Standard Model of Particle Physics

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Lecture 5 — Flavourdynamics and Non-Perturbative QCD II

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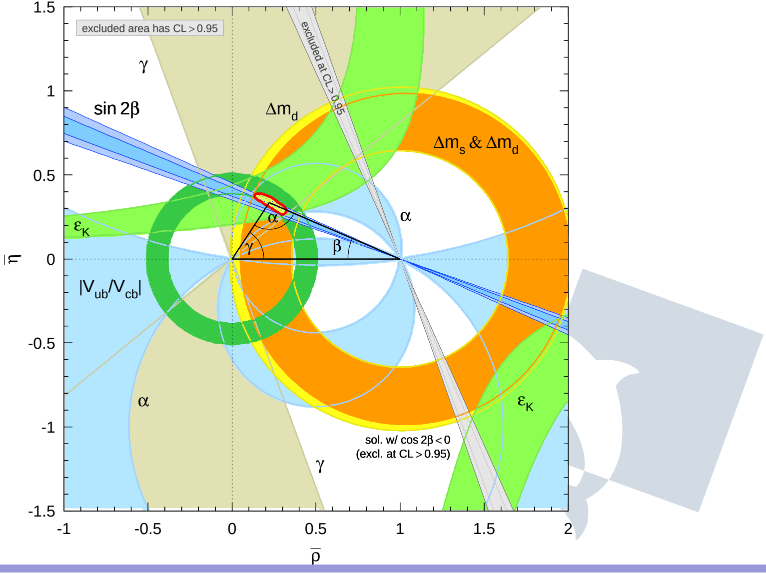
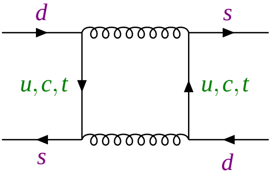
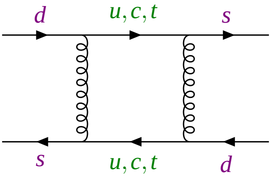
PDG2006 Unitarity Triangle



$K^0 - \bar{K}^0$ Mixing





- The CP-eigenstates (K_1 and K_2) are linear combinations of the two strong-interaction eigenstates:

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \qquad CP|K_1\rangle = |K_1\rangle$$

and

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \qquad CP|K_2\rangle = -|K_2\rangle\ .$$

  - I use the phase convention so that $CP|K^0\rangle = |\bar{K}^0\rangle$.

K^0 – $\bar{K}^0$ Mixing Cont.

- Because of the complex phase in the CKM-matrix, the physical states (the mass eigenstates) differ from $|K_1\rangle$ and $|K_2\rangle$ by a small admixture of the other state:

$$|K_S\rangle = \frac{|K_1\rangle + \bar{\varepsilon}\,|K_2\rangle}{(1+|\bar{\varepsilon}|^2)^{\frac{1}{2}}} \quad \text{and} \quad |K_L\rangle = \frac{|K_2\rangle + \bar{\varepsilon}\,|K_1\rangle}{(1+|\bar{\varepsilon}|^2)^{\frac{1}{2}}} \ ,$$

  - The parameter $\bar{\varepsilon}$ depends on the phase convention chosen for $|K^0\rangle$ and $|\bar{K}^0\rangle$.

$K^0 - \bar{K}^0$ Mixing Cont.

- For $K \to \pi\pi$ and $K \to \pi\pi\pi$ decays, the two pion states are CP-even and the three-pion states are CP-odd $\Rightarrow$ the dominant decays are:

$$K_S \to \pi\pi \quad \text{and} \quad K_L \to 3\pi \ .$$

  - This is the reason why K_L is much longer lived than K_S.

- K_L and K_S are not CP-eigenstates, however $\Rightarrow$ $K_L \to 2\pi$ and $K_S \to 3\pi$ decays may occur.

- CP-violating decays which occur due to the fact that the mass eigenstates are not CP-eigenstates are called *indirect CP-violating decays*.

A measure of the strength of indirect CP-violation is given by the physical parameter ε_K defined by the ratio:

$$\varepsilon_K \equiv \frac{A\left(K_L \to (\pi\pi)_{I=0}\right)}{A\left(K_S \to (\pi\pi)_{I=0}\right)} = (2.280 \pm 0.013)\, 10^{-3}\, e^{i\frac{\pi}{4}} \ .$$

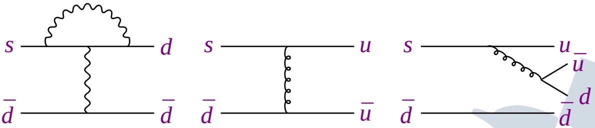
- Directly CP-violating decays are those in which a CP-even (-odd) state decays into a CP-odd (-even) one:

$$K_L \propto K_2 + \bar{\varepsilon} K_1 \,.$$

Direct (ε'): $K_2 \to \pi\pi$

Indirect (ε_K): $K_1 \to \pi\pi$

- Consider the following contributions to $K \to \pi\pi$ decays:



$I = 0$, Complex (a) $I = 0$, Real (b) $I = 0$ or 2, Real (c)

- Thus direct CP-violation in kaon decays manifests itself as a non-zero relative phase between the $I = 0$ and $I = 2$ amplitudes.
- We also have *strong phases*, δ_0 and δ_2 which are independent of the form of the weak Hamiltonian.

$K \to \pi\pi$ Decays Cont.

$$
\begin{aligned}
A(K^0 \to \pi^+\pi^-) &= \sqrt{\frac{2}{3}} A_0\, e^{i\delta_0} + \sqrt{\frac{1}{3}} A_2\, e^{i\delta_2} \\
A(K^0 \to \pi^0\pi^0) &= \sqrt{\frac{2}{3}} A_0\, e^{i\delta_0} - 2\sqrt{\frac{1}{3}} A_2\, e^{i\delta_2} \ .
\end{aligned}
$$

- The parameter ε', which is used as a measure of CP-violation is defined by:

$$
\varepsilon' = \frac{\omega}{\sqrt{2}}\, e^{i\phi} \left(\frac{\mathrm{Im} A_2}{\mathrm{Re} A_2} - \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0} \right) ,
$$

where

$$
\omega \equiv \frac{\mathrm{Re} A_2}{\mathrm{Re} A_0} \quad \text{and} \quad \phi = \frac{\pi}{2} + \delta_2 - \delta_0 \simeq \frac{\pi}{4} \ .
$$

  - ε' is manifestly zero if the phases of the $I = 0$ and $I = 2$ weak amplitudes are the same.
  - The $\Delta I = 1/2$ rule puzzle - *Why is ω^{-1} so large?* $(\omega^{-1} \simeq 22.)$

• Experimentally the two parameters ε_K (which, following standard conventions I rename from now on as ε, $\varepsilon \equiv \varepsilon_K$) and ε' can be determined by measuring the ratios:

$$\begin{aligned} \eta_{00} &\equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \simeq \varepsilon - 2\varepsilon' \\ \eta_{+-} &\equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \simeq \varepsilon + \varepsilon' \, . \end{aligned}$$

• Direct CP-violation is found to be considerably smaller than indirect violation. By measuring the decays and using

$$\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \simeq 1 - 6\,\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) + \cdots ,$$

The NA31 and E371 experiments have measured ε'/ε, and the combined result is:

$$\varepsilon'/\varepsilon = (17.2 \pm 1.8)\,10^{-4} \, .$$

ε and the Unitarity Triangle

- We need to know the matrix element:

$$\langle \bar{K}^0 \,|\, \mathscr{H}_{\text{eff}}^{\Delta S=2} \,|\, K^0 \rangle \;.$$

The form of the effective Hamiltonian is

$$\mathscr{H}_{\text{eff}}^{\Delta S=2} = \frac{G_F^2}{16\pi^2} M_W^2 \, \mathscr{X} \, O^{\Delta S=2}(\mu)$$

where $\mathscr{X}$ is a function of the CKM-matrix elements, with coefficients which can be calculated perturbatively and which depend on the $(u,)c,t$ masses.

- The non-perturbative QCD corrections are contained in the matrix element:

$$\langle \bar{K}^0 \,|\, \bar{s}\gamma^\mu(1-\gamma^5)d \; \bar{s}\gamma_\mu(1-\gamma^5)d \,|\, K^0 \rangle \equiv \frac{8}{3} m_K^2 f_K^2 B_K(\mu) \;.$$

- Uncertainty in B_K is a major restriction on the Unitarity Triangle analysis.

Recent Lattice Results for B_K

- Recent summaries of the quenched value of B_K include:

$$\begin{aligned} B_K^{\overline{\rm MS}}(2\,{\rm GeV}) &= 0.58(4) \qquad \text{S.Hashimoto (ICHEP 2004)} \\ B_K^{\overline{\rm MS}}(2\,{\rm GeV}) &= 0.58(3) \qquad \text{C.Dawson (Lattice 2005)}\,. \end{aligned}$$

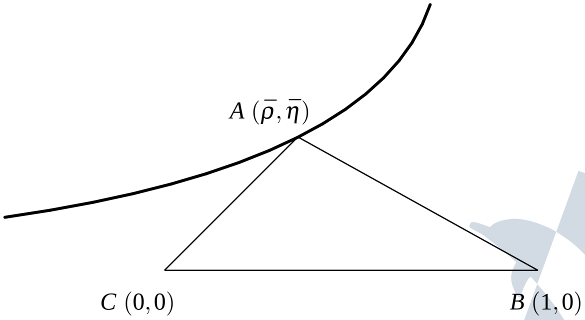
- Dynamical computations of B_K are underway by a number of collaborations, but so far the results are very preliminary.

  C.Dawson's guesstimate (from comparison of unquenched & quenched results at similar masses and lattice spacings)

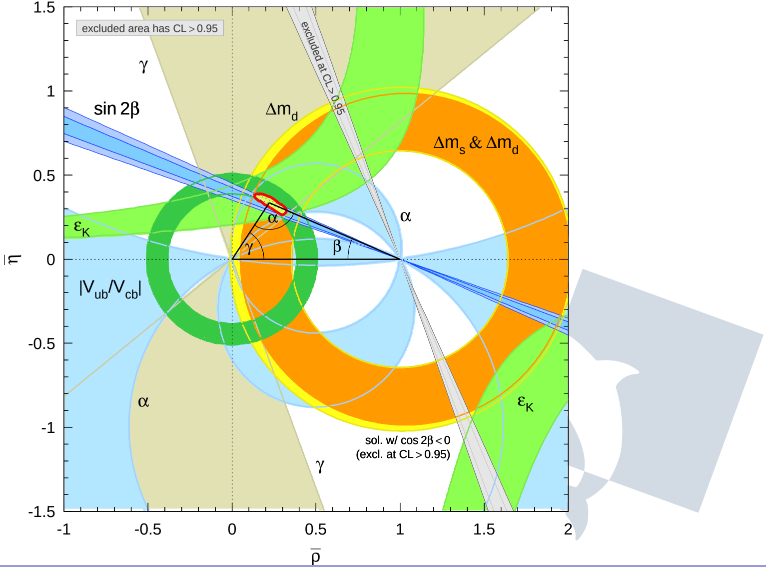
$$B_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.58(3)(6) \qquad \text{C.Dawson (Lattice 2005)}\,.$$

  We need to wait until reliable dynamical results are available in the next year or two.

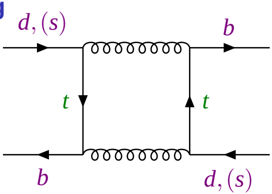
- A precise determination of ε would fix the vertex A to lie on a hyperbola

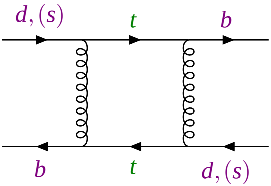


PDG2006 Unitarity Triangle



$B^0 - \overline{B}^0$ Mixing





- In $B^0 - \overline{B}^0$ mixing, the top quark dominates and hence from the measured mass differences $\Rightarrow V_{td}$ and V_{ts}.
- The non-perturbative QCD effects are contained in the matrix element of the $\Delta B = 2$ operator:

$$O^{\Delta B=2} = \bar{b}\gamma^\mu(1-\gamma^5)d\ \bar{b}\gamma_\mu(1-\gamma^5)d\ \equiv \frac{8}{3}m_B^2 f_B^2 B_B(\mu)\,.$$

  The uncertainty in this matrix element dominates that in the final answer for $|V_{td}|$.
- PDG2006 use $\Delta m_d = 0.507 \pm 0.004$ and take the lattice value $f_{B_d}\sqrt{\hat{B}_{B_d}} = (244 \pm 11 \pm 24)$ MeV to obtain

$$|V_{td}| = (7.4 \pm 0.8) \times 10^{-3}$$

$B^0 - \overline{B}^0$ Mixing Cont.

- The uncertainties are reduced in the lattice calculation of the ratio

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} = 1.21 \pm 0.04^{+0.04}_{-0.01} \quad \Rightarrow \left|\frac{V_{td}}{V_{ts}}\right| = 0.208^{+0.008}_{-0.006},$$

  where the new Tevatron result of $\Delta m_s = (17.31^{+0.33}_{-0.18} \pm 0.07)\,\mathrm{ps}^{-1}$ has been used.

- From a comprehensive unitarity triangle analysis **without** using the lattice result for the $\Delta B = 2$ matrix element:

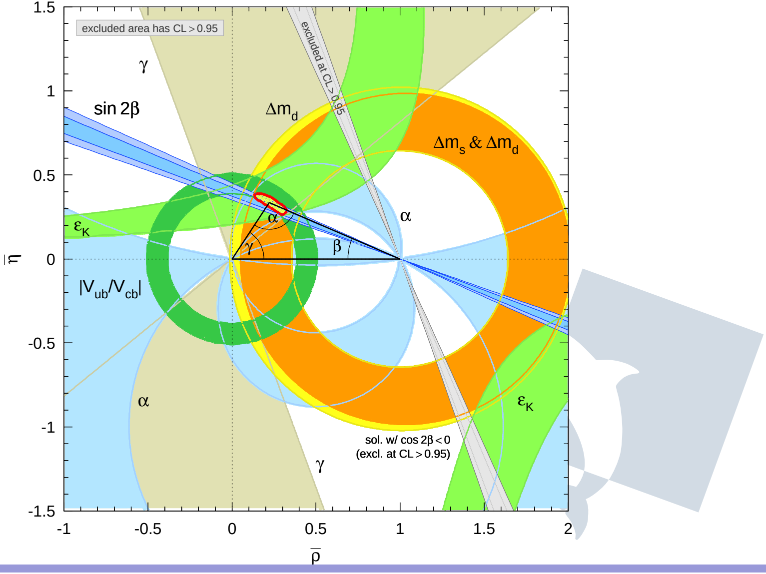
  G.Martinelli (for UTfit Collaboration) - Ringberg April 2006

  - CDF (2006) $\Delta m_S = (17.33^{+.42}_{-.21}(\mathrm{stat}) \pm 0.07\mathrm{syst})\,\mathrm{ps}^{-1}$

$$\begin{aligned} &\Rightarrow \quad \xi = 1.15 \pm 0.08 \quad (V_{ub}\ \text{exclusive}) \\ &\text{or} \quad \xi = 1.05 \pm 0.10 \quad (V_{ub}\ \text{inclusive}) \\ &\text{or} \quad \xi = 1.06 \pm 0.09 \quad (V_{ub}\ \text{combined}) \end{aligned}$$

- $V_{td} \propto 1 - \bar{\rho} - i\bar{\eta}\,.$

PDG2006 Unitarity Triangle



The Golden Mode - $B \to K_S J/\Psi$

Mixing Induced CP-Violating Decays

- In order to study CP-violation we need to be sensitive to the weak phase $\Rightarrow$ *interference*.
- The strong interactions also generate phases, so, in general, we need to be able to control the hadronic effects.
- For the golden-mode $B \to J/\Psi K_s$ this is possible to a great degree of accuracy $\Rightarrow$ precise determination of $\sin(2\beta)$. I will now review the theoretical background behind this statement.
- The two neutral mass-eigenstates are given by

$$|B_L\rangle = \frac{1}{\sqrt{p^2+q^2}}\left(p\,|B^0\rangle + q\,|\bar{B}^0\rangle\right)$$

and

$$|B_H\rangle = \frac{1}{\sqrt{p^2+q^2}}\left(p\,|B^0\rangle - q\,|\bar{B}^0\rangle\right).$$

where p and q are complex parameters.

- The 2×2 mass-matrix takes the form

$$M - \frac{i\Gamma}{2} = \begin{pmatrix} A & p^2 \\ q^2 & A \end{pmatrix} .$$

where A, p and q are complex parameters.

- Starting with a B^0 meson at time $t = 0$, its subsequent evolution is governed by the Schrödinger equation:

$$|B^0_{\rm phys}(t)\rangle = g_+(t)\,|B^0\rangle + \left(\frac{q}{p}\right) g_-(t)|\bar{B}^0\rangle\ ,$$

where

$$\begin{aligned} g_+(t) &= \exp\left[-\frac{\Gamma t}{2}\right]\exp[-iMt]\cos\left(\frac{\Delta M\,t}{2}\right), \\ g_-(t) &= \exp\left[-\frac{\Gamma t}{2}\right]\exp[-iMt]\,i\sin\left(\frac{\Delta M\,t}{2}\right), \end{aligned}$$

and $M = (M_H + M_L)/2$.

- Starting with a $\bar{B}^0$ meson at $t = 0$, the time evolution is

$$|\bar{B}^0_{\rm phys}(t)\rangle = (p/q)\,g_-(t)|\bar{B}^0\rangle + g_+(t)\,|\bar{B}^0\rangle.$$

Decays of Neutral B-Mesons into CP-Eigenstates

- Let f_{CP} be a CP-eigenstate and $A, \bar{A}$ be the amplitudes

$$A \equiv \langle f_{CP} | \mathscr{H} | B^0 \rangle \;\text{ and }\; \bar{A} \equiv \langle f_{CP} | \mathscr{H} | \bar{B}^0 \rangle .$$

- Defining

$$\lambda \equiv \frac{q}{p} \frac{\bar{A}}{A}$$

we have

$$\langle f_{CP} \,|\, \mathscr{H} \,|\, B^0_{\text{phys}} \rangle = A\,[g_+(t) + \lambda\, g_-(t)] \quad \text{and} \quad \langle f_{CP} \,|\, \mathscr{H} \,|\, \bar{B}^0_{\text{phys}} \rangle = A\,\frac{p}{q}\,[g_-(t) + \lambda\, g_+(t)].$$

- The time-dependent rates for initially pure B^0 or $\bar{B}^0$ states to decay into the CP-eigenstate f_{CP} at time t are given by:

$$\Gamma(B^0_{\text{phys}}(t) \to f_{CP}) = |A|^2 e^{-\Gamma t} \times \left[\frac{1+|\lambda|^2}{2} + \frac{1-|\lambda|^2}{2} \cos(\Delta M\, t) - \text{Im}\,\lambda\, \sin(\Delta M\, t) \right]$$

$$\Gamma(\bar{B}^0_{\text{phys}}(t) \to f_{CP}) = |A|^2 e^{-\Gamma t} \times \left[\frac{1+|\lambda|^2}{2} - \frac{1-|\lambda|^2}{2} \cos(\Delta M\, t) + \text{Im}\,\lambda\, \sin(\Delta M\, t) \right].$$

- The time-dependent asymmetry is defined as:

$$\mathscr{A}_{f_{CP}}(t) \equiv \frac{\Gamma(B^0_{\rm phys}(t)\to f_{CP}) - \Gamma(\bar{B}^0_{\rm phys}(t)\to f_{CP})}{\Gamma(B^0_{\rm phys}(t)\to f_{CP}) + \Gamma(\bar{B}^0_{\rm phys}(t)\to f_{CP})} = \frac{(1-|\lambda|^2)\cos(\Delta M\,t) - 2{\rm Im}\,\lambda\,\sin(\Delta M\,t)}{1+|\lambda|^2}.$$

- If $|q/p| = 1$ (which is the case if $\Delta\Gamma \ll \Delta M$) and $|\bar{A}/A| = 1$ (examples of this will be presented below), then $|\lambda| = 1$ and the first term on the right-hand side above vanishes.
- The form of the amplitudes A and $\bar{A}$ is:

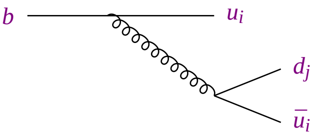
$$A = \sum_i A_i\, e^{i\delta_i}\, e^{i\phi_i} \quad \text{and} \quad \bar{A} = \sum_i A_i\, e^{i\delta_i}\, e^{-i\phi_i}$$

  - Sum is over all the contributions to the process;
  - the A_i are real;
  - the δ_i are the strong phases;
  - the ϕ_i are the phases from the CKM matrix.

$$A = \sum_i A_i \, e^{i\delta_i} \, e^{i\phi_i} \quad \text{and} \quad \bar{A} = \sum_i A_i \, e^{i\delta_i} \, e^{-i\phi_i}$$

- In the most favourable situation, all the contributions have a single CKM phase (ϕ_D say) and
$$\frac{\bar{A}}{A} = \exp(-2i\phi_D).$$
- Since $\Gamma_{12} << M_{12}$, $q/p = \sqrt{M_{12}^*/M_{12}} \equiv \exp(-2i\phi_M)$, and
$$\lambda = \exp(-2i(\phi_D + \phi_M)).$$
Thus
$$\text{Im}\,\lambda = -\sin(2(\phi_D + \phi_M)).$$
- From the box diagrams:
$$\left(\frac{q}{p}\right)_{B_d} = \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}} \quad \text{and} \quad \left(\frac{q}{p}\right)_{B_s} = \frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}}.$$

- Consider processes in which the b-quark decays through the subprocess $b \to d_j u_i \bar{u}_i$. The corresponding tree-level diagram is



for which

$$\frac{\bar{A}}{A} = \frac{V_{ib} V_{ij}^*}{V_{ib}^* V_{ij}} \, .$$

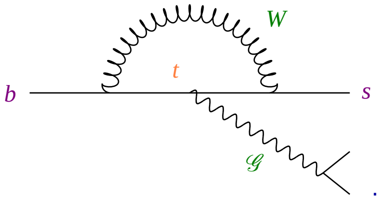
- $B_d \to J/\Psi K_S$ – In this case

$$\lambda(B \to J/\Psi K_S) = \frac{V_{td} V_{tb}^*}{V_{td}^* V_{tb}} \; \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} \; \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \; = -\sin(2\beta)$$

  - The first factor is $(q/p)_{B_d}$;
  - the second factor is the analogous one for the final state kaon;
  - the third factor is $\bar{A}/A$, with $u_i = c$ and $d_j = s$.
  - Recall that

$$\beta = \arg\left(-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right) .$$

- There is also a small penguin contribution to this process:



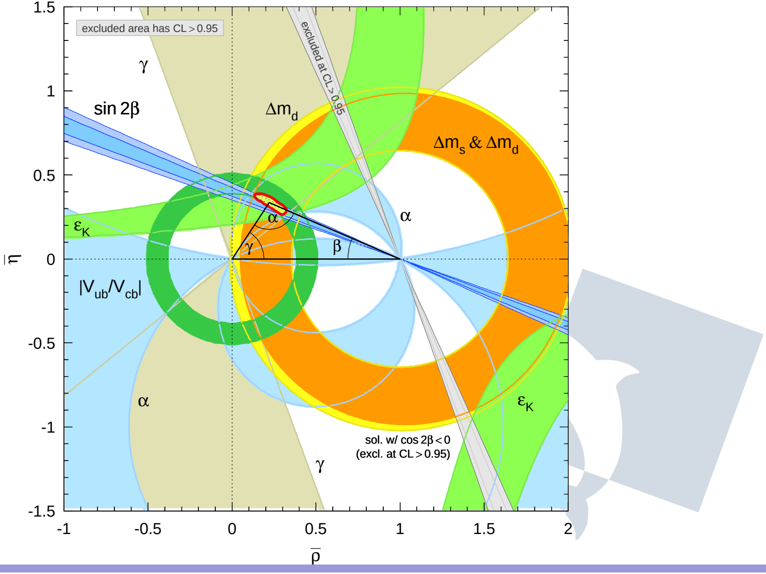
  - Phase is that of $V_{tb}V_{ts}^*$, which is is equal (to an excellent approximation) to that of $V_{cb}V_{cs}^*$.
  - Thus we have a single weak phase and hence hadronic uncertainties are negligible in the determination of the $\sin(2\beta)$ from this process (*golden mode*).
  - This is an (almost) ideal situation but one which is very rare.

- PDG 2006 average the results from BaBar and Belle and obtain

$$\sin(2\beta) = 0.687 \pm 0.032\,.$$

- In PDG 2000, $\sin(2\beta) = 0.78 \pm 0.08\,.$

PDG2006 Unitarity Triangle



Summary and Conclusions

- In these lectures I have tried to remind you of the main elements of the Standard Model of Particle Physics and to describe some of the attempts to explore its limits in the quark sector.

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- The observation of ν oscillations $\Rightarrow$ window on BSM .
- Flavour Physics will continue to be a powerful tool with which to unravel the structure of BSM physics.
- Warm thanks to the organisers for inviting me to such an enjoyable school, to the students for the stimulating questions and to everyone for your excellent company.