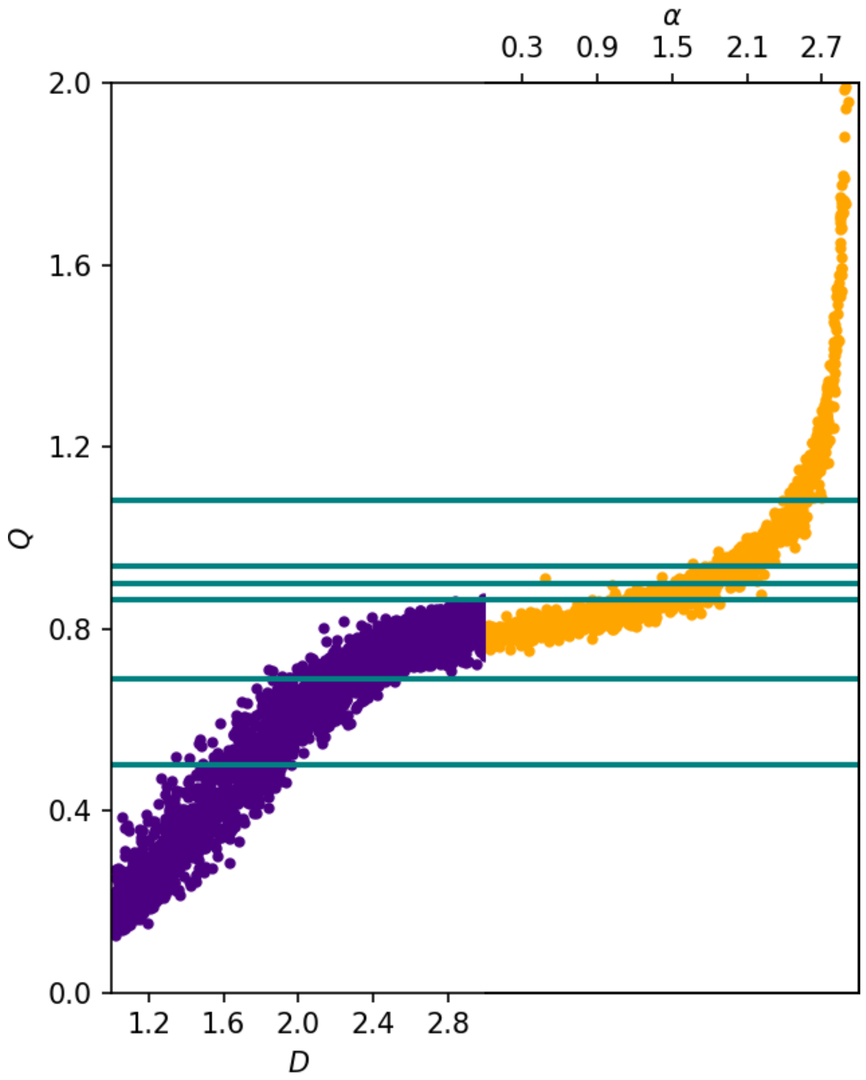
$$\bar{m} = \frac{\sum_{i=1}^{N_m} m_i}{(N_m + 1)^{\frac{E-1}{E}} R}$$

$$\bar{s} = \frac{\sum_{i=1}^{N_s} s_i}{N_s R}$$

Cartwright and Whitworth, 2004, MNRAS, 348, 589

Q parameter

Cluster	#	N
Lupus 3	1	67
IC 348	2	350
ρ Oph	3	198
IC 2391	4	200
Cha I	5	234
Taurus	6	335



$$Q = \frac{\bar{m}}{\bar{s}}$$

$$\bar{m} = \frac{\sum_{i=1}^{N_m} m_i}{(N_m + 1)^{\frac{E-1}{E}} R}$$

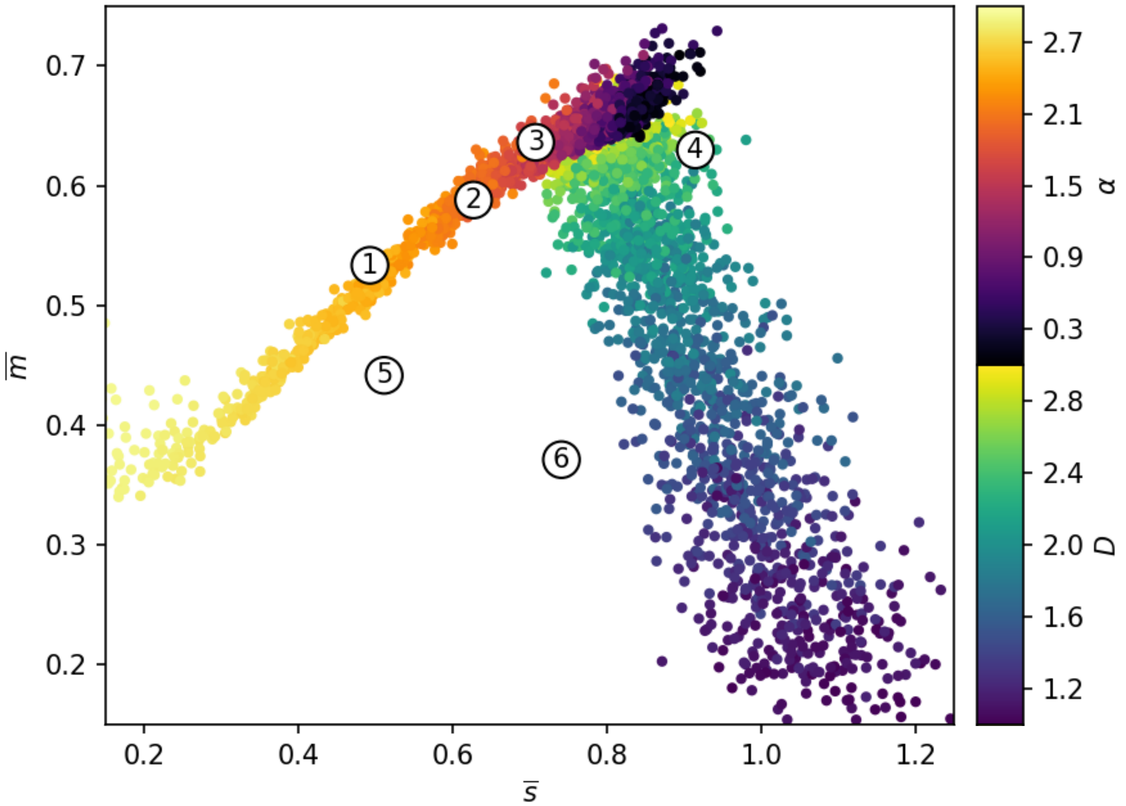
$$\bar{s} = \frac{\sum_{i=1}^{N_s} s_i}{N_s R}$$

Comerón (2008)
Lada et al. (2006); Muench et al. (2007)
Bontemps et al. (2001)
Barrado y Navascués et al. (2001)
Luhman (2007)
Luhman et al. (2010)

Cartwright and Whitworth, 2004, MNRAS, 348, 589

$\bar{m}$-$\bar{s}$ plots

Cluster	#	*N*
Lupus 3	1	67
IC 348	2	350
ρ Oph	3	198
IC 2391	4	200
Cha I	5	234
Taurus	6	335

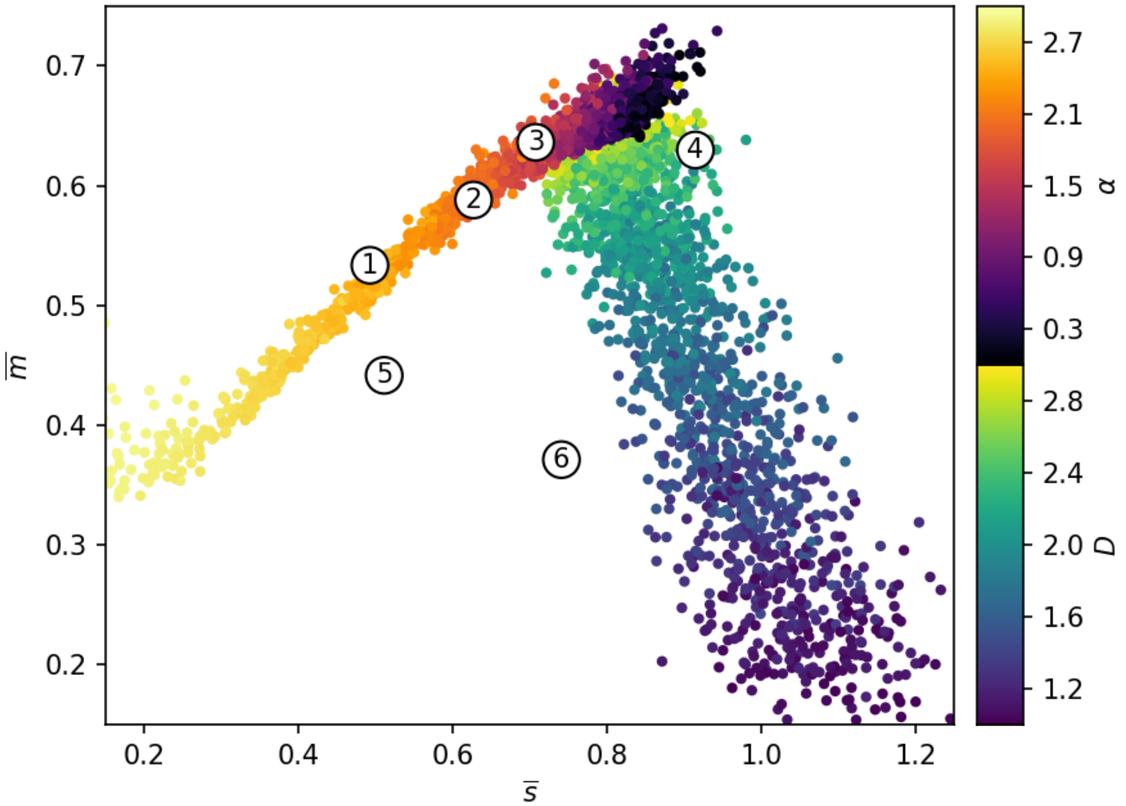


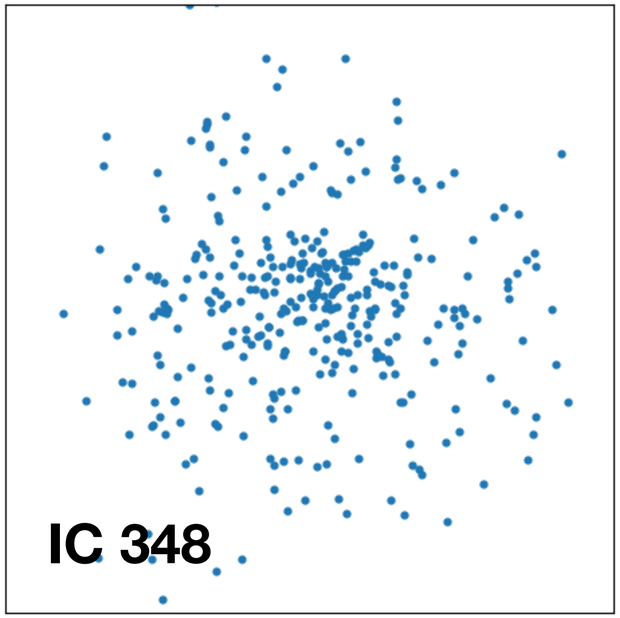
Comerón (2008)
Lada et al. (2006); Muench et al. (2007)
Bontemps et al. (2001)
Barrado y Navascués et al. (2001)
Luhman (2007)
Luhman et al. (2010)

Cartwright, 2009, MNRAS, 400, 1427

$\bar{m}$-$\bar{s}$ plots

Cluster	#	N
Lupus 3	1	67
IC 348	2	350
ρ Oph	3	198
IC 2391	4	200
Cha I	5	234
Taurus	6	335



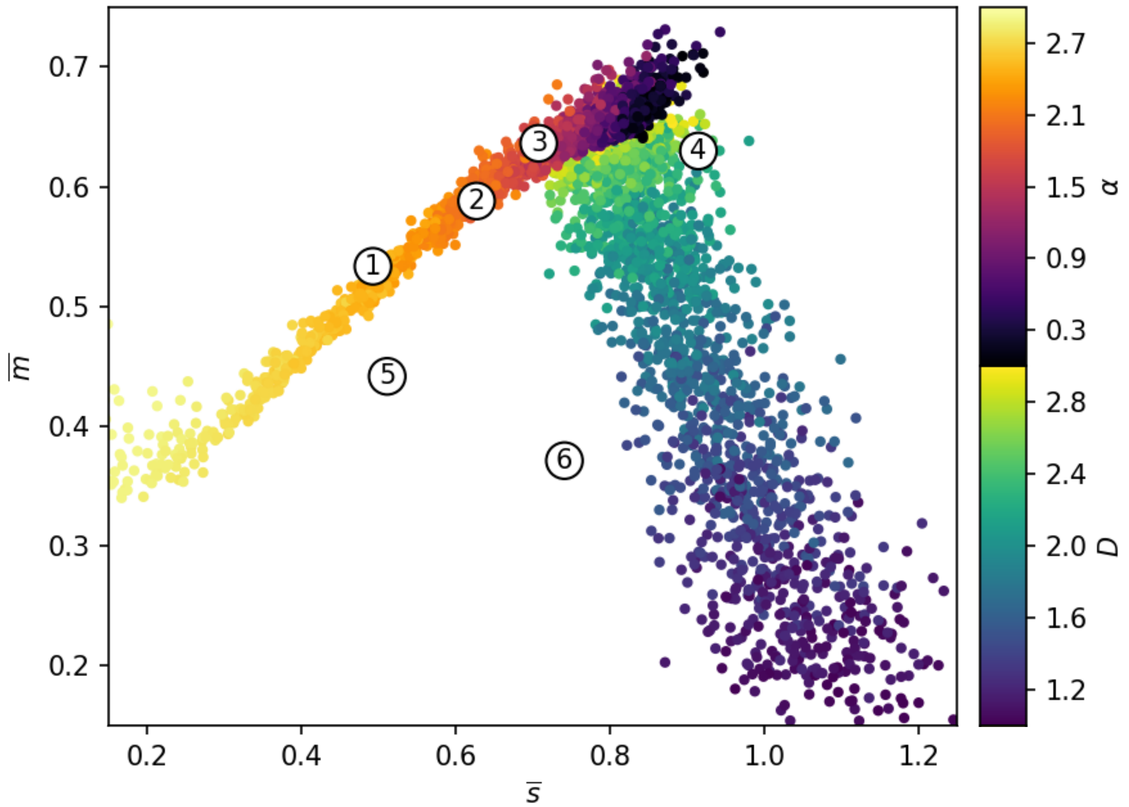


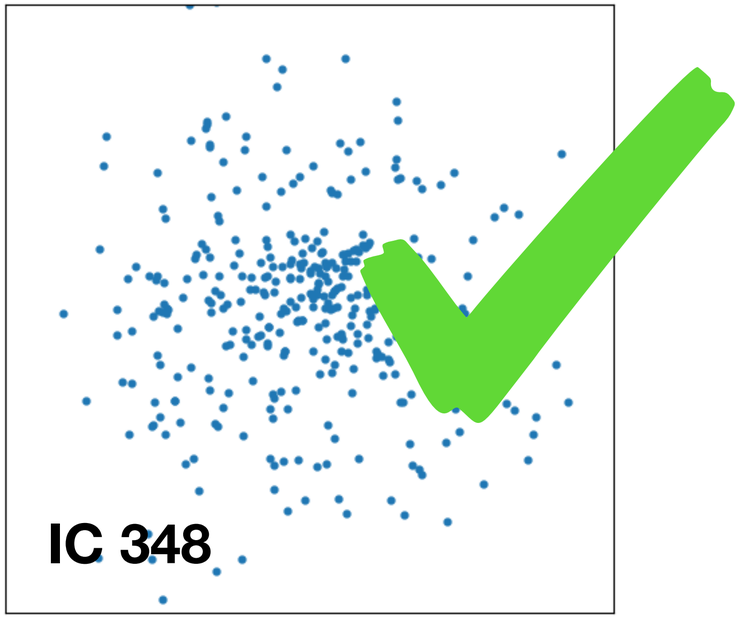
Comerón (2008)
Lada et al. (2006); Muench et al. (2007)
Bontemps et al. (2001)
Barrado y Navascués et al. (2001)
Luhman (2007)
Luhman et al. (2010)

Cartwright, 2009, MNRAS, 400, 1427

$\bar{m}$-$\bar{s}$ plots

Cluster	#	N
Lupus 3	1	67
IC 348	2	350
ρ Oph	3	198
IC 2391	4	200
Cha I	5	234
Taurus	6	335



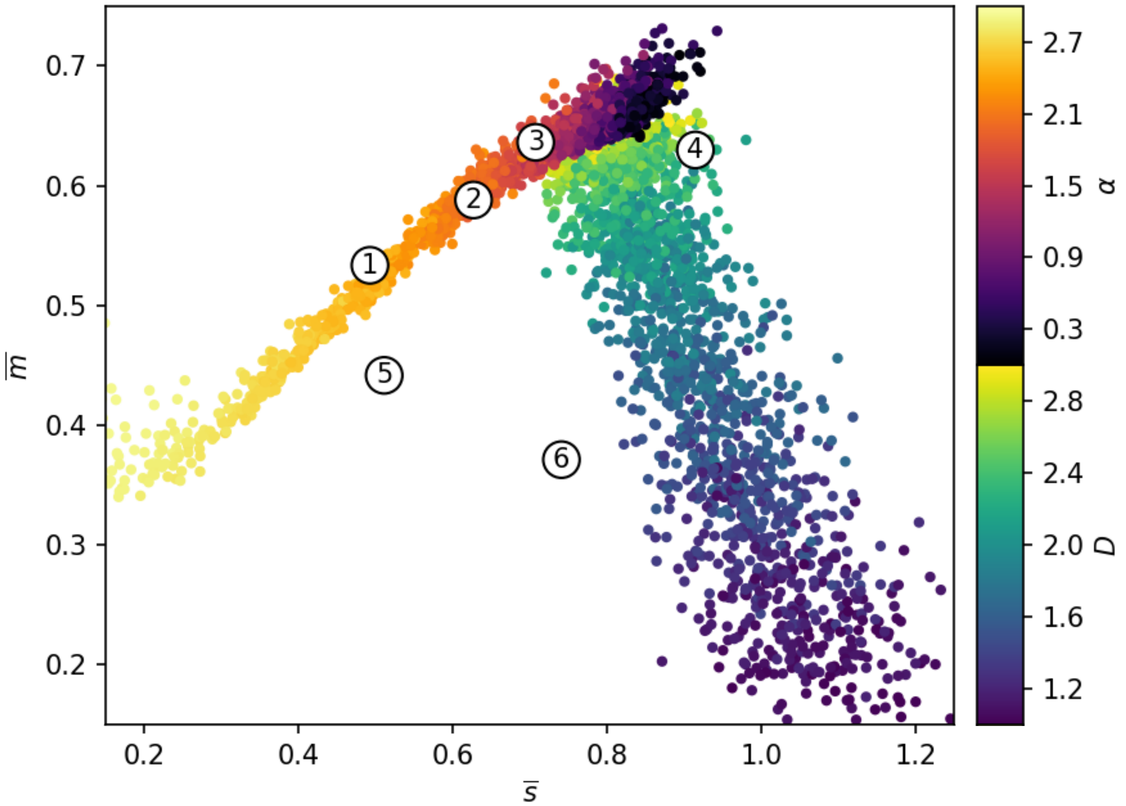


Comerón (2008)
Lada et al. (2006); Muench et al. (2007)
Bontemps et al. (2001)
Barrado y Navascués et al. (2001)
Luhman (2007)
Luhman et al. (2010)

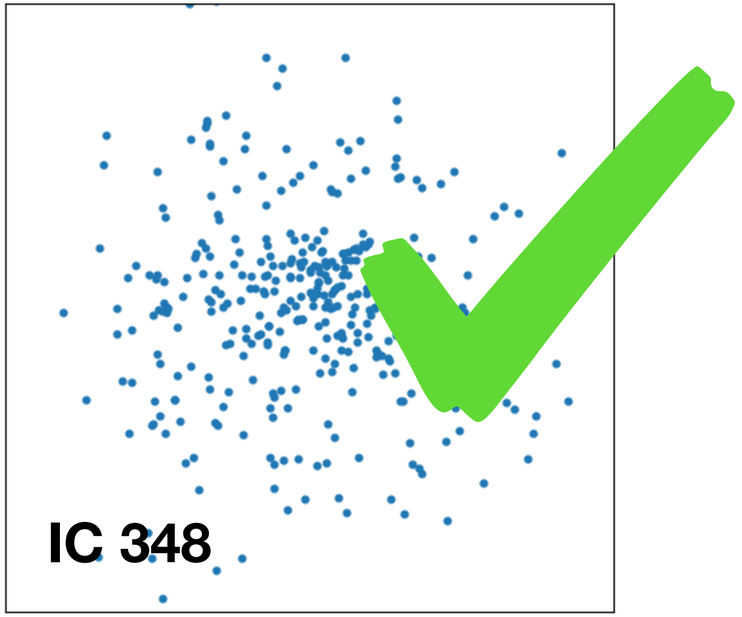
Cartwright, 2009, MNRAS, 400, 1427

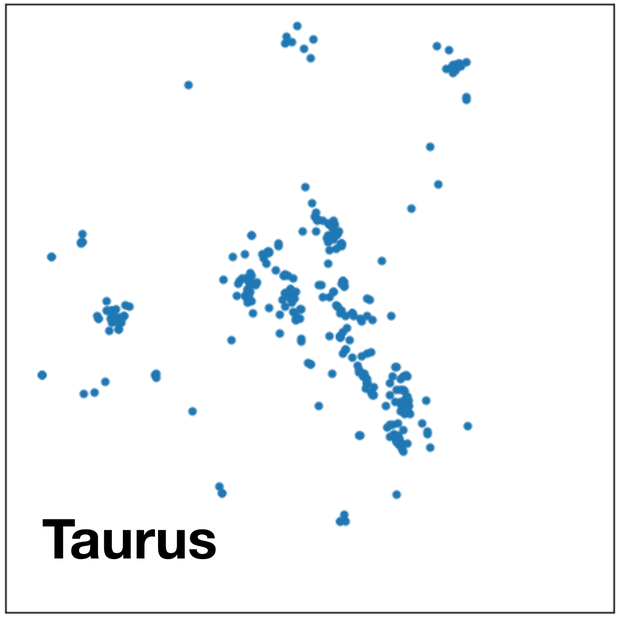
$\bar{m}$-$\bar{s}$ plots

Cluster	#	N
Lupus 3	1	67
IC 348	2	350
ρ Oph	3	198
IC 2391	4	200
Cha I	5	234
Taurus	6	335



Cartwright, 2009, MNRAS, 400, 1427

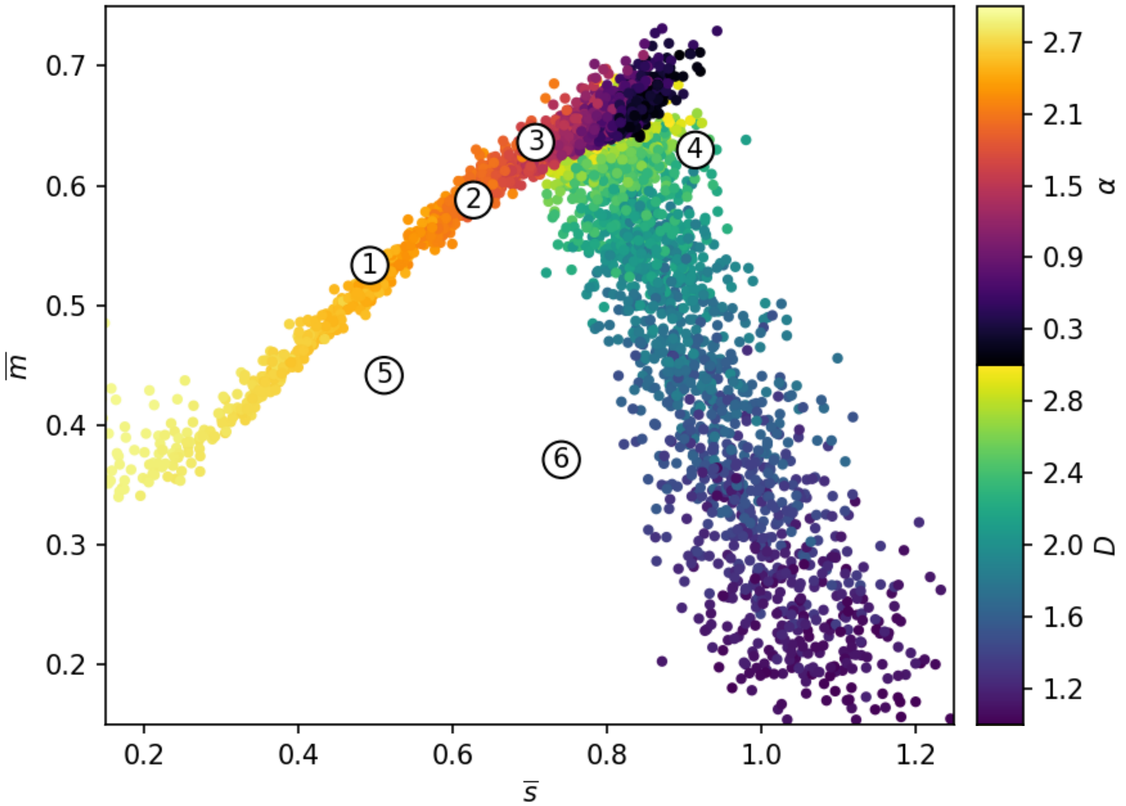


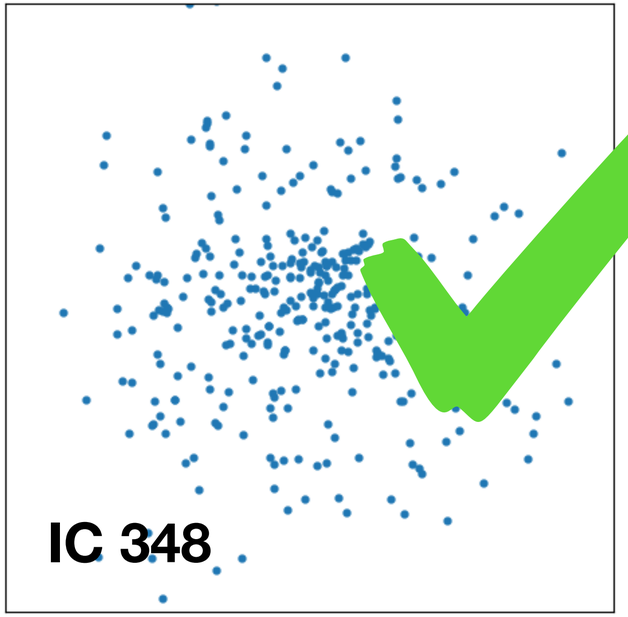


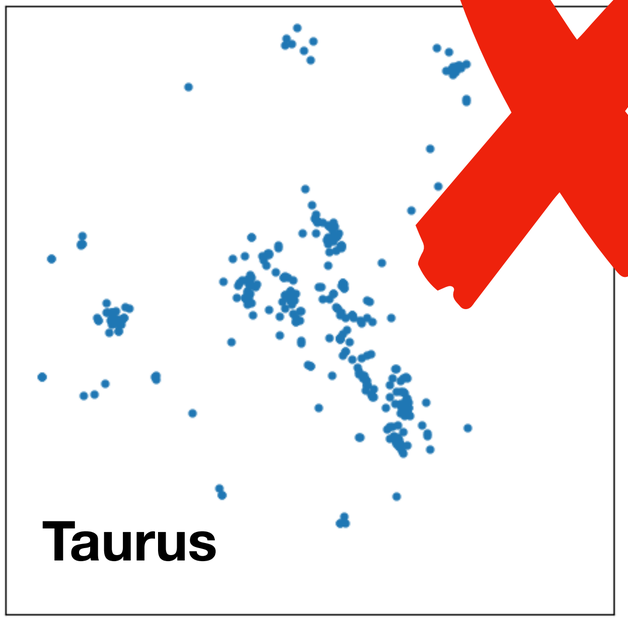
Comerón (2008)
Lada et al. (2006); Muench et al. (2007)
Bontemps et al. (2001)
Barrado y Navascués et al. (2001)
Luhman (2007)
Luhman et al. (2010)

$\bar{m}$-$\bar{s}$ plots

Cluster	#	*N*
Lupus 3	1	67
IC 348	2	350
ρ Oph	3	198
IC 2391	4	200
Cha I	5	234
Taurus	6	335





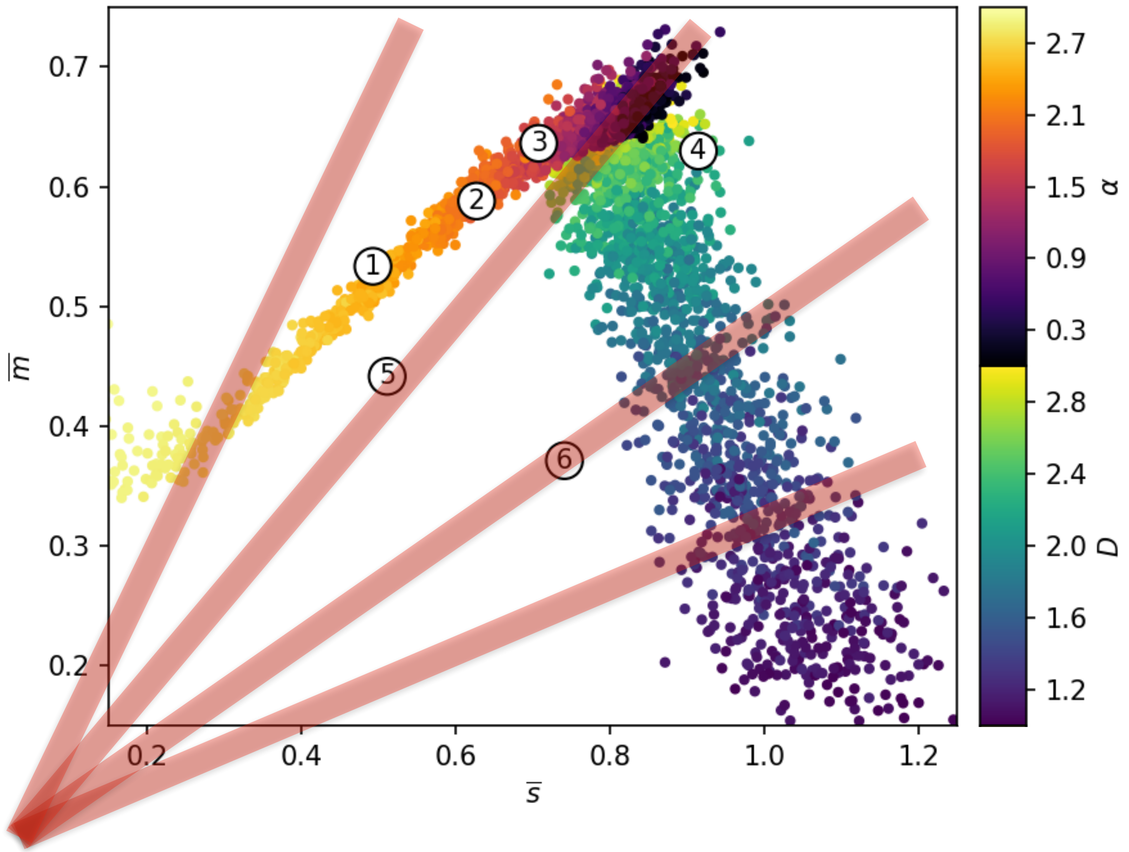


Comerón (2008)
Lada et al. (2006); Muench et al. (2007)
Bontemps et al. (2001)
Barrado y Navascués et al. (2001)
Luhman (2007)
Luhman et al. (2010)

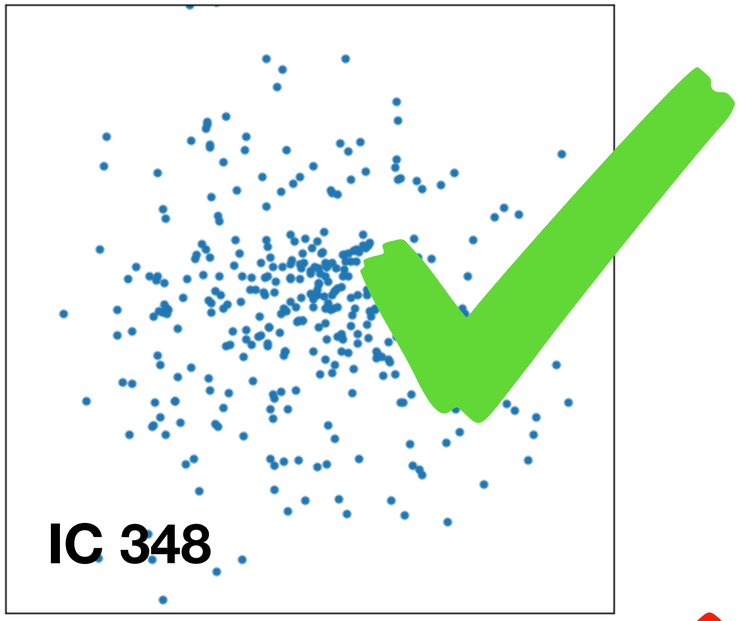
Cartwright, 2009, MNRAS, 400, 1427

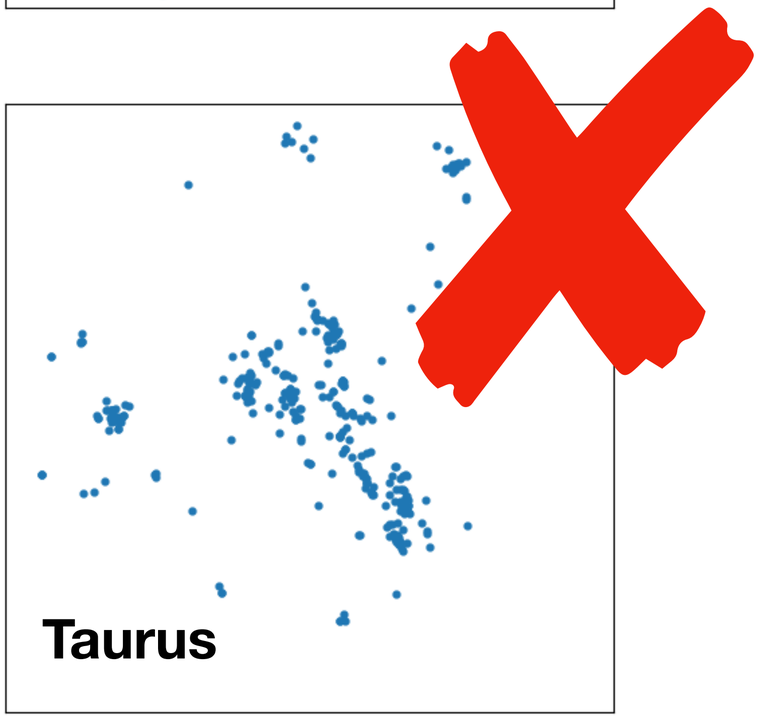
$\bar{m}$-$\bar{s}$ plots

Cluster	#	N
Lupus 3	1	67
IC 348	2	350
ρ Oph	3	198
IC 2391	4	200
Cha I	5	234
Taurus	6	335



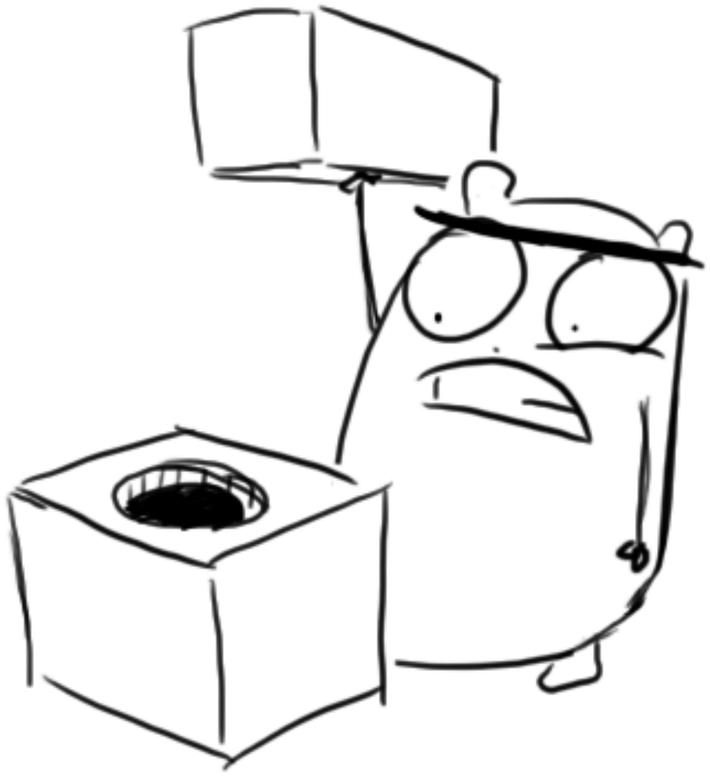
Cartwright, 2009, MNRAS, 400, 1427



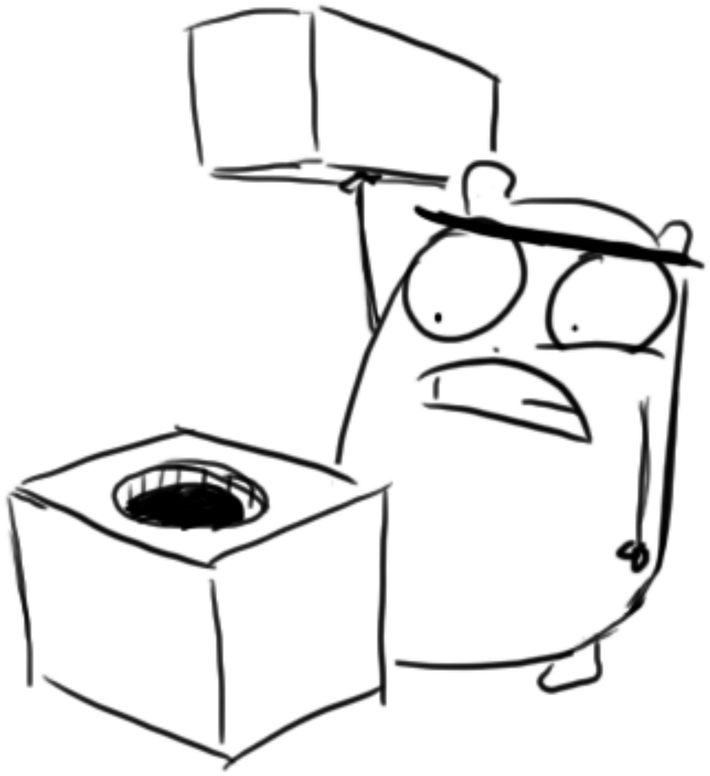


Comerón (2008)
Lada et al. (2006); Muench et al. (2007)
Bontemps et al. (2001)
Barrado y Navascués et al. (2001)
Luhman (2007)
Luhman et al. (2010)

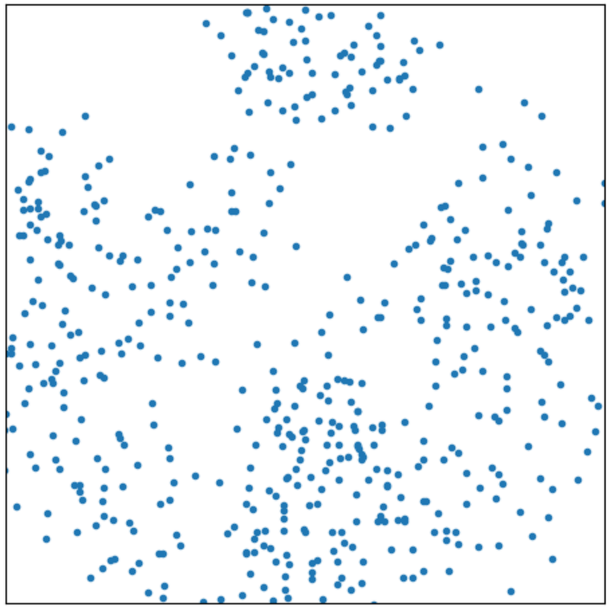
Stop using “square” models…



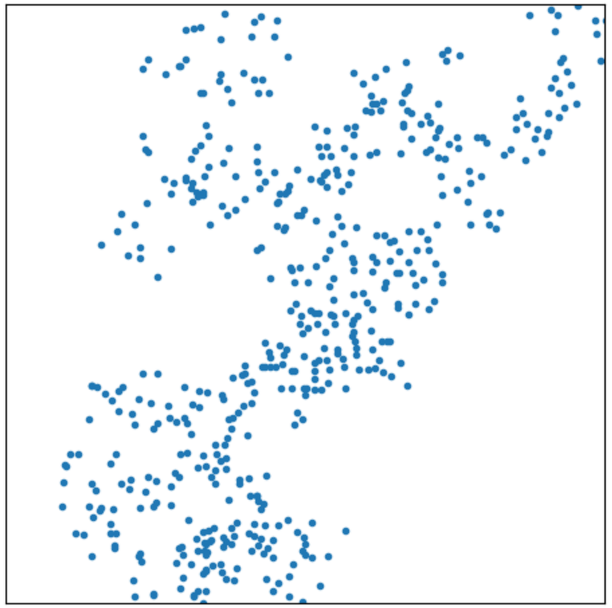
Stop using “square” models…



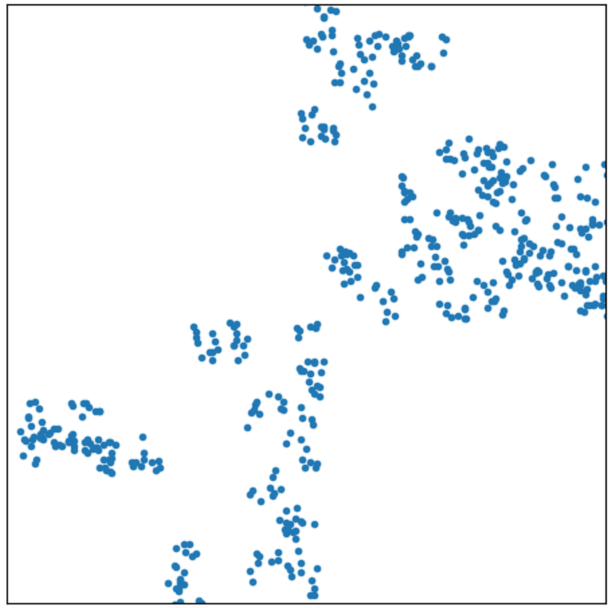
Stop using “square” models…



***D*=2.4**



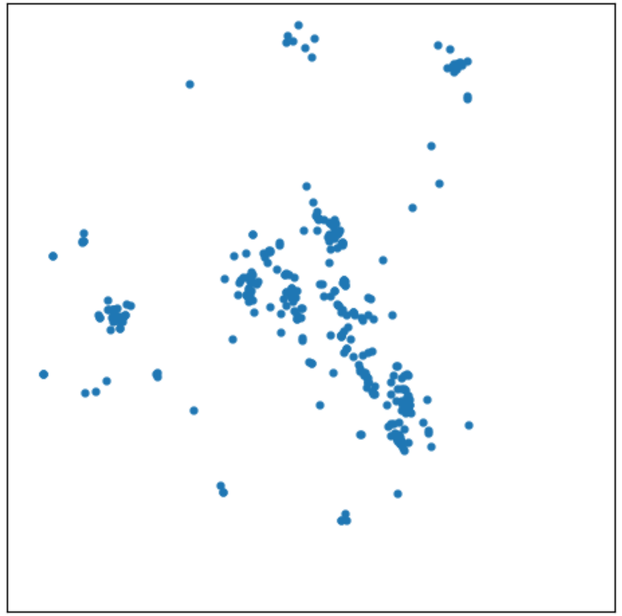
***D*=1.8**



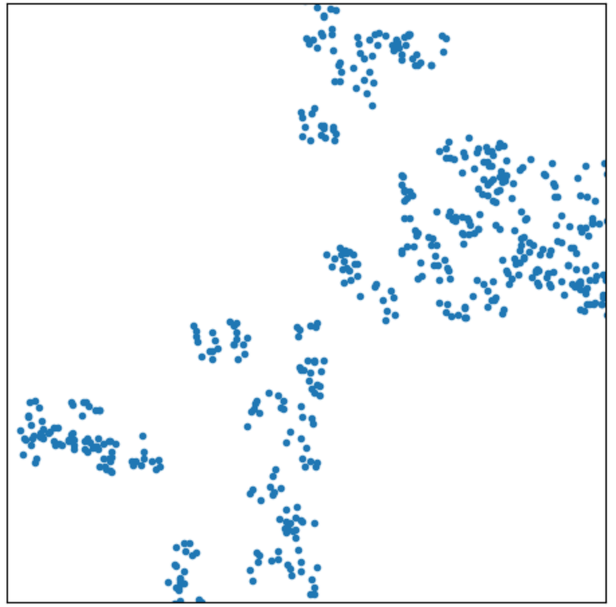
***D*=1.5**

Goodwin and Whitworth, 2004, A&A, 413, 929

Stop using “square” models…



Taurus



D=1.5

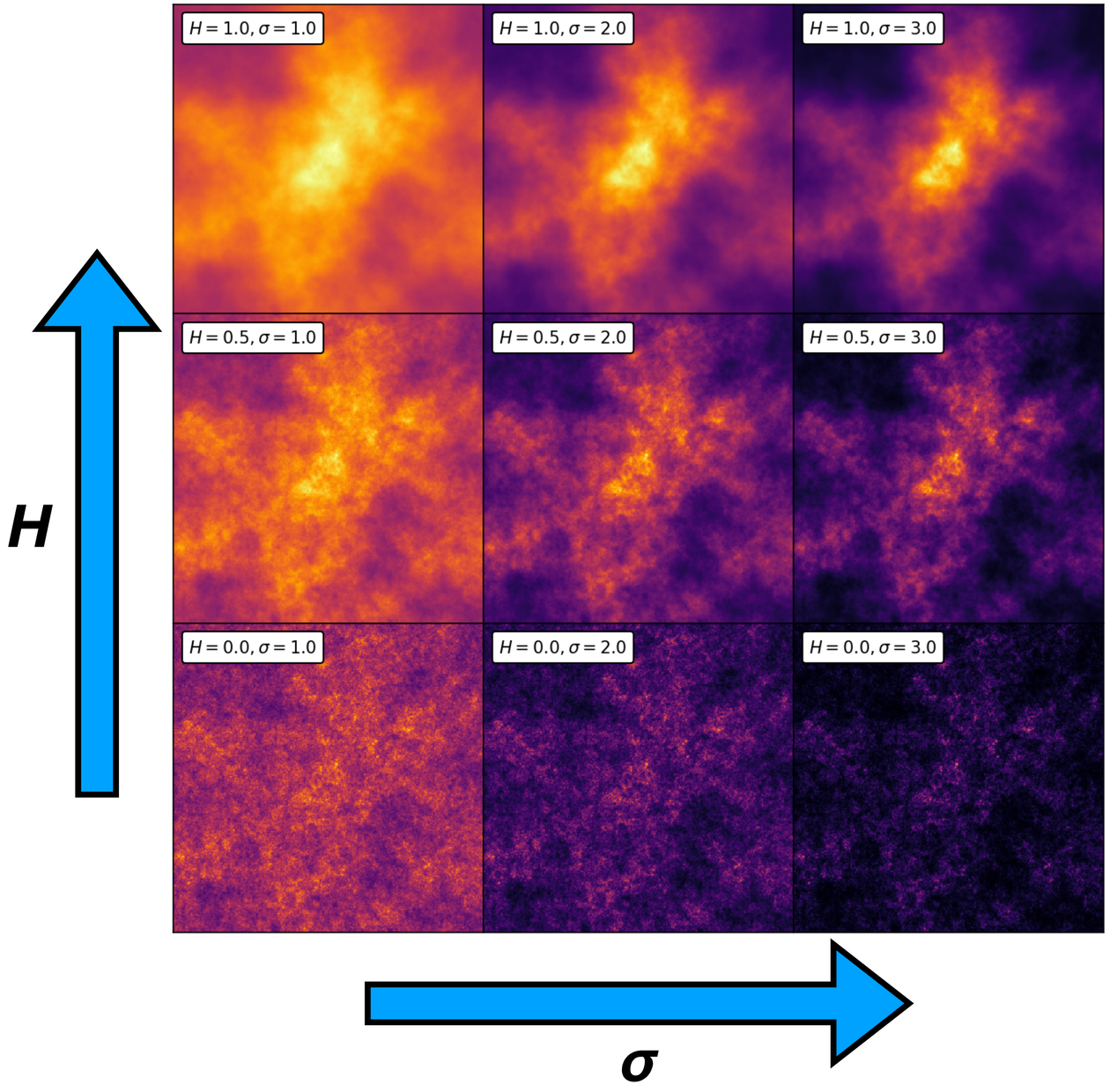
Goodwin and Whitworth, 2004, A&A, 413, 929

Fractional Brownian Motion

- A generalisation of classical Brownian motion.
- Used to interpret properties of molecular clouds (e.g. Stutzki et al., 1998, Elia et al., 2014).
- Parameterised by the “Hurst index” *H* and a standard deviation *σ*.
- Relatively simple to generate in Fourier space:

$$P(k) \propto k^{-\beta}$$

$$\beta = E + 2H$$

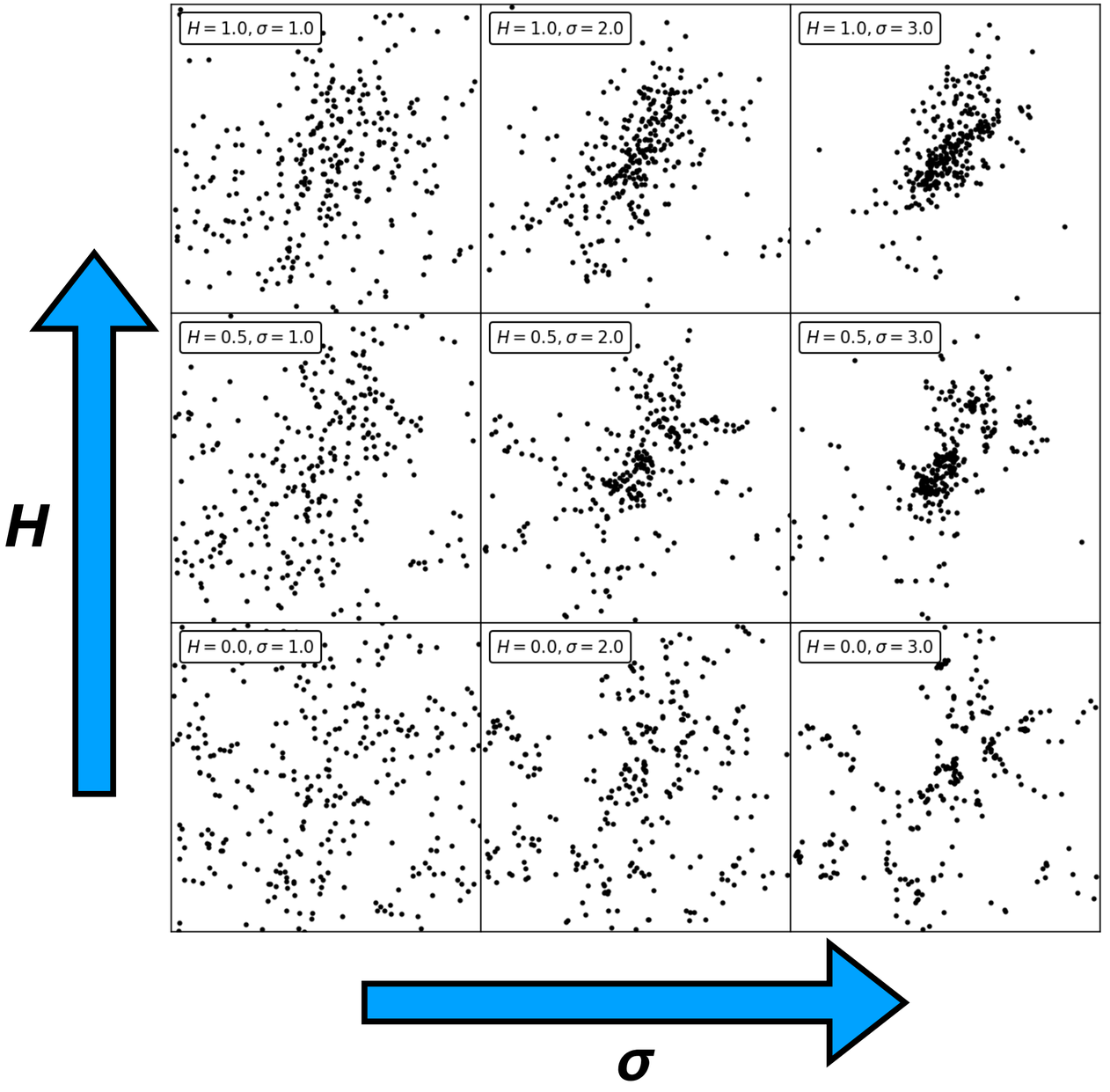


Fractional Brownian Motion

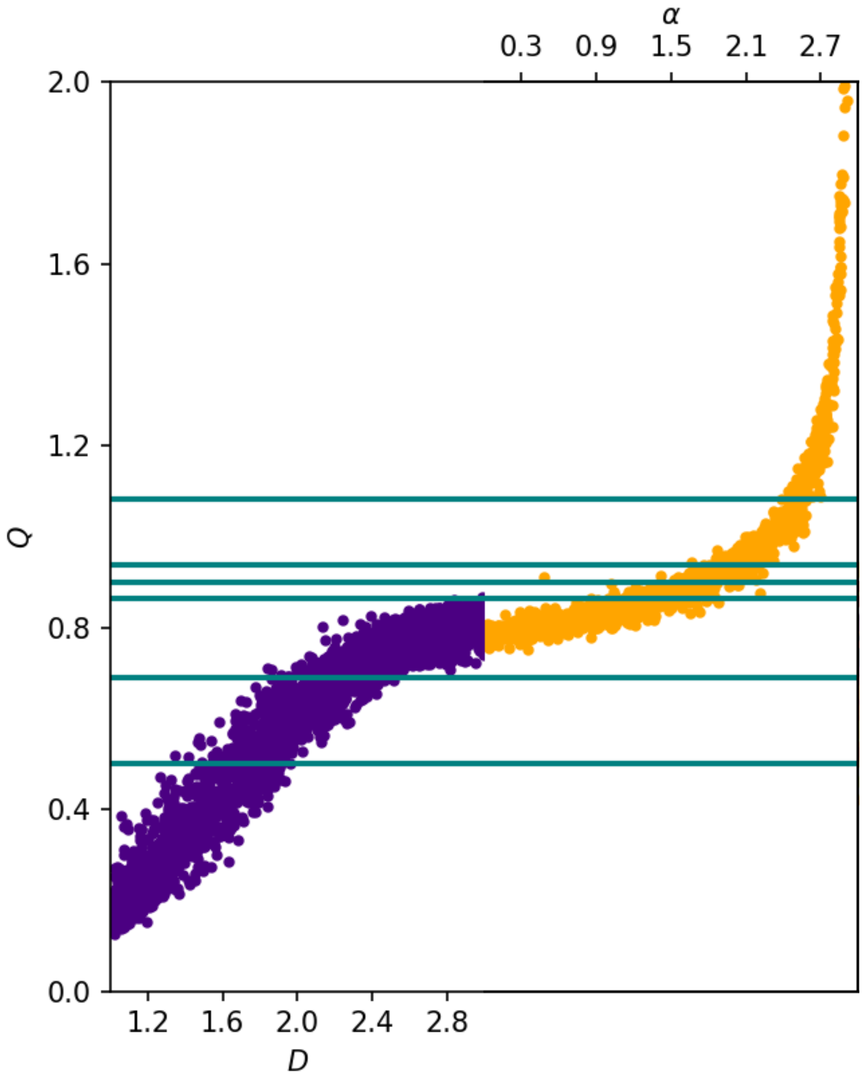
- A generalisation of classical Brownian motion.
- Used to interpret properties of molecular clouds (e.g. Stutzki et al., 1998, Elia et al., 2014).
- Parameterised by the “Hurst index” *H* and a standard deviation *σ*.
- Relatively simple to generate in Fourier space:

$$P(k) \propto k^{-\beta}$$

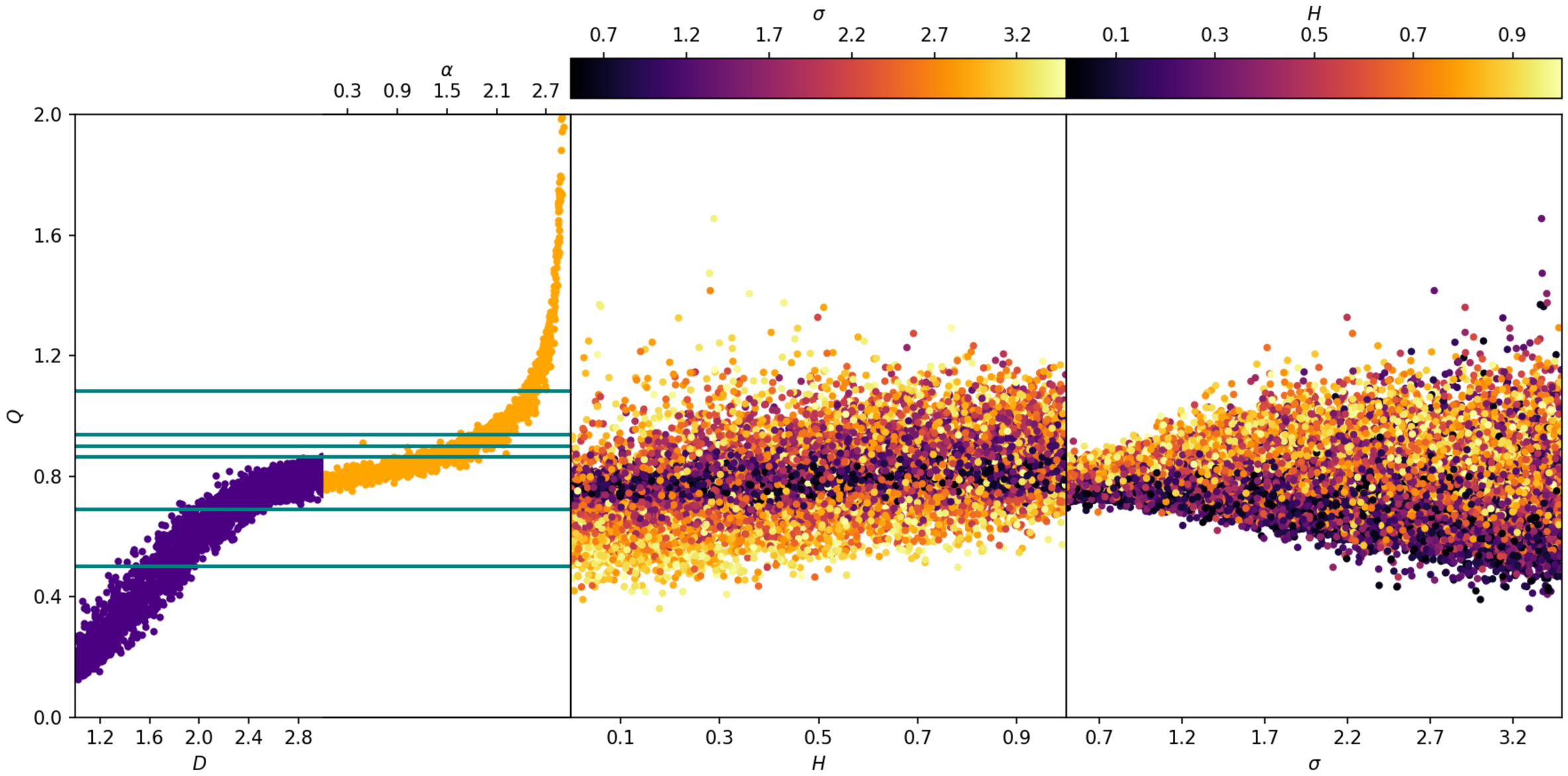
$$\beta = E + 2H$$



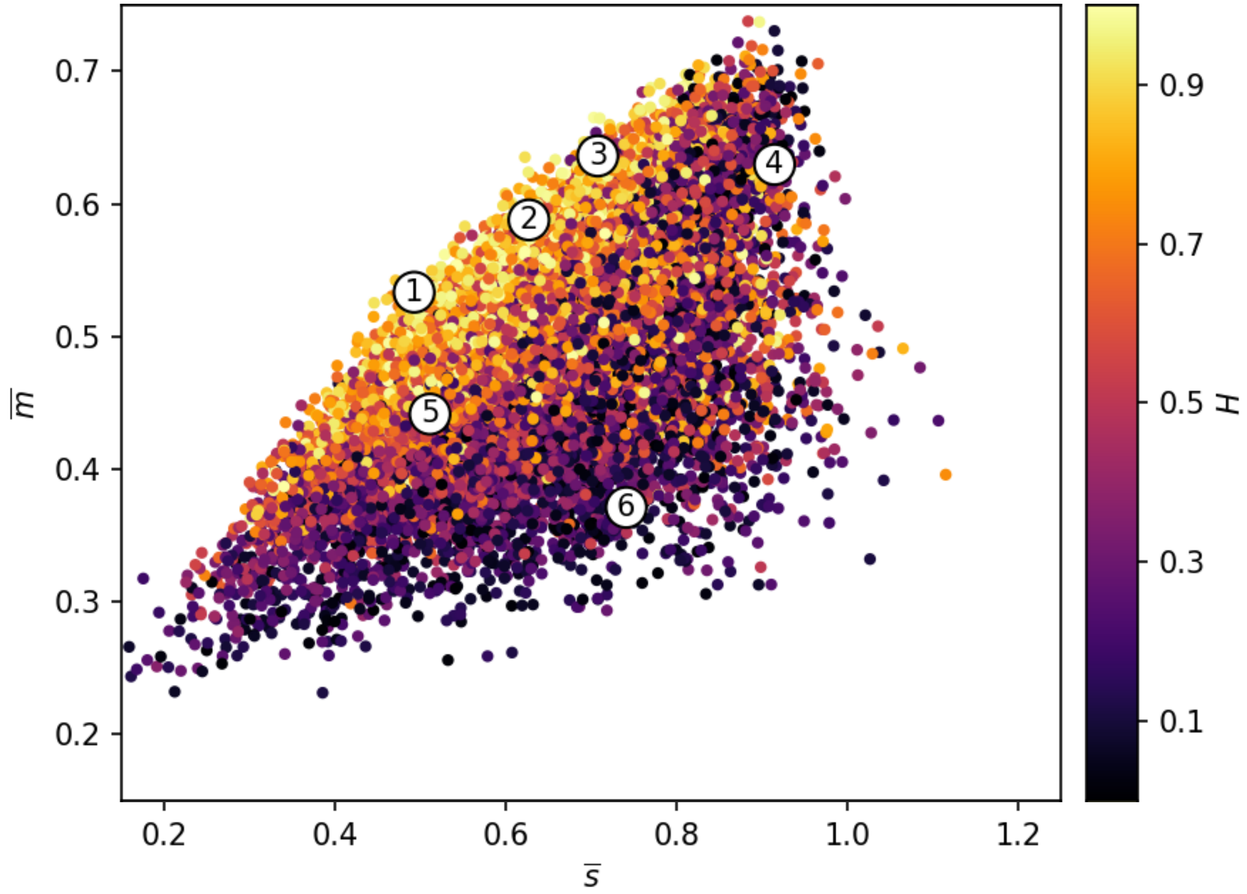
Q parameter

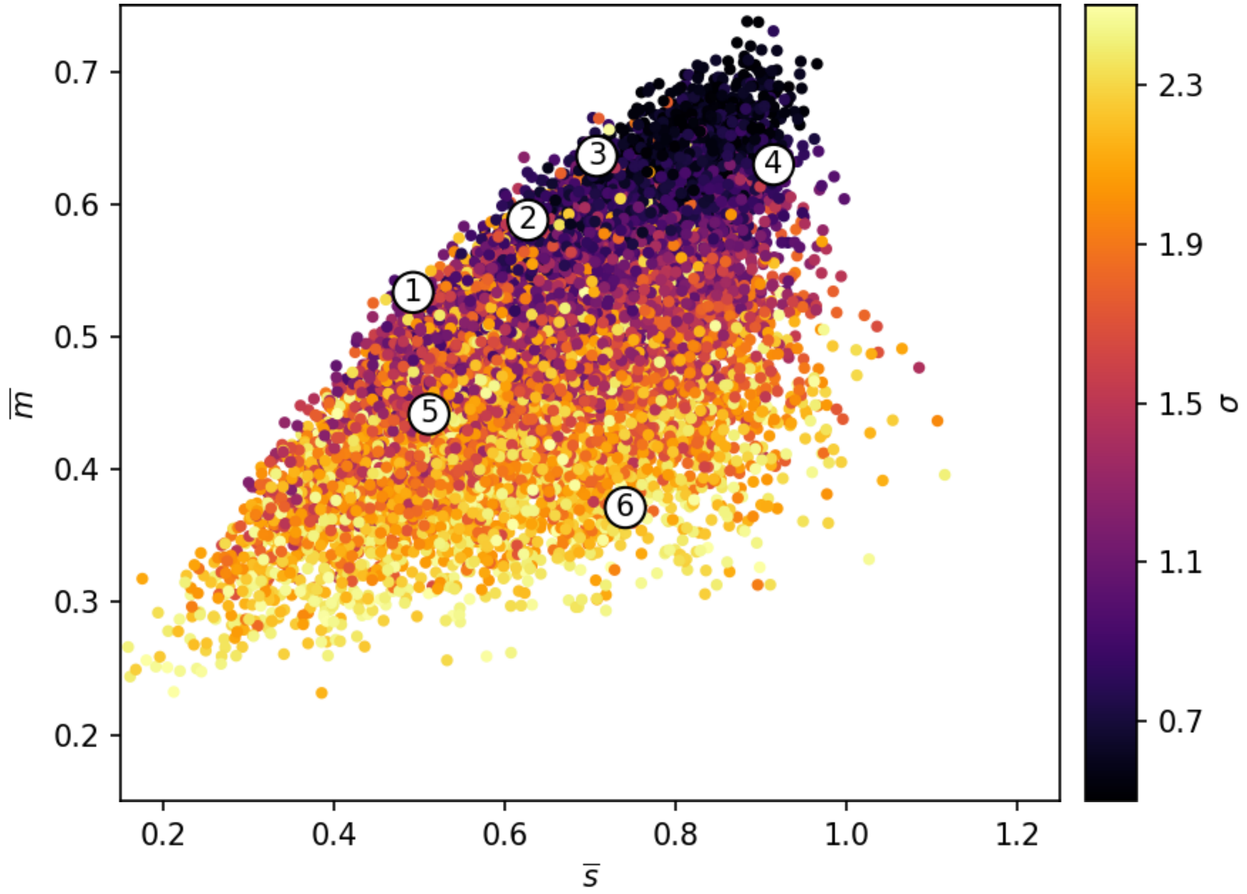


Q parameter is rubbish



$\bar{m}$-$\bar{s}$ plots



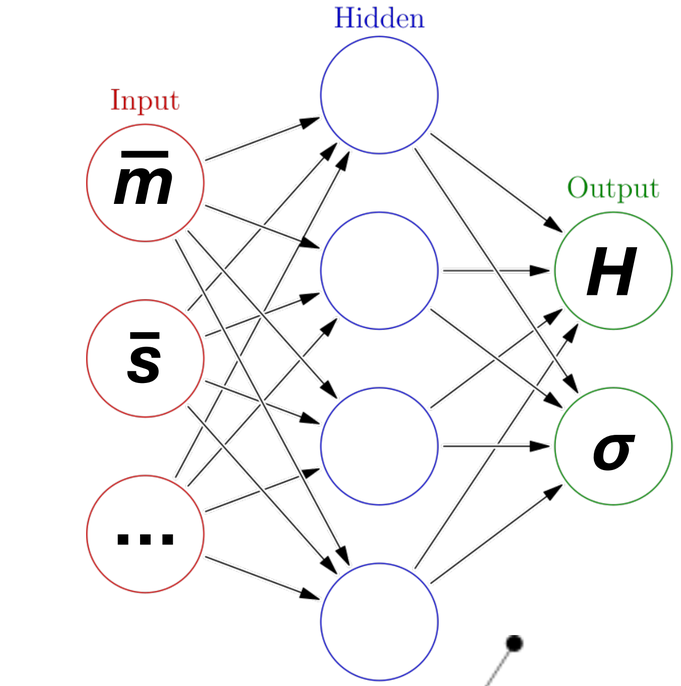


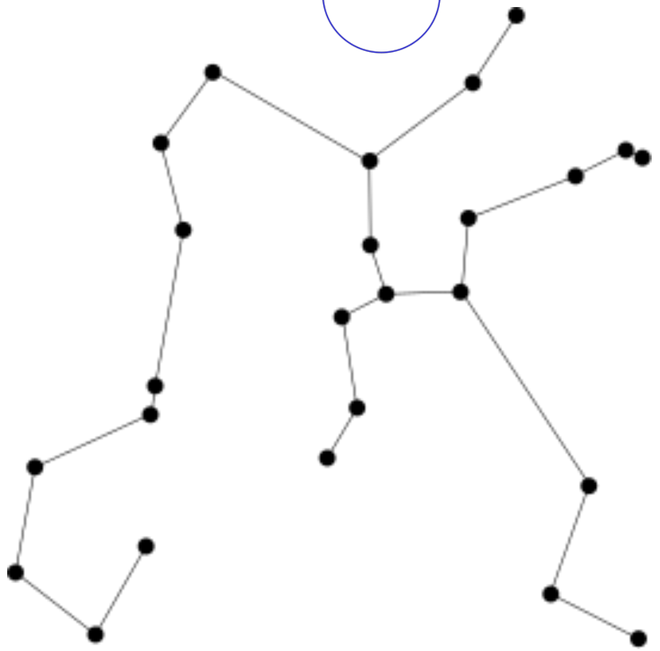
Machine learning

- Cluster distribution is “whitened”, i.e. Cov(***x***)=**I**
- Parameters estimated using minimum spanning tree and complete graph.
- Neural net method used to map edge length moments of graphs to underlying parameters.

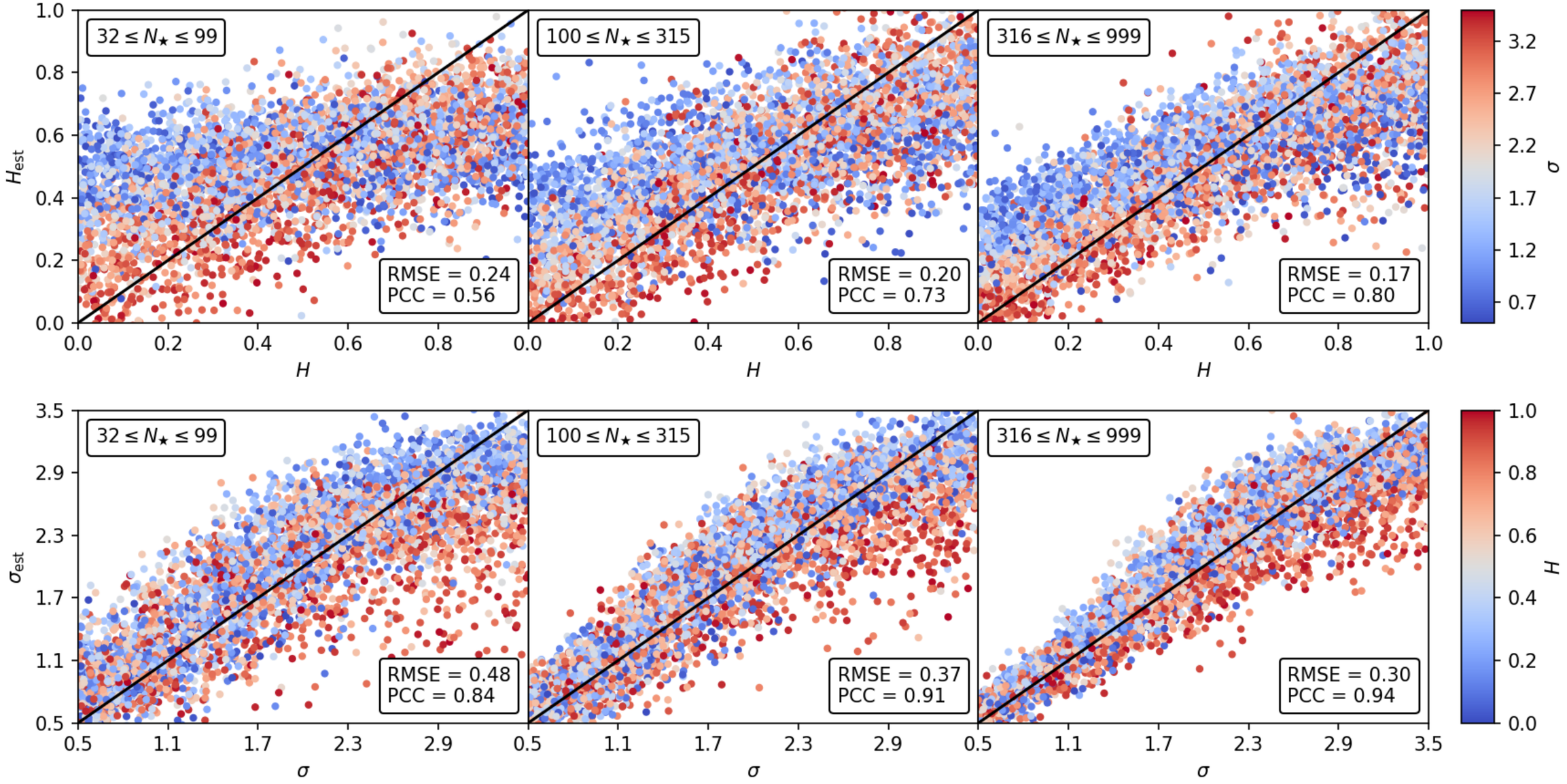
$$M_n(\boldsymbol{x}) = \left(\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu_{\boldsymbol{x}})^n \right)^{1/n}$$

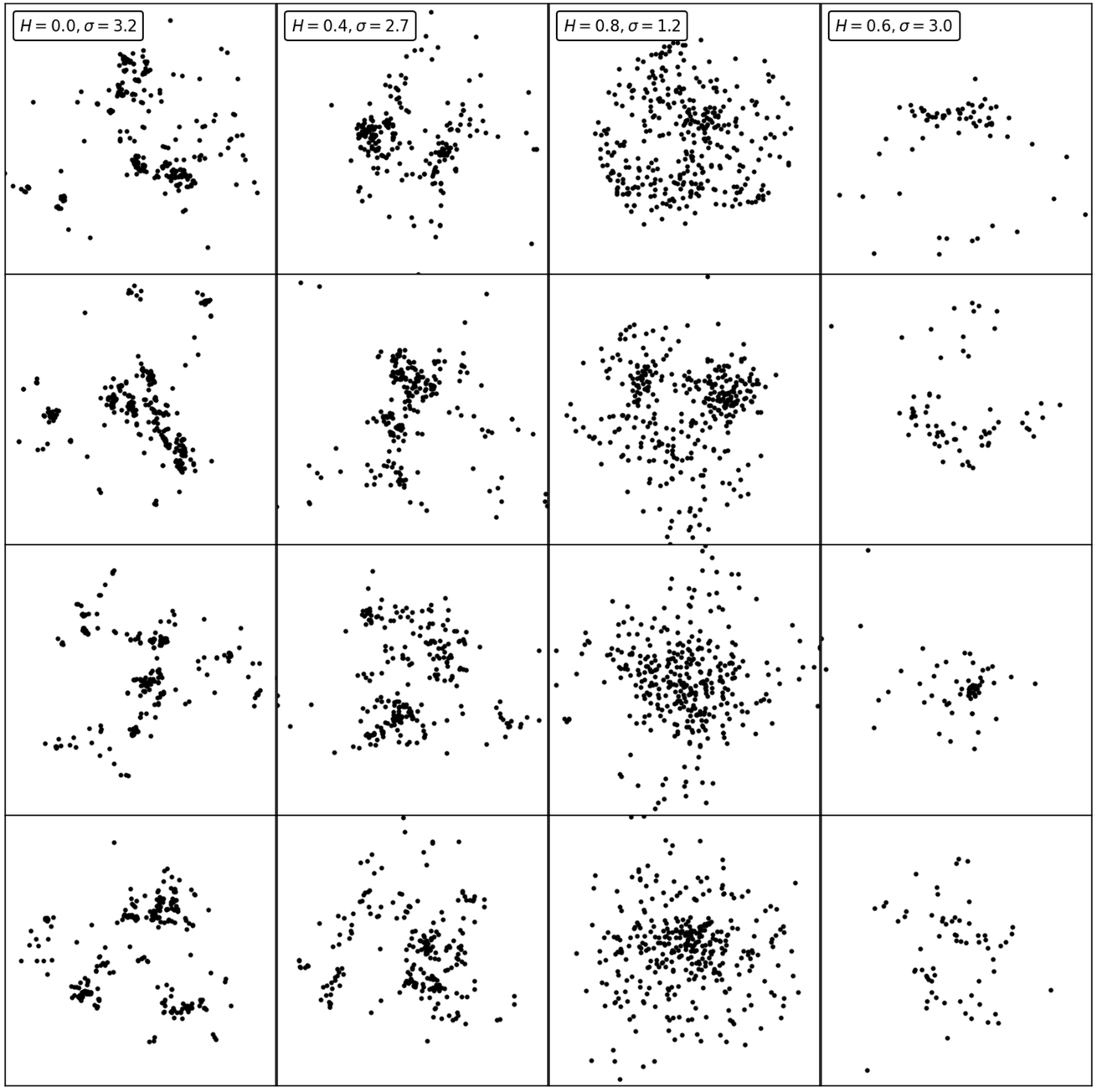
GitHub: *github.com/odlomax/clusterfrac*

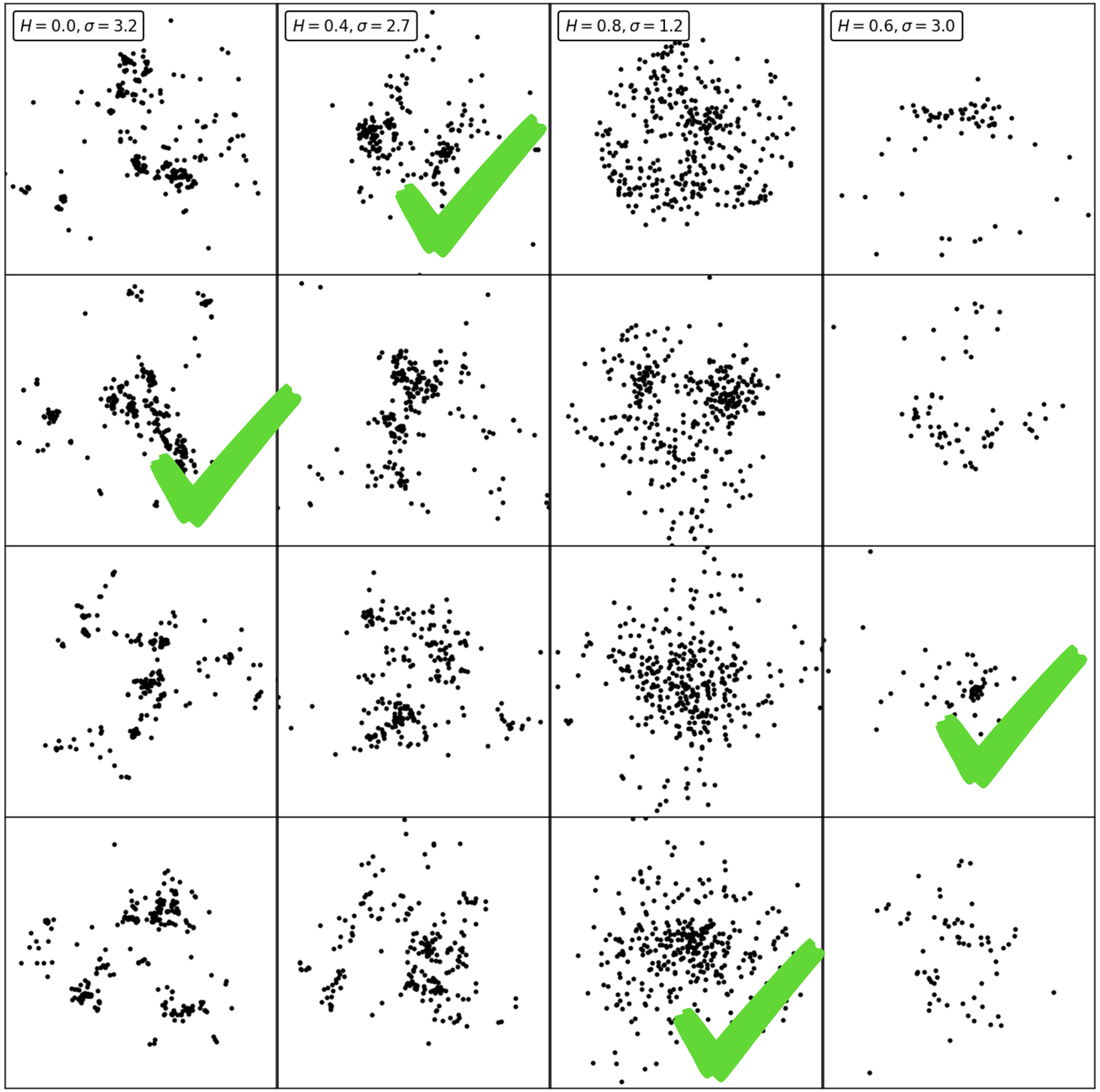




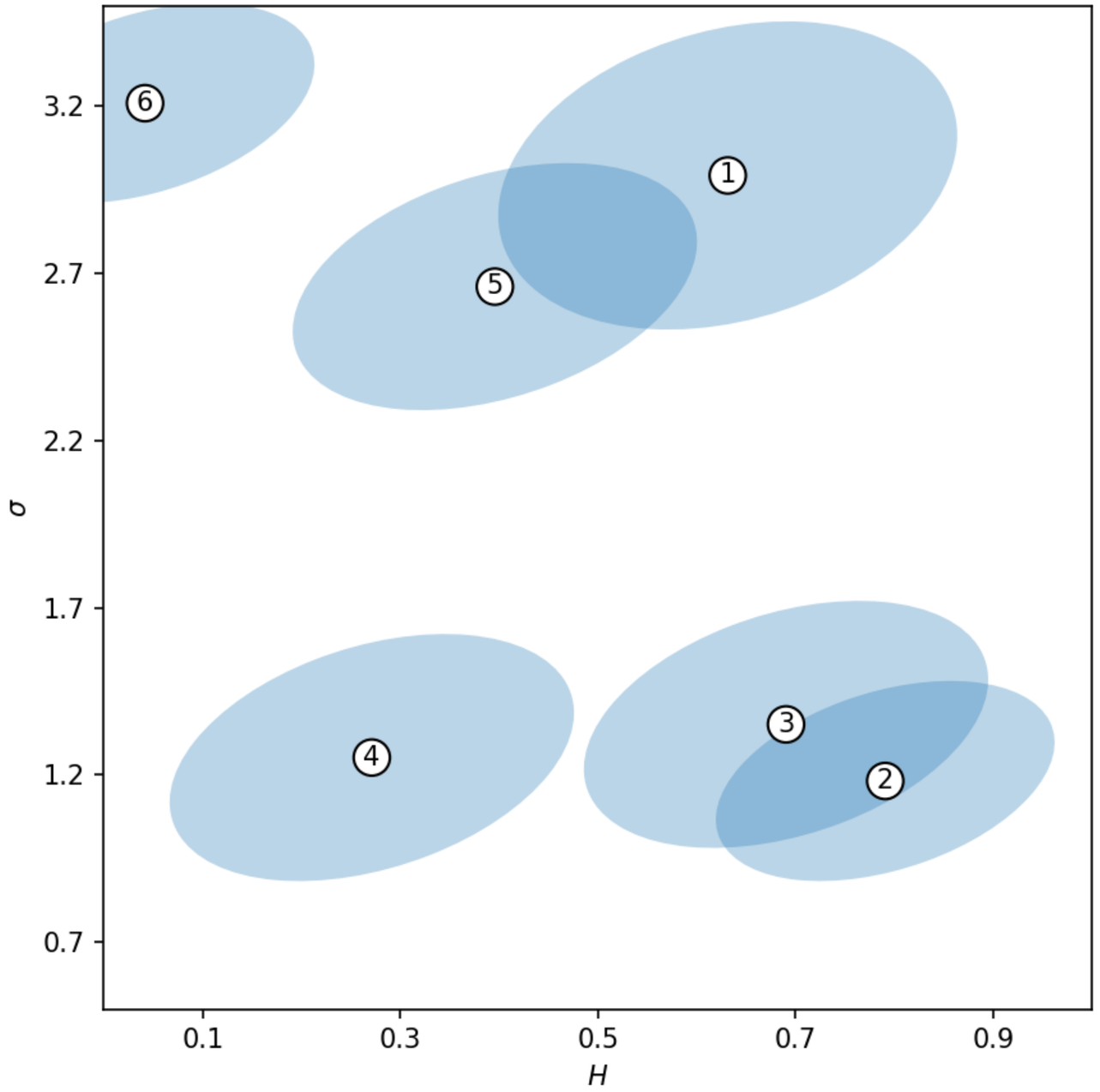
Parameter Estimation




$H = 0.0, \sigma = 3.2$
$H = 0.4, \sigma = 2.7$
$H = 0.8, \sigma = 1.2$
$H = 0.6, \sigma = 3.0$

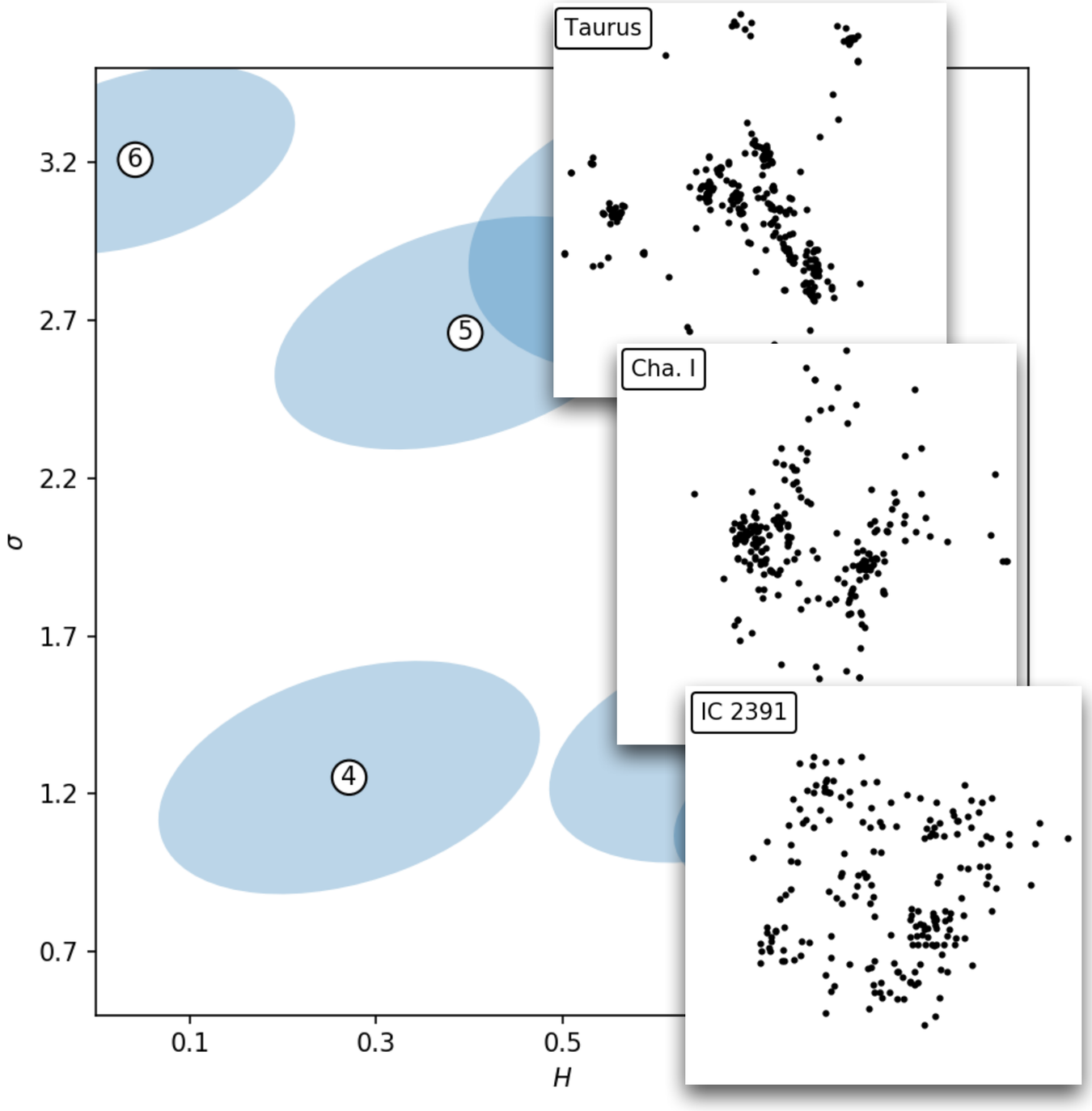

$H = 0.0, \sigma = 3.2$
$H = 0.4, \sigma = 2.7$
$H = 0.8, \sigma = 1.2$
$H = 0.6, \sigma = 3.0$

Cluster	#	*N*
Lupus 3	1	67
IC 348	2	350
ρ Oph	3	198
IC 2391	4	200
Cha I	5	234
Taurus	6	335



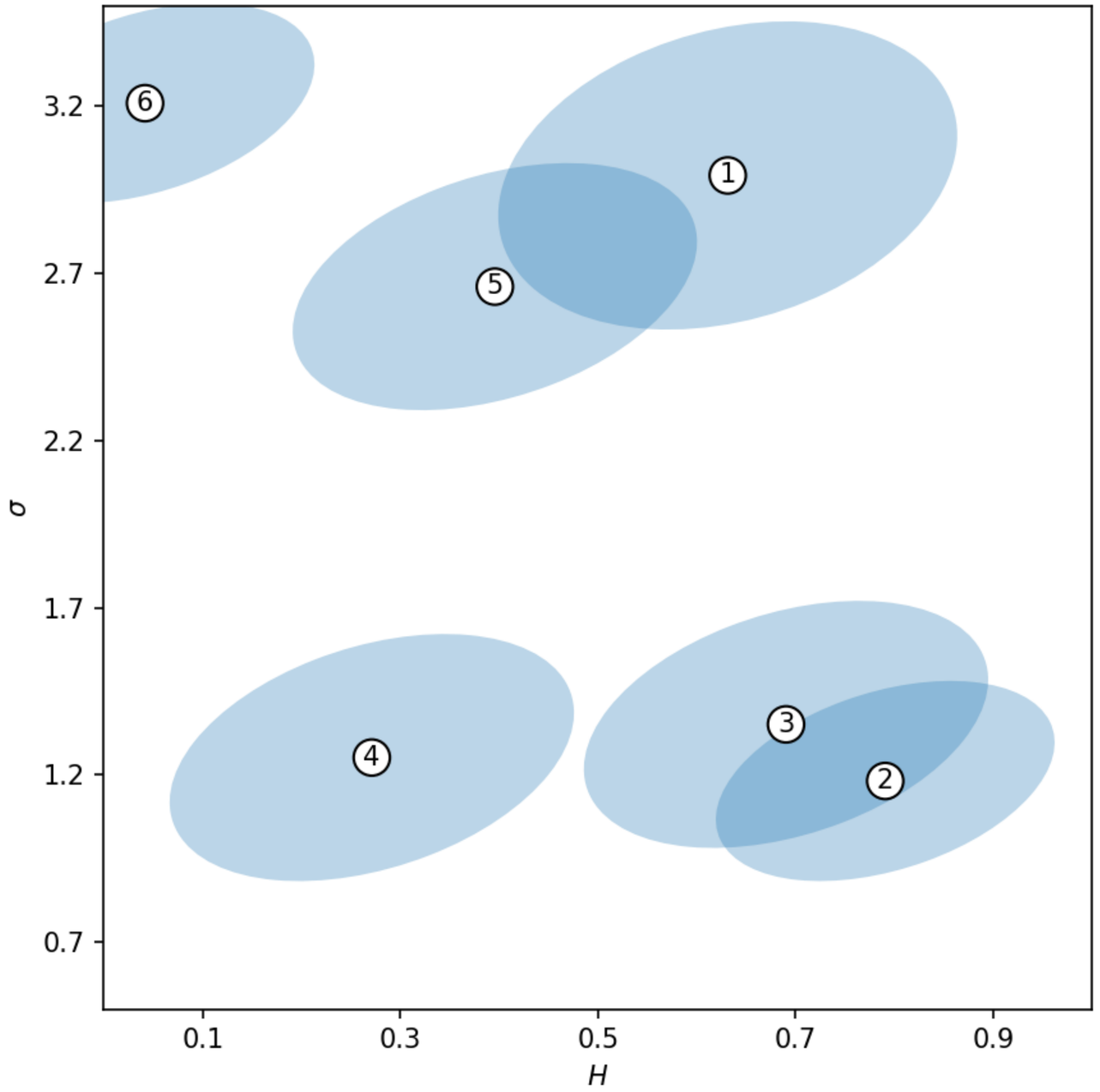
Comerón (2008)
Lada et al. (2006); Muench et al. (2007)
Bontemps et al. (2001)
Barrado y Navascués et al. (2001)
Luhman (2007)
Luhman et al. (2010)

Cluster	#	*N*
Lupus 3	1	67
IC 348	2	350
ρ Oph	3	198
IC 2391	4	200
Cha I	5	234
Taurus	6	335



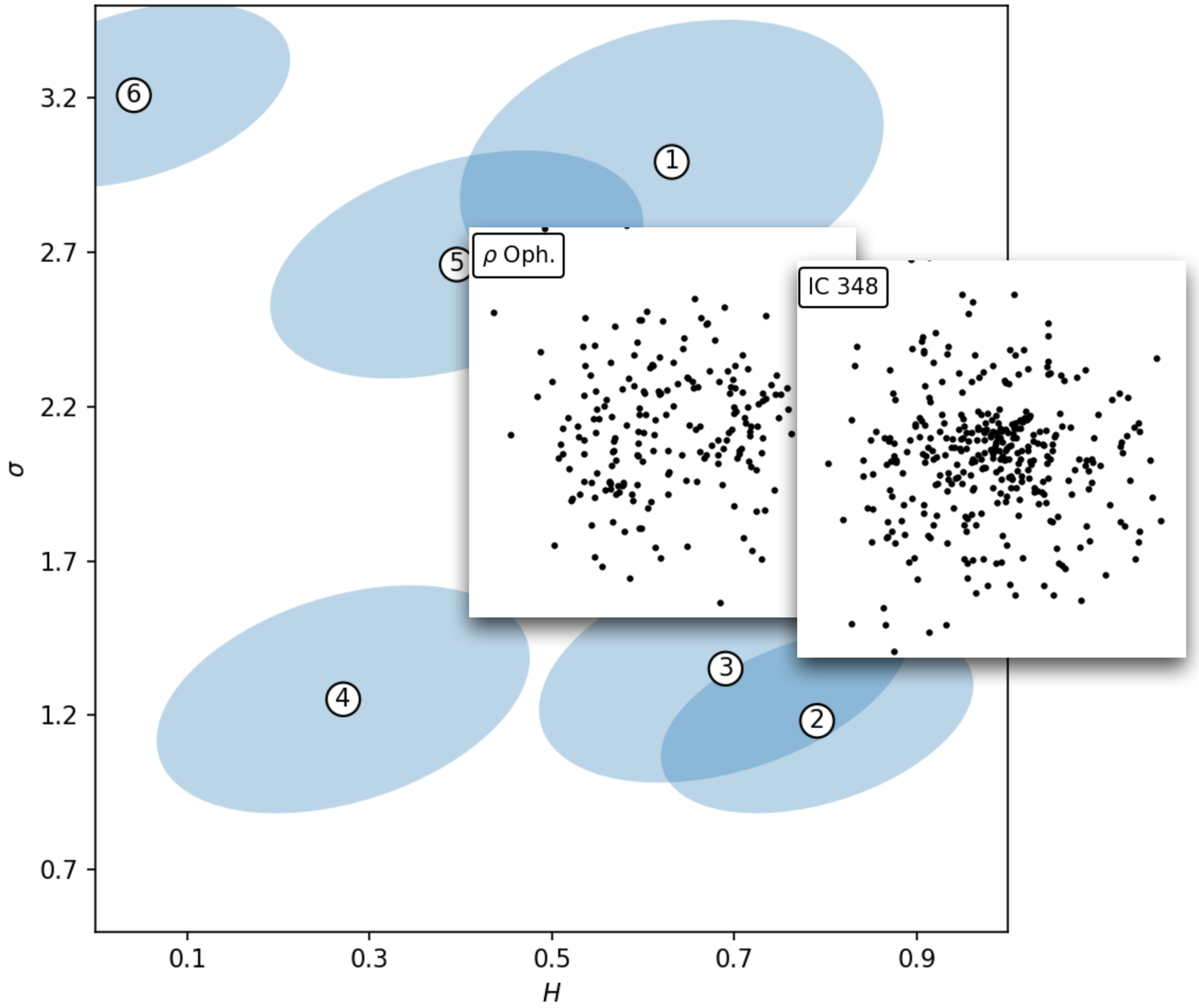
Comerón (2008)
Lada et al. (2006); Muench et al. (2007)
Bontemps et al. (2001)
Barrado y Navascués et al. (2001)
Luhman (2007)
Luhman et al. (2010)

Cluster	#	*N*
Lupus 3	1	67
IC 348	2	350
ρ Oph	3	198
IC 2391	4	200
Cha I	5	234
Taurus	6	335



Comerón (2008)
Lada et al. (2006); Muench et al. (2007)
Bontemps et al. (2001)
Barrado y Navascués et al. (2001)
Luhman (2007)
Luhman et al. (2010)

Cluster	#	*N*
Lupus 3	1	67
IC 348	2	350
ρ Oph	3	198
IC 2391	4	200
Cha I	5	234
Taurus	6	335



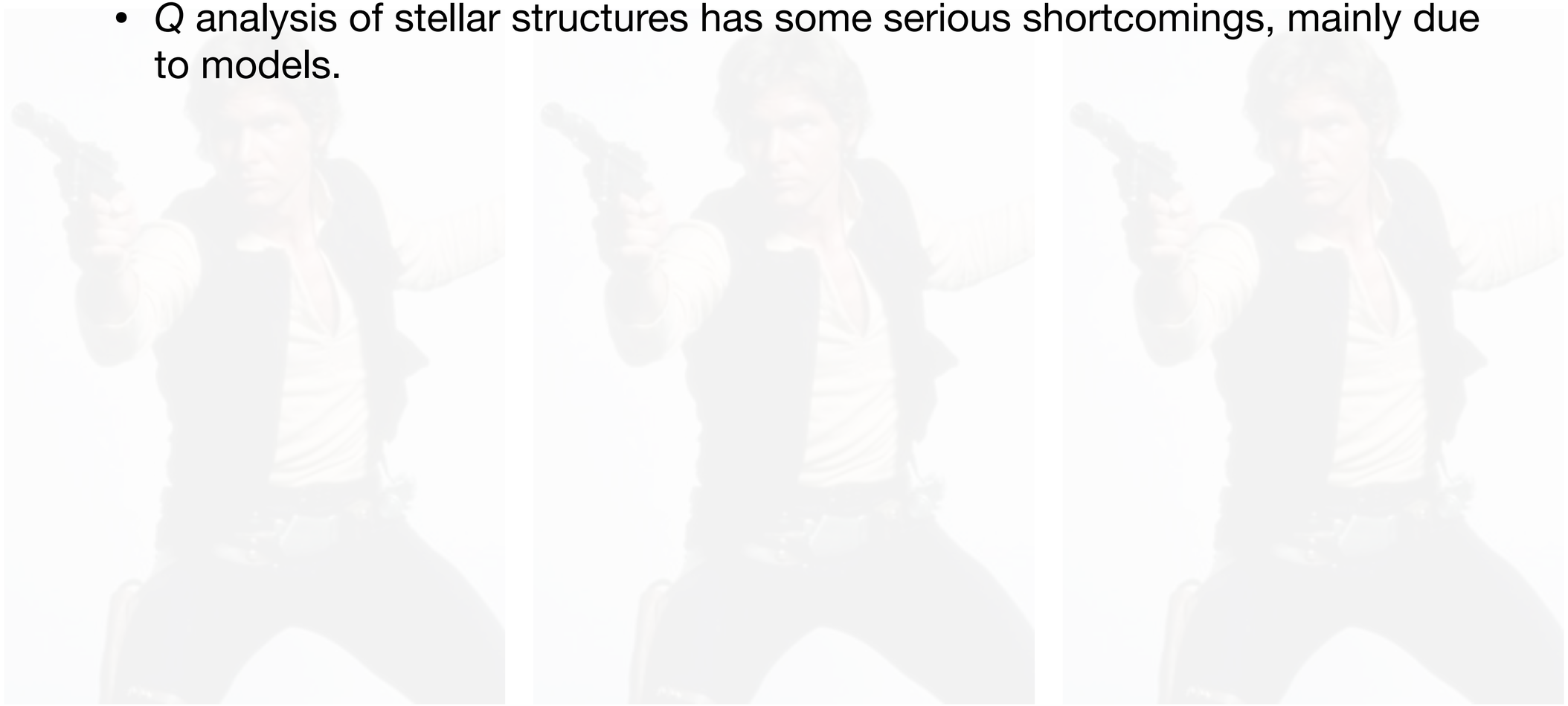
Comerón (2008)
Lada et al. (2006); Muench et al. (2007)
Bontemps et al. (2001)
Barrado y Navascués et al. (2001)
Luhman (2007)
Luhman et al. (2010)

HANS!

Conclusions

Conclusions

- *Q* analysis of stellar structures has some serious shortcomings, mainly due to models.



Conclusions

- *Q* analysis of stellar structures has some serious shortcomings, mainly due to models.
- Fractional Brownian motion (FBM) provides a convenient model of star cluster morphology, related to molecular clouds.

Conclusions

- *Q* analysis of stellar structures has some serious shortcomings, mainly due to models.
- Fractional Brownian motion (FBM) provides a convenient model of star cluster morphology, related to molecular clouds.
- Machine learning techniques can be used to modernise *Q* analysis.

Conclusions

- *Q* analysis of stellar structures has some serious shortcomings, mainly due to models.
- Fractional Brownian motion (FBM) provides a convenient model of star cluster morphology, related to molecular clouds.
- Machine learning techniques can be used to modernise *Q* analysis.
- So far, we can classify clusters, and generate synthetic analogues. Simulations needed to better understand physical meaning of parameters.

Conclusions

- *Q* analysis of stellar structures has some serious shortcomings, mainly due to models.
- Fractional Brownian motion (FBM) provides a convenient model of star cluster morphology, related to molecular clouds.
- Machine learning techniques can be used to modernise *Q* analysis.
- So far, we can classify clusters, and generate synthetic analogues. Simulations needed to better understand physical meaning of parameters.

Article: *Lomax et al., 2018, MNRAS, 480, 371*
arXiv: *1804.06844*
GitHub: *github.com/odlomax/clusterfrac*