Some recent work by our group
G.A., F. Feruglio, I. Masina, hep-ph/0402155,
Reviews:
For ν masses and mixings we do not have so far a Standard Model: many possibilities are still open.

In fact this is the case also for quark and charged leptons; we do not have a theory of flavour that explains the observed spectrum, mixings and CP violation.

ν's are interesting because they can provide new clues on this important problem

In my lectures I will review what we have learnt and the ideas that are being used in model building.

Lecture 1: Basic concepts and experimental facts.
Lecture 2,3: A survey of different classes of models
Summary of basic concepts and experimental facts.

Also, a number of assumptions to restrict the subject of my lectures:

- no exotic interpretation of data
- only 3 active $\nu$'s
- CPT invariance
- \ldots
\[\nu\text{ Oscillations Imply Different }\nu\text{ Masses}\]

\[\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = U \begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}\]

\(U: \text{mixing matrix}\)

\[\nu_e = \cos\theta \nu_1 + \sin\theta \nu_2\]
\[\nu_\mu = -\sin\theta \nu_1 + \cos\theta \nu_2\]

\(\nu_{1,2}: \text{different mass, different x-dep:}\)

\[\nu_a(x) = e^{i p_a x} \nu_a\]

\(p_a^2 = E^2 - m_a^2\)

\[P(\nu_e \leftrightarrow \nu_\mu) = |\langle \nu_\mu(L) | \nu_e \rangle|^2 = \sin^2(2\theta) \sin^2(\Delta m^2 L/4E)\]

At a distance \(L\), \(\nu_\mu\) from \(\mu^-\) decay can produce \(e^-\) via charged weak interact's
Solid evidence for solar and atm. $\nu$ oscillations (+LSND unclear)

$\Delta m^2$ values fixed:
$\Delta m^2_{\text{atm}} \sim 2.5 \times 10^{-3} \, \text{eV}^2,$
$\Delta m^2_{\text{sol}} \sim 8 \times 10^{-5} \, \text{eV}^2$
($\Delta m^2_{\text{LSND}} \sim 1 \, \text{eV}^2$)

mixing angles:
$\theta_{12}$ (solar) large
$\theta_{23}$ (atm) large, ~maximal
$\theta_{13}$ (CHOOZ) small
ν oscillations measure $\Delta m^2$. What is $m^2$?

$\Delta m^2_{\text{atm}} \sim 2.5 \times 10^{-3} \text{ eV}^2$; $\Delta m^2_{\text{sun}} \sim 8 \times 10^{-5} \text{ eV}^2$

- **Direct limits**
  
  $m''_{\nu e} < 2.2 \text{ eV}$
  $m''_{\nu \mu} < 170 \text{ KeV}$
  $m''_{\nu \tau} < 18.2 \text{ MeV}$

  $m_{ee} = |\sum U_{ei}^2 m_i|$

- **0νββ**
  
  $m_{ee} < 0.3 - 0.7 - \text{ ? eV}$ (nucl. matrix elmnts)

  Evidence of signal? Klapdor-Kleingrothaus

- **Cosmology**
  
  $\Omega_\nu h^2 \sim \sum_i m_i / 94\text{ eV}$ (h$^2$~1/2)

  $\sum_i m_i < 0.17 - 0.68 - 2.1 \text{ eV}$ (dep. on data&priors)

Any $\nu$ mass $< 0.06 - 0.23 - 0.7 \text{ eV}$

WMAP, 2dFGRS, Ly-α
$0\nu\beta\beta$ experiments

$$<m_\nu>^2 = \frac{1}{G(Q, Z) IM_{\text{nucl}} l^2 \tau}$$

phase space

matrix element

large uncertainties

Pavan

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Isotope</th>
<th>$\tau_{1/2}^{0\nu}$</th>
<th>range $&lt;m_\nu&gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heidelberg Moscow 2001</td>
<td>$^{76}$Ge</td>
<td>$1.9 \times 10^{25}$</td>
<td>0.3-2.5</td>
</tr>
<tr>
<td>IGEX 2002</td>
<td>$^{76}$Ge</td>
<td>$1.57 \times 10^{25}$</td>
<td>0.3-2.5</td>
</tr>
<tr>
<td>Cuoricino 2005</td>
<td>$^{130}$Te</td>
<td>$2.1 \times 10^{24}$</td>
<td>0.3-0.7</td>
</tr>
<tr>
<td>NEMO 2005</td>
<td>$^{100}$Mo</td>
<td>$4.6 \times 10^{23}$</td>
<td>0.6-1.0</td>
</tr>
</tbody>
</table>

claimed evidence only by a part of the collaboration

started in 2003

$m_{ee} = |\sum U_{ej}^2 m_j e^{i\alpha_j}|$

Future: a factor $\sim 10$ improvement in next decade
\( \nu \) oscillations measure \( \Delta m^2 \). What is \( m^2 \)?

\[
\Delta m^2_{\text{atm}} \sim 2.5 \times 10^{-3} \text{ eV}^2; \quad \Delta m^2_{\text{sun}} \sim 8 \times 10^{-5} \text{ eV}^2
\]

- **Direct limits**
  
  \[
  m_{\nu e}'' < 2.2 \text{ eV} \\
  m_{\nu \mu}'' < 170 \text{ KeV} \\
  m_{\nu \tau}'' < 18.2 \text{ MeV}
  \]

  \[
  m_{ee} = |\sum U_{ei}^2 m_i|<0.3 - 0.7 - \text{? eV} \quad \text{(nucl. matrix elmnts)}
  \]

  Evidence of signal? Klapdor-Kleingrothaus

- **Cosmology**
  
  \[
  \Omega_\nu h^2 \sim \sum_i m_i / 94 \text{eV} \quad (h^2 \sim 1/2)
  \]

  \[
  \sum_i m_i < 0.17-0.68-2.1 \text{ eV} \quad \text{(dep. on data\&priors)}
  \]

- **Any \( \nu \) mass \( < 0.06-0.23-0.7 \text{eV} \)**

- **End-point tritium \( \beta \) decay (Mainz, Troitsk)**
By itself CMB (eg WMAP) is only mildly sensitive to $\sum_i m_i$
Only in combination with Large Scale Structure (2dFGRS, SDSS) the limit becomes stronger.
And even stronger by adding the Lyman alpha forest data (but some tension among the data).

\[
\begin{align*}
\text{CMB only} & \quad \sum m_{\nu_i} < 2.11 \text{ eV} \\
(CMB +) \text{ LSS} & \quad \sum m_{\nu_i} < 0.68 \text{ eV} \\
+ \text{ Ly-} \alpha & \quad \sum m_{\nu_i} < 0.17 \text{ eV}
\end{align*}
\]

95\% cl

Seljac et al ‘06
Neutrino masses are really special!

\[ m_t/(\Delta m^2_{\text{atm}})^{1/2} \approx 10^{12} \]

Massless \( \nu \)'s?
- no \( \nu_R \)
- L conserved

Small \( \nu \) masses?
- \( \nu_R \) very heavy
- L not conserved

Upper limit on \( m_\nu \)

- \( (\Delta m^2_{\text{sol}})^{1/2} \)
- \( (\Delta m^2_{\text{atm}})^{1/2} \)

WMAP

KamLAND
How to guarantee a massless neutrino?

1) $\nu_R$ does not exist

No Dirac mass

$\bar{\nu}_L\nu_R + \nu_R\bar{\nu}_L$

and

2) Lepton Number is conserved

No Majorana mass

$\nu^c \nu \rightarrow \nu^T_R C \nu_R$ or $\nu^T_L C \nu_L$

$C = i\gamma^0\gamma^2$
Neutrinos:  Dirac mass:  $\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L$

(needs $\nu_R$)

$\nu$'s have no electric charge. Their only charge is lepton number $L$.

**IF** $L$ is not conserved (not a good quantum number)

$\nu$ and $\bar{\nu}$ are not really different

TCP, "Lorentz"

$| \nu, h = -1/2 \rangle$  \hspace{1cm}  $| \bar{\nu}, h = +1/2 \rangle$

Majorana mass:  $\nu_R^T \nu_R$ or $\nu_L^T \nu_L$

(we omit the charge conj. matrix $C$)

Violates L, B-L by $|\Delta L| = 2$
Weak isospin I

\( \nu_L \Rightarrow I = 1/2, I_3 = 1/2 \)

\( \nu_R \Rightarrow I = 0, I_3 = 0 \)

**Dirac Mass:**

\[ \bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L \]

Can be obtained from Higgs doublets: \( \nu_L \bar{\nu}_R H \)

**Majorana Mass:**

\( \nu^T \nu_L \quad |\Delta I| = 1 \)

Non ren. dim. 5 operator: \( \nu^T_L \nu_L HH \)

\( \nu^T_R \nu_R \quad |\Delta I| = 0 \)

Directly compatible with SU(2)xU(1)!
See-Saw Mechanism

\[ M \nu^T_R \nu_R \] allowed by SU(2)xU(1)
Large Majorana mass \( M \) (as large as the cut-off)

\[ m_D \bar{\nu}_L \nu_R \] Dirac mass \( m \) from Higgs doublet(s)

\[ \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \]

M \gg m_D

\[ \nu_{light} = -\frac{m^2_D}{M} \quad , \quad \nu_{heavy} = M \]

sign conventional for fermions
In general $\nu$ mass terms are:

$$L_\nu = \bar{L}_h \nu_R H + \text{h.c.} + \nu_R^T M_R \nu_R + \nu_L^T \frac{\lambda}{M_L} \nu_L H H$$

$$m = \frac{\lambda \nu^2}{M_L}$$

More general see-saw mechanism:

$$\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} = \begin{pmatrix} \frac{\lambda \nu^2}{M_L} & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

$$m_{\text{light}} \sim \frac{m_D^2}{M_R}$$

$$m_{\text{heavy}} \sim M_R$$

$$m_{\text{eff}} = \nu_L^T m_{\text{light}} \nu_L$$
Neutrinos are (probably) Majorana particles: $\nu_L^T M \nu_L$

See-saw

$\nu_L$ $m_D$ $H$ $\nu_R$ $m_D$ $H$ $\nu_L$

mass $M$

$m_\nu = m_D^T M^{-1} m_D$

connection with $m_D$

More in general: non ren. $O_5$ operator

$O_5 = \frac{\lambda}{M} L^T H H^T L$

например from

$\nu_L$ $H$ $N$ $H$ $\nu_L$

mass $M$

$N$: new particle $l_w = 0, 1$

Whatever the underlying dynamics $O_5$ is a more general effective description of light Majorana neutrino masses

$\nu$ oscillations point to very large values of $M$
ν's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale $M \sim M_{\text{GUT}}$.

$$m_\nu \sim \frac{m^2}{M}$$

$m \leq m_t \sim v \sim 200 \text{ GeV}$

$M$: scale of L non cons.

Note:

$$m_\nu \sim (\Delta m^2_{\text{atm}})^{1/2} \sim 0.05 \text{ eV}$$

$m \sim v \sim 200 \text{ GeV}$

$M \sim 10^{15} \text{ GeV}$

Neutrino masses are a probe of physics at $M_{\text{GUT}}$!
The current experimental situation is still unclear

- LSND: true or false?  -> MiniBooNE soon will tell
- what is the absolute scale of $\nu$ masses?
- no detection of $0\nu\beta\beta$ (proof that $\nu$'s are Majorana)

Different classes of models are still possible:

If LSND true
- sterile $\nu(s)$??
- CPT violat'n??

If LSND false
- 3 light $\nu$'s are OK

- Degenerate ($m^2 \gg \Delta m^2$)
- Inverse hierarchy
- Normal hierarchy

We assume this case here
3-ν Models

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = U
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

flavour mass

In basis where e\(^-\), \(\mu\(^-\), \(\tau\(^-\) are diagonal:

\[
U = U_{\text{P-MNS}}
\]

Pontecorvo
Maki, Nakagawa, Sakata

\[\delta: \text{CP violation}\]

\[s = \text{solar: large}\]

CHOOZ: \(|s_{13}| < \sim 0.2\]

atm.: \(\sim \max\)

(some signs are conventional)

In general: \(U = U^+_e U_\nu\)
In general 9 parameters:
3 masses, 3 angles,
3 phases

Note:
• $m_\nu$ is symmetric
• phases included in $m_i$

Relation between masses and frequencies:

$P(\nu_e \leftrightarrow \nu_\mu) = P(\nu_\mu \leftrightarrow \nu_\tau) = \frac{1}{2} \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

$P(\nu_\mu \leftrightarrow \nu_\tau) = \sin^2 \Delta_{atm} - \frac{1}{4} \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

\[
\Delta_{sun} = \frac{m_2^2 - m_1^2}{4E} L ; \quad \Delta_{atm} = \frac{m_3^2 - m_1^2}{4E} L
\]

In our def.: $\Delta_{sun} > 0$, $\Delta_{atm} > 0$ or $< 0$
Defining:
\[
\Delta m^2_{\text{atm}} = m_3^2 - m_2^2 > 0 \\
\Delta m^2_{\text{sol}} = m_2^2 - m_1^2 > 0
\]

one has:
\[
m_3^2 = m^2 + \frac{2}{3} \Delta m^2_{\text{atm}} + \frac{1}{3} \Delta m^2_{\text{sol}}
\]
\[
m_2^2 = m^2 - \frac{1}{3} \Delta m^2_{\text{atm}} + \frac{1}{3} \Delta m^2_{\text{sol}}
\]
\[
m_1^2 = m^2 - \frac{1}{3} \Delta m^2_{\text{atm}} - \frac{2}{3} \Delta m^2_{\text{sol}}
\]

and
\[
\frac{2}{m^2} > > \left| \Delta m^2_{\text{atm}} \right| > \Delta m^2_{\text{sol}} \quad \text{degenerate}
\]
\[
\Delta m^2_{\text{atm}} < 0 \quad \text{inverse hierarchy}
\]
\[
\Delta m^2_{\text{atm}} > 0 \quad \text{normal hierarchy}
\]
Parameters in the lepton sector

$$\mathbf{U} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & -s_{23} c_{12} - s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 \\ e^{i \alpha} \\ e^{i \beta} \end{pmatrix}$$

$$0 \leq \theta_{12}, \theta_{23}, \theta_{13} \leq \pi/2, \ 0 \leq \delta \leq 2\pi$$

$$\Delta m_{21}^2$$

$$\left| \Delta m_{32}^2 \right|$$

$$m_{\text{lightest}}$$

$$\alpha$$

$$\beta$$

$$m_{e, \mu, \tau}$$

$$\text{sign}(\Delta m_{32}^2)$$

$$\theta_{12}, \theta_{23}, \theta_{13}, \delta$$
### Neutrino oscillation parameters

- 2 distinct frequencies
- 2 large angles, 1 small

<table>
<thead>
<tr>
<th>parameter</th>
<th>best fit</th>
<th>2σ</th>
<th>3σ</th>
<th>5σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m^2_{21}$ $[10^{-5}\text{eV}^2]$</td>
<td>6.9</td>
<td>6.0–8.4</td>
<td>5.4–9.5</td>
<td>2.1–28</td>
</tr>
<tr>
<td>$\Delta m^2_{31}$ $[10^{-3}\text{eV}^2]$</td>
<td>2.6</td>
<td>1.8–3.3</td>
<td>1.4–3.7</td>
<td>0.77–4.8</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>0.30</td>
<td>0.25–0.36</td>
<td>0.23–0.39</td>
<td>0.17–0.48</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>0.52</td>
<td>0.36–0.67</td>
<td>0.31–0.72</td>
<td>0.22–0.81</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>0.006</td>
<td>$\leq 0.035$</td>
<td>$\leq 0.054$</td>
<td>$\leq 0.11$</td>
</tr>
</tbody>
</table>

Maltoni et al ’04
$2\sigma$ ranges 95%

very precise (KamLAND)

very close to 1/3

\[
\begin{align*}
\delta m^2 &= 7.92 \left(1^{+0.09}_{-0.09}\right) \times 10^{-5} \text{ eV}^2 \\
\Delta m^2 &= 2.4 \left(1^{+0.21}_{-0.26}\right) \times 10^{-3} \text{ eV}^2 \\
\sin^2 \theta_{12} &= 0.314 \left(1^{+0.18}_{-0.15}\right) \\
\sin^2 \theta_{23} &= 0.44 \left(1^{+0.41}_{-0.22}\right) \\
\sin^2 \theta_{13} &< 3.2 \times 10^{-2}
\end{align*}
\]
$\theta_{13}$ bounds

3$\nu$ oscillation parameter bounds on $\theta_{13}$

Fogli et al '05

$\lambda_C^2$

$\sin^2 \theta_{13}$

number of sigma

KamLAND

SOLAR

Sol + Kam

Atm + K2K + CHOOZ

ALL
Measuring $\theta_{13}$ is crucial for future $\nu$-oscill’s experiments (eg CP violation)
A possible time map for $\sin^2 2\theta_{13}$
Improvement of $\Delta m^2_{31}$ and $\sin^2\theta_{23}$
Abs. scale  Normal hierarchy...  OR... Inverted hierarchy  mass^2 splittings

\[ m^2_\nu \]

\[ \delta m^2 \sim 8 \times 10^{-5} \text{ eV}^2 \]
\[ \Delta m^2 \sim 3 \times 10^{-3} \text{ eV}^2 \]
\[ m_\nu < O(1) \text{ eV} \]
\[ \sin^2 \theta_{12} \sim 0.3 \]
\[ \sin^2 \theta_{23} \sim 0.5 \]
\[ \sin^2 \theta_{13} < \text{few\%} \]
\[ \text{sign}(\pm \Delta m^2) \text{ unknown} \]
\[ \delta (\text{CP}) \text{ unknown} \]
Only moderate degeneracy allowed
$0\nu\beta\beta$ would prove that L is not conserved and $\nu$'s are Majorana. Also can tell degenerate, inverted or normal hierarchy.

$$|m_{ee}| = c_{13}^2 [m_1 c_{12}^2 + e^{i\alpha} m_2 s_{12}^2] + m_3 e^{i\beta} s_{13}^2$$

**Degenerate:** $|m||c_{12}^2 + e^{i\alpha} s_{12}^2| \sim |m|(0.3-1)$

$$|m_{ee}| \sim |m| (0.3-1) \leq 0.23 - 1 \text{ eV}$$

**IH:** $\sim (\Delta m_{\text{atm}}^2)^{1/2}|c_{12}^2 + e^{i\alpha} s_{12}^2|$

$$|m_{ee}| \sim (1.6-5) \times 10^{-2} \text{ eV}$$

**NH:** $\sim (\Delta m_{\text{sol}}^2)^{1/2}s_{12}^2 + (\Delta m_{\text{atm}}^2)^{1/2}e^{i\beta}s_{13}^2$

$$|m_{ee}| \sim (\text{few}) \times 10^{-3} \text{ eV}$$

Full dependence on $\min m_\nu$

Present exp. limit: $m_{ee} < 0.3-0.5 \text{ eV}$ (and a hint of signal?????? Klapdor Kleingrothaus)
A most attractive possibility:

**BG via Leptogenesis near the GUT scale**

\[ T \sim 10^{12\pm3} \text{ GeV} \text{ (after inflation)} \]

Only survives if \( \Delta(B-L) \neq 0 \)

(otherwise is washed out at \( T_{\text{ew}} \) by instantons)

Main candidate: decay of lightest \( \nu_R \) (\( M \sim 10^{12} \text{ GeV} \))

L non conserv. in \( \nu_R \) out-of-equilibrium decay:

B-L excess survives at \( T_{\text{ew}} \) and gives the obs. B asymmetry.

Quantitative studies confirm that the range of \( m_i \) from
\( \nu \) oscill's is compatible with BG via (thermal) LG

In particular the bound

\[ m_i < 10^{-1} \text{ eV} \]

Can be relaxed for degenerate neutrinos

So fully compatible with oscill’n data!!
Summary

Cirelli
Model building

Quality factors for models:

• Based on the most general lagrangian compatible with some simple symmetry or dynamical principle

• Should be complete: address at least charged leptons and neutrinos ($U_{\text{P-NMS}} = U^+_e U_\nu$, and the gauge symmetry connects ch. leptons and LH neutrinos)

• As many as possible small parameters (masses and mixings) should be naturally explained as a consequence.

• The necessary vev configuration should be a minimum of the most general potential for a region of parameter space

• The stability under radiative corrections and higher dim operators must be checked

• Simplicity, economy of fields and parameters, predictivity
General remarks

• After KamLAND, SNO and WMAP.... not too much hierarchy is needed for $\nu$ masses:

$$r \sim \Delta m^2_{\text{sol}} / \Delta m^2_{\text{atm}} \sim 1/30$$

Precisely at $2\sigma$: $0.025 < r < 0.049$

or

$m_{\text{heaviest}} < 0.2 - 0.7 \text{ eV}$

$m_{\text{next}} > 8 \times 10^{-3} \text{ eV}$

For a hierarchical spectrum: $\frac{m^2_2}{m^2_3} \sim \sqrt{r} \sim 0.2$

Comparable to: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m^u}{m^e}} \approx 0.24$

Suggests the same “hierarchy” parameters for $q$, $l$, $\nu$

e.g. $\theta_{13}$ not too small!
• Still large space for non maximal 23 mixing

\[ 2-\sigma \text{ interval } 0.32 < \sin^2 \theta_{23} < 0.62 \]

Maximal \( \theta_{23} \) theoretically hard

• \( \theta_{13} \) not necessarily too small
  probably accessible to exp.

Very small \( \theta_{13} \) theoretically hard

"Normal" models: \( \theta_{23} \) large but not maximal,
\( \theta_{13} \) not too small (\( \theta_{13} \) of order \( \lambda_C \) or \( \lambda_C^2 \))

"Exceptional" models: \( \theta_{23} \) very close to maximal and/or \( \theta_{13} \)
very small
or: a special value for \( \theta_{12} \)....
Natural models of the "normal" type are not too difficult to build up.

It is reasonable to attribute hierarchies in masses and mixings to differences in some flavour quantum number(s).

A simplest flavour (or horizontal) symmetry is $U(1)_F$.

For example, some simple models based on see-saw and $U(1)_F$ work for all quark and lepton masses and mixings, are natural and compatible with (SUSY) GUT's, e.g. $SU(5) \times U(1)_F$.

Larger flavour symmetry groups have been studied. They are more predictive but less flexible. The problem of the "best" flavour group is still open.

The most ambitious models try to combine (SUSY) $SO(10)$ GUT's with a suitable flavour group.
Hierarchy for masses and mixings via horizontal $U(1)_F$ charges.

Froggatt, Nielsen '79

**Principle:** A generic mass term

$$\bar{R}_1 m_{12} L_2 H$$

is forbidden by $U(1)$ if $q_1 + q_2 + q_H$ not 0

$U(1)$ broken by vev of "flavon" field $\theta$ with $U(1)$ charge $q_\theta = -1$. If vev $\theta = w$, and $w/M = \lambda$ we get for a generic interaction:

$$\bar{R}_1 m_{12} L_2 H (\theta/M) \Delta_{\text{charge}}$$

$$m_{12} \rightarrow m_{12} \lambda^{q_1+q_2+q_H}$$

**Hierarchy:** More $\Delta_{\text{charge}} \rightarrow$ more suppression ($\lambda$ small)

One can have more flavons ($\lambda, \lambda', ...$) with different charges ($>0$ or $<0$) etc $\rightarrow$ many versions