

St. Andrews, August '06

Models of Neutrino Masses & Mixings

G. Altarelli

Universita' di Roma Tre/CERN

Some recent work by our group

G.A., F. Feruglio, I. Masina, hep-ph/0402155,

G.A., F. Feruglio, hep-ph/0504165, hep-ph/0512103

G.A., R. Franceschini, hep-ph/0512202.

Reviews:

G.A., F. Feruglio, New J.Phys.6:106,2004 [hep-ph/0405048];

G.A., hep-ph/0410101; F. Feruglio, hep-ph/0410131

For ν masses and mixings we do not have so far a Standard Model: many possibilities are still open.

In fact this is the case also for quark and charged leptons; we do not have a theory of flavour that explains the observed spectrum, mixings and CP violation.

ν 's are interesting because they can provide new clues on this important problem

In my lectures I will review what we have learnt and the ideas that are being used in model building.

Lecture 1: Basic concepts and experimental facts.

Lecture 2,3: A survey of different classes of models



Lecture 1

Summary of basic concepts and experimental facts.

Also, a number of assumptions to restrict the subject of my lectures:

e.g. no exotic interpretation of data
only 3 active ν 's
CPT invariance
.....



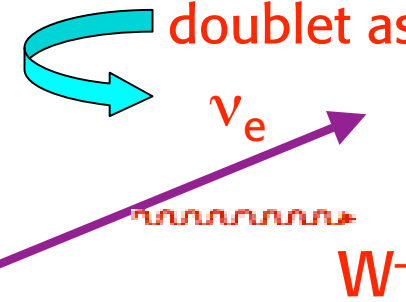
ν Oscillations Imply Different ν Masses

flavour

mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

U: mixing matrix



ν_e : same weak isospin doublet as e^-

$U = U_{\text{P-MNS}}$
Pontecorvo
Maki, Nakagawa, Sakata

$$\begin{aligned} \nu_e &= \cos\theta \nu_1 + \sin\theta \nu_2 \\ \nu_\mu &= -\sin\theta \nu_1 + \cos\theta \nu_2 \end{aligned}$$

e.g 2 flav.

Stationary source:

Stodolsky

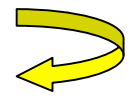
$\nu_{1,2}$: different mass, different x-dep:

$$\nu_a(x) = e^{i p_a x} \nu_a$$

$$p_a^2 = E^2 - m_a^2$$

$$P(\nu_e \leftrightarrow \nu_\mu) = |\langle \nu_\mu(L) | \nu_e \rangle|^2 = \sin^2(2\theta) \sin^2(\Delta m^2 L / 4E)$$

At a distance L , ν_μ from μ^- decay can produce e^- via charged weak interact's



Solid evidence for solar and atmosph. ν oscillations (+LSND unclear)

Δm^2 values fixed:

$$\Delta m^2_{\text{atm}} \sim 2.5 \cdot 10^{-3} \text{ eV}^2,$$

$$\Delta m^2_{\text{sol}} \sim 8 \cdot 10^{-5} \text{ eV}^2$$

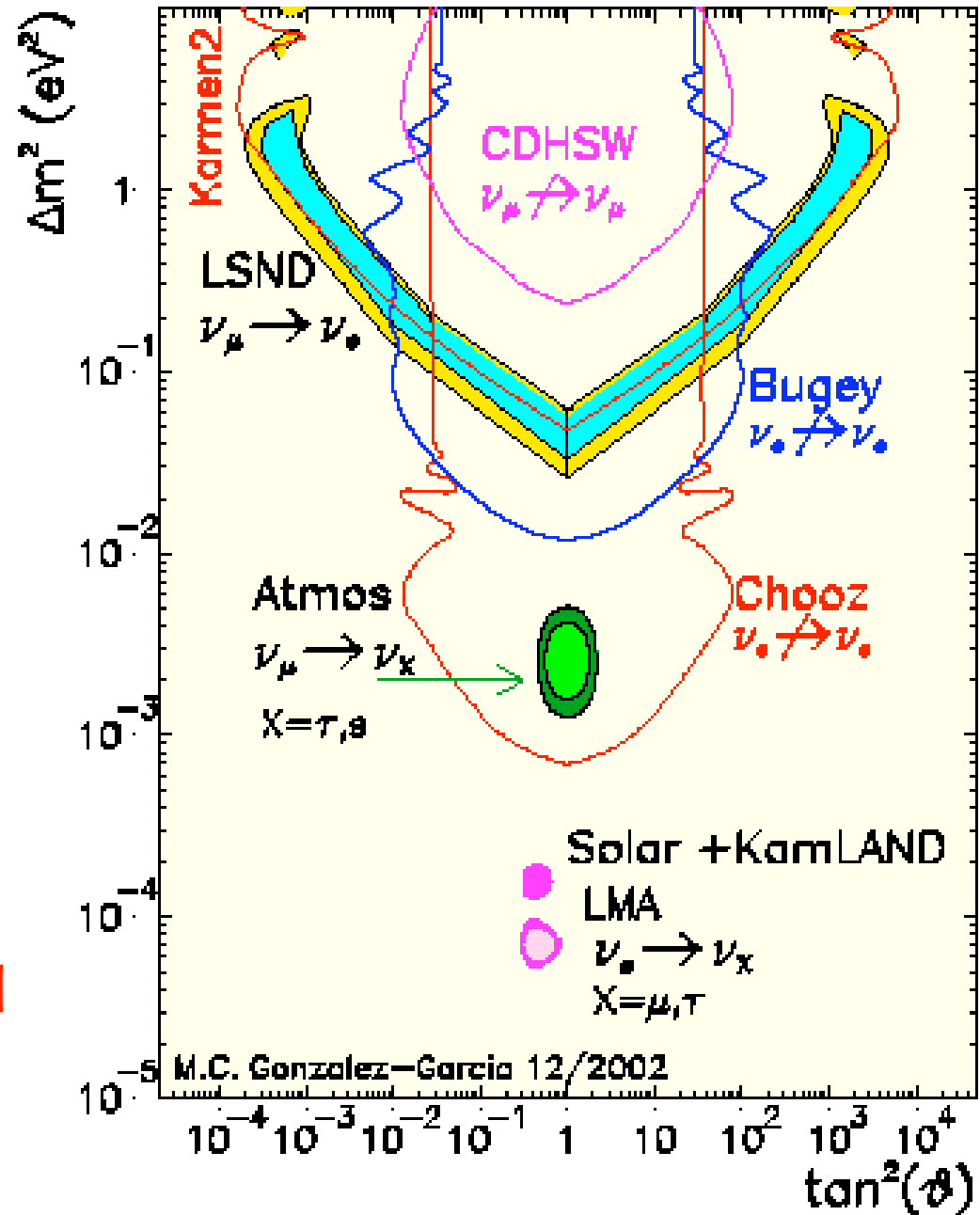
$$(\Delta m^2_{\text{LSND}} \sim 1 \text{ eV}^2)$$

mixing angles:

θ_{12} (solar) large

θ_{23} (atm) large, \sim maximal

θ_{13} (CHOOZ) small



ν oscillations measure Δm^2 . What is m^2 ?

$\Delta m^2_{\text{atm}} \sim 2.5 \cdot 10^{-3} \text{ eV}^2; \quad \Delta m^2_{\text{sun}} \sim 8 \cdot 10^{-5} \text{ eV}^2$

- Direct limits

$m_{\nu e} < 2.2 \text{ eV}$

$m_{\nu \mu} < 170 \text{ KeV}$

$m_{\nu \tau} < 18.2 \text{ MeV}$

End-point tritium β decay (Mainz, Troitsk)

$m_{ee} = |\sum U_{ei}^2 m_i|$

- $0\nu\beta\beta$ $m_{ee} < 0.3 - 0.7 - ? \text{ eV}$ (nucl. matrix elmnts)

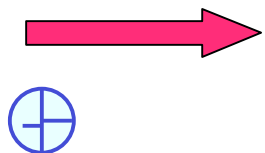
Evidence of signal?

Klapdor-Kleingrothaus

- Cosmology

$\Omega_\nu h^2 \sim \sum_i m_i / 94 \text{ eV} \quad (h^2 \sim 1/2)$

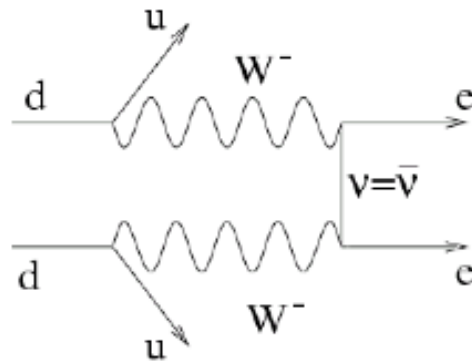
$\sum_i m_i < 0.17 - 0.68 - 2.1 \text{ eV}$ (dep. on data&priors)



Any ν mass $< 0.06 - 0.23 - 0.7 \text{ eV}$

WMAP,
2dFGRS,
Ly- α

$0\nu\beta\beta$ experiments



$$\langle m_\nu \rangle^2 = \frac{1}{G(Q,Z) |M_{\text{nucl}}|^2 \tau}$$

phase space

matrix elmnt
large uncrtns

Pavan

Experiment	Isotope	$\tau_{1/2}^{0\nu}$ [y]	range $\langle m_\nu \rangle$ [eV]
Heidelberg Moscow 2001	^{76}Ge	$1.9 \cdot 10^{25}$	0.3-2.5
IGEX 2002	^{76}Ge	$1.57 \cdot 10^{25}$	0.3-2.5
Cuoricino 2005	^{130}Te	$2 \cdot 10^{24}$	0.3-0.7
NEMO 2005	^{100}Mo	$4.6 \cdot 10^{23}$	0.6-1.0

claimed evidence only by a part of the collaboration

started in 2003

$$m_{ee} = \left| \sum U_{ej}^2 m_j e^{i\alpha_j} \right|$$



Future: a factor ~ 10 improvement in next decade

ν oscillations measure Δm^2 . What is m^2 ?

$\Delta m^2_{\text{atm}} \sim 2.5 \cdot 10^{-3} \text{ eV}^2; \quad \Delta m^2_{\text{sun}} \sim 8 \cdot 10^{-5} \text{ eV}^2$

- Direct limits

$m_{\nu e} < 2.2 \text{ eV}$

$m_{\nu \mu} < 170 \text{ KeV}$

$m_{\nu \tau} < 18.2 \text{ MeV}$

End-point tritium β decay (Mainz, Troitsk)

$m_{ee} = |\sum U_{ei}^2 m_i|$

- $0\nu\beta\beta$ $m_{ee} < 0.3 - 0.7 - ? \text{ eV}$ (nucl. matrix elmnts)

Evidence of signal?

Klapdor-Kleingrothaus

- Cosmology

$\Omega_\nu h^2 \sim \sum_i m_i / 94 \text{ eV} \quad (h^2 \sim 1/2)$

$\sum_i m_i < 0.17 - 0.68 - 2.1 \text{ eV}$ (dep. on data&priors)

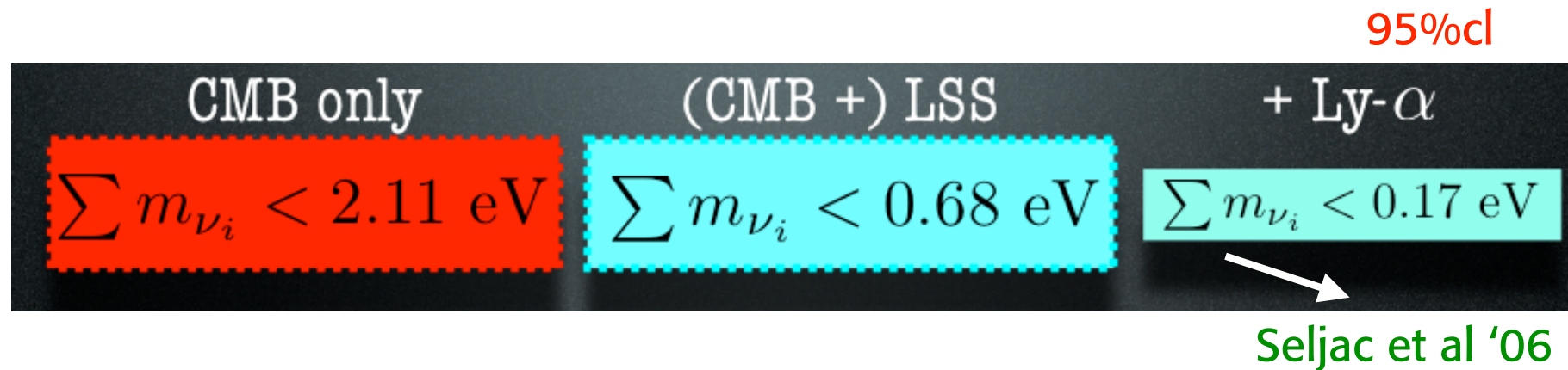
→ Any ν mass $< 0.06 - 0.23 - 0.7 \text{ eV}$

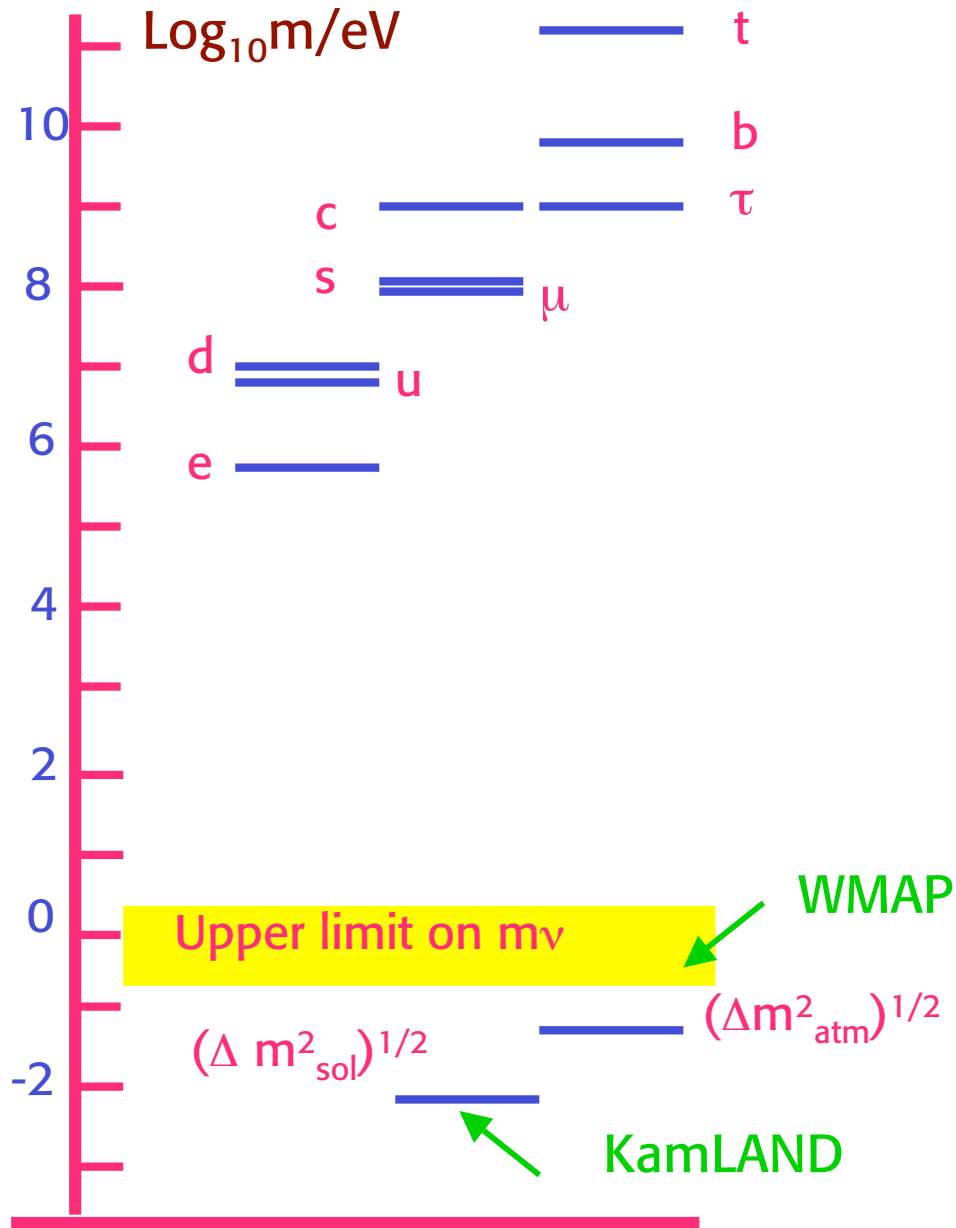


WMAP, SDSS,
2dFGRS,
Ly- α

By itself CMB (eg WMAP) is only mildly sensitive to $\sum_i m_i$
Only in combination with Large Scale Structure (2dFGRS, SDSS) the limit becomes stronger.

And even stronger by adding the Lyman alpha forest data
(but some tension among the data).





Neutrino masses are really special!

$m_t / (\Delta m^2_{atm})^{1/2} \sim 10^{12}$

Massless ν 's?

- no ν_R
- L conserved

Small ν masses?

- ν_R very heavy
- L not conserved



How to guarantee a massless neutrino?

1) ν_R does not exist



No Dirac mass

$$\bar{\nu}_L \nu_R + \nu_R \bar{\nu}_L$$

and

2) Lepton Number is conserved



No Majorana mass

$$\bar{\nu}^c \nu \rightarrow \nu_R^T C \nu_R \text{ or } \nu_L^T C \nu_L$$

$$C = i\gamma^0 \gamma^2$$

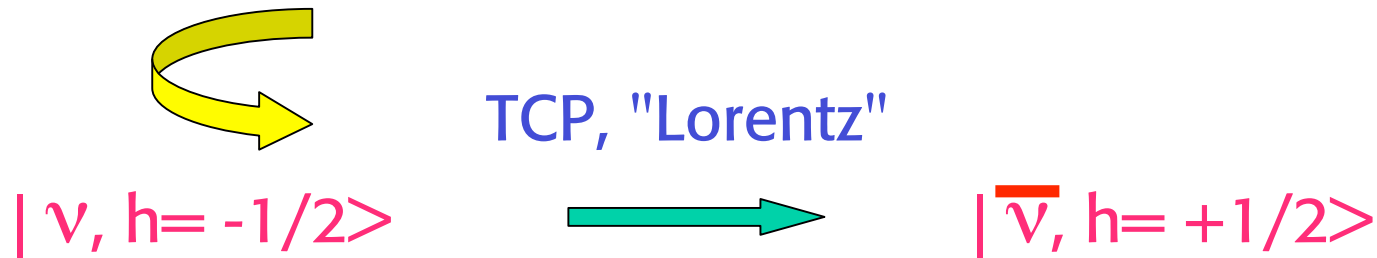


Neutrinos:

Dirac mass: $\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L$
(needs ν_R)

ν 's have no electric charge. Their only charge is lepton number L.

IF L is not conserved (not a good quantum number)
 ν and $\bar{\nu}$ are not really different



Majorana mass: $\nu_R^T \nu_R$ or $\nu_L^T \nu_L$
(we omit the charge conj. matrix C)

Violates L, B-L by $|\Delta L| = 2$



Weak isospin I

$$\nu_L \Rightarrow I = 1/2, I_3 = 1/2$$

$$\nu_R \Rightarrow I = 0, I_3 = 0$$

Dirac Mass:

$$\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L \quad |\Delta I| = 1/2$$

Can be obtained from Higgs doublets: $\nu_L \bar{\nu}_R H$

Majorana Mass:

- $\nu_L^T \nu_L \quad |\Delta I| = 1$

Non ren., dim. 5 operator: $\nu_L^T \nu_L H H$

- $\nu_R^T \nu_R \quad |\Delta I| = 0$

Directly compatible with $SU(2) \times U(1)$!



See-Saw Mechanism

Minkowski;
Yanagida; Gell-Mann, Ramond, Slansky;
Glashow; Mohapatra, Senjanovic.....

 $M \nu_R^T \nu_R$ allowed by $SU(2) \times U(1)$
Large Majorana mass M (as large as the cut-off)

$m_D \bar{\nu}_L \nu_R$ Dirac mass m from Higgs doublet(s)

$$\begin{array}{cc} & \begin{array}{cc} \nu_L & \nu_R \end{array} \\ \begin{array}{c} \nu_L \\ \nu_R \end{array} & \left[\begin{array}{cc} 0 & m_D \\ m_D & M \end{array} \right] \end{array} \quad M \gg m_D$$

Eigenvalues

$$\nu_{\text{light}} = \frac{-m_D^2}{M}, \quad \nu_{\text{heavy}} = M$$

sign conventional
for fermions



In general ν mass terms are:

$$\mathcal{L}_\nu = \bar{L}h\nu_R H + \text{h.c.} + \nu_R^T M_R \nu_R + \nu_L^T \frac{\lambda}{M_L} \nu_L H H$$

Dirac $m_D = hv$
 $v = \langle 0 | H | 0 \rangle$

Majorana $m = \frac{\lambda v^2}{M_L}$

More general see-saw mechanism:

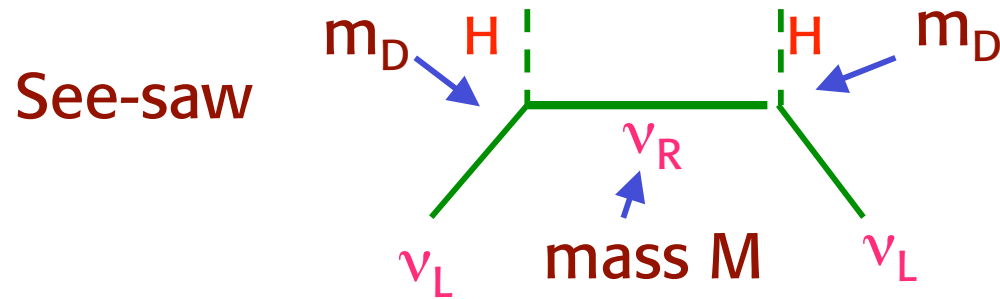
$$\begin{matrix} \nu_L & \nu_R \\ \nu_L & \begin{bmatrix} \lambda v^2 / M_L & m_D \\ m_D & M_R \end{bmatrix} \\ \nu_R & \end{matrix}$$

$m_{\text{light}} \sim \frac{m_D^2}{M_R}$ and/or $\frac{\lambda v^2}{M_L}$
 $m_{\text{heavy}} \sim M_R$ $m_{\text{eff}} = \nu_L^T m_{\text{light}} \nu_L$



Neutrinos are (probably) Majorana particles:

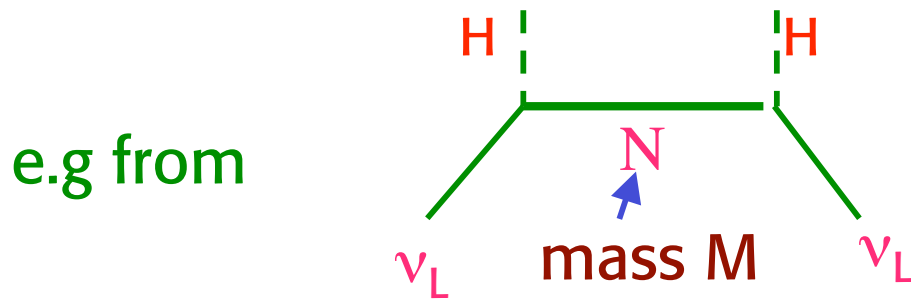
$$\nu_L^T m_\nu \nu_L$$



$$m_\nu = m_D^T M^{-1} m_D$$

connection with m_D

More in general: non ren. O_5 operator $O_5 = \lambda/M L^T H H^T L$



N: new particle $I_w=0,1$

Whatever the underlying dynamics O_5 is a more general effective description of light Majorana neutrino masses



ν oscillations point to very large values of M

A very natural and appealing explanation:

ν 's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale $M \sim M_{\text{GUT}}$

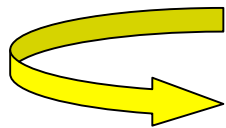
$$m_\nu \sim \frac{m^2}{M}$$

$m: \leq m_t \sim v \sim 200 \text{ GeV}$
 $M: \text{ scale of L non cons.}$

Note:

$$m_\nu \sim (\Delta m_{\text{atm}}^2)^{1/2} \sim 0.05 \text{ eV}$$

$$m \sim v \sim 200 \text{ GeV}$$



$$M \sim 10^{15} \text{ GeV}$$

Neutrino masses are a probe of physics at M_{GUT} !



The current experimental situation is still unclear

- LSND: true or false? -> MiniBooNE soon will tell
- what is the absolute scale of ν masses?
- no detection of $0\nu\beta\beta$ (proof that ν 's are Majorana)

Different classes of models are still possible:

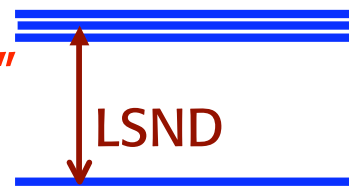
If LSND true

sterile ν (s)??

CPT violat'n??

• "3-1" or "3-n"

$\nu_{sterile}$



$m^2 \sim 1-2 \text{ eV}^2$

If LSND false



3 light ν 's are OK

We assume this case here

- Degenerate ($m^2 \gg \Delta m^2$)  $m^2 < o(1) \text{ eV}^2$

- Inverse hierarchy



- Normal hierarchy

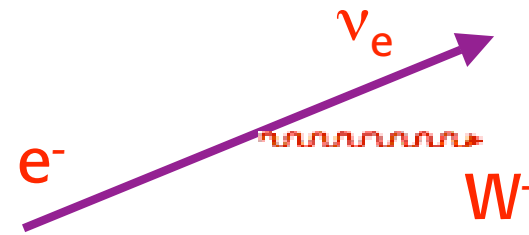


3-ν Models

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavour

mass



$$U = U_{\text{P-MNS}}$$

Pontecorvo

Maki, Nakagawa, Sakata

In basis where e^- , μ^- , τ^- are diagonal:

δ : CP violation

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \sim$$

s = solar: large

$$\sim \begin{pmatrix} c_{13} & c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ \dots & \dots & \dots & c_{13}s_{23} \\ \dots & \dots & \dots & c_{13}c_{23} \end{pmatrix}$$

CHOOZ: $|s_{13}| < \sim 0.2$

atm.: $\sim \text{max}$



$$U \cong \begin{pmatrix} c & -s & 0 \\ s & c & -1 \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(some signs are conventional)

In general: $U = U_e^+ U_\nu$



$m_\nu \sim U^* \begin{bmatrix} e^{i\alpha_1} m_1 & 0 & 0 \\ 0 & e^{i\alpha_2} m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} U^+$

In general 9 parameters:
 3 masses, 3 angles,
 3 phases

$L^T m_\nu L$ For $s_{13} \sim 0$: $0\nu\beta\beta \longrightarrow$

$m_\nu \sim \begin{bmatrix} m_1 c^2 + m_2 s^2 & (m_1 - m_2) cs / \sqrt{2} & (m_1 - m_2) cs / \sqrt{2} \\ \dots & (m_1 s^2 + m_2 c^2 + m_3) / 2 & (m_1 s^2 + m_2 c^2 - m_3) / 2 \\ \dots & \dots & (m_1 s^2 + m_2 c^2 + m_3) / 2 \end{bmatrix}$

- Note:
- m_ν is symmetric
 - phases included in m_i

Relation between masses and frequencies:

$$P(\nu_e \leftrightarrow \nu_\mu) = P(\nu_e \leftrightarrow \nu_\tau) = 1/2 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{\text{sun}}$$

$$P(\nu_\mu \leftrightarrow \nu_\tau) = \sin^2 \Delta_{\text{atm}} - 1/4 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{\text{sun}}$$

$$\Delta_{\text{sun}} = \frac{m_2^2 - m_1^2}{4E} L \quad ; \quad \Delta_{\text{atm}} = \frac{m_3^2 - m_{1,2}^2}{4E} L$$

In our def.: $\Delta_{\text{sun}} > 0$, $\Delta_{\text{atm}} >$ or < 0



Defining:

$$\Delta m_{atm}^2 = m_3^2 - m_2^2 > \text{or} < 0$$

$$\Delta m_{sol}^2 = m_2^2 - m_1^2 > 0$$

one has:

$$m_3^2 = \overline{m^2} + \frac{2}{3}\Delta m_{atm}^2 + \frac{1}{3}\Delta m_{sol}^2$$

$$m_2^2 = \overline{m^2} - \frac{1}{3}\Delta m_{atm}^2 + \frac{1}{3}\Delta m_{sol}^2$$

$$m_1^2 = \overline{m^2} - \frac{1}{3}\Delta m_{atm}^2 - \frac{2}{3}\Delta m_{sol}^2$$

and

$$\overline{m^2} \gg |\Delta m_{atm}^2| > \Delta m_{sol}^2 \quad \text{degenerate}$$

$$\Delta m_{atm}^2 < 0 \quad \text{inverse hierarchy}$$

$$\Delta m_{atm}^2 > 0 \quad \text{normal hierarchy}$$



Parameters in the lepton sector

Romanino

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 \\ e^{i\alpha} \\ e^{i\beta} \end{pmatrix}$$

$$(0 \leq \theta_{12}, \theta_{23}, \theta_{13} \leq \pi/2, \quad 0 \leq \delta \leq 2\pi)$$

$$\begin{array}{l} m_{e,\mu,\tau} \\ \Delta m_{21}^2 \\ |\Delta m_{32}^2| \\ \text{sign}(\Delta m_{32}^2) \\ \theta_{12}, \theta_{23}, \theta_{13}, \delta \end{array} \quad \begin{array}{l} m_{\text{lightest}} \\ \alpha \\ \beta \end{array}$$

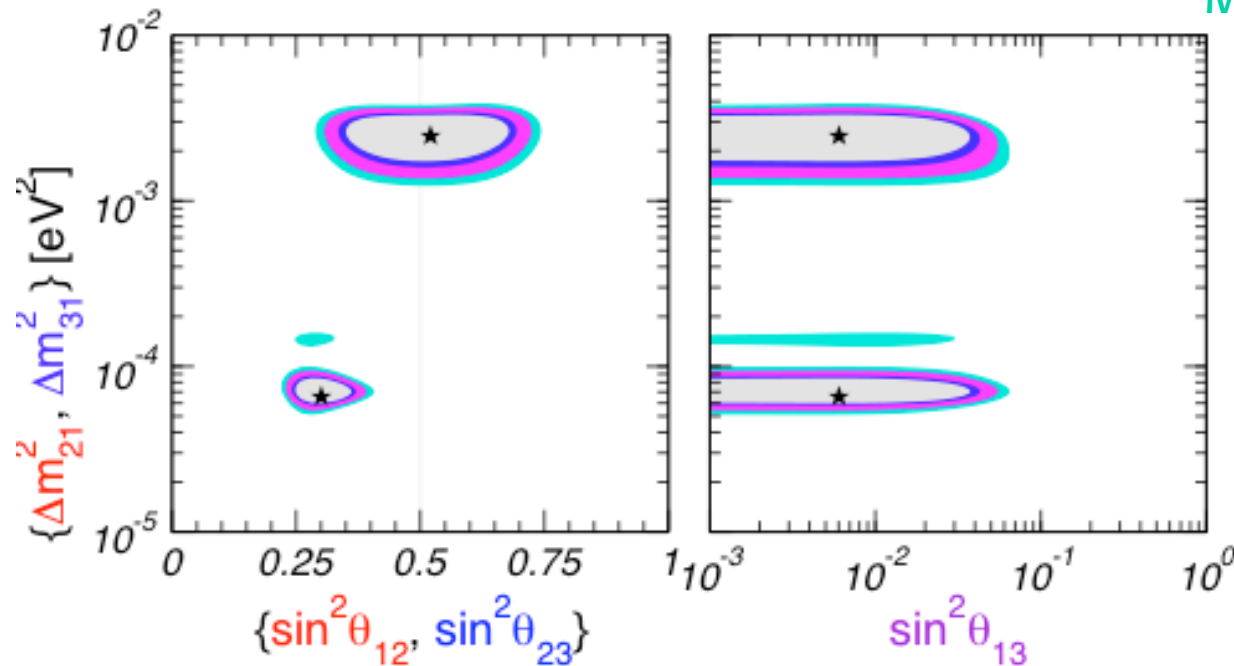


Neutrino oscillation parameters

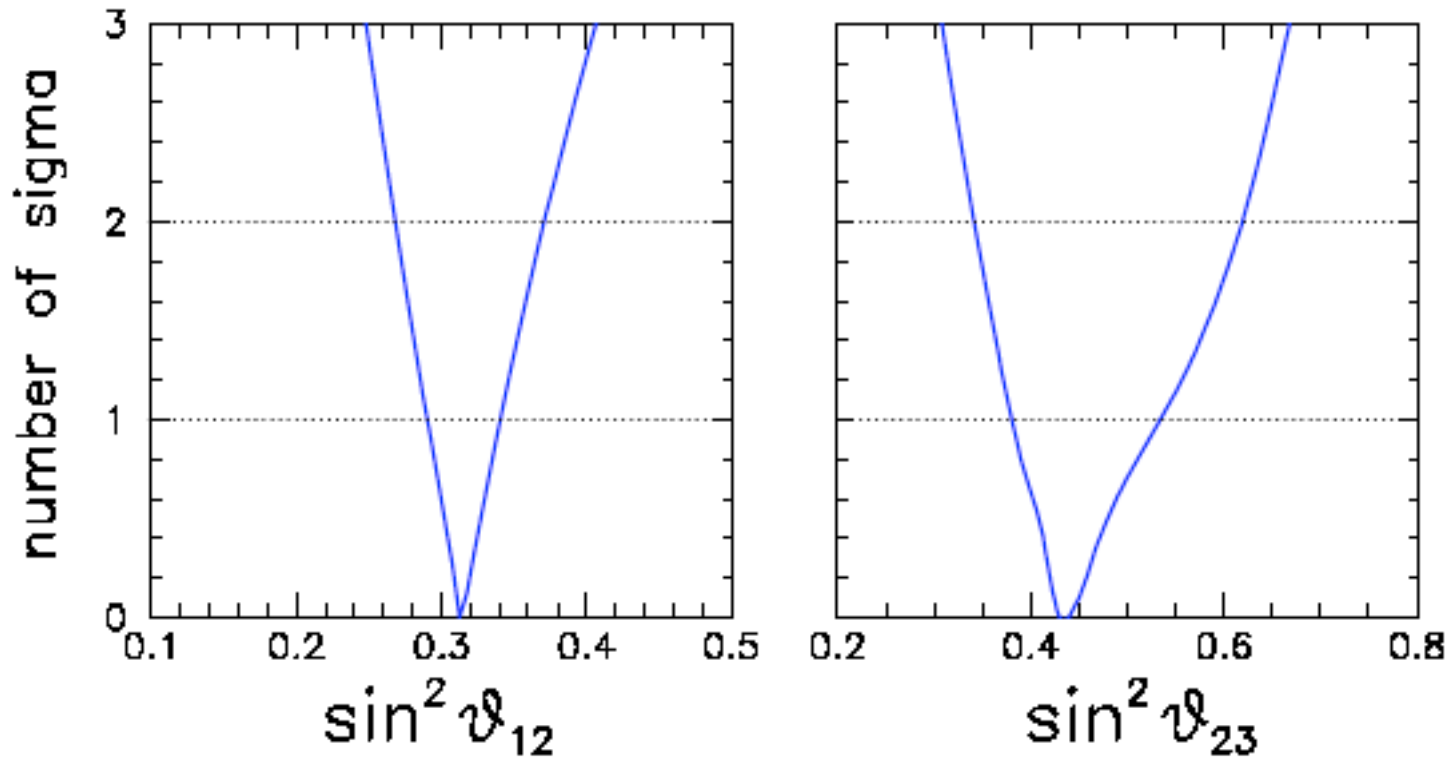
- 2 distinct frequencies
- 2 large angles, 1 small

parameter	best fit	2σ	3σ	5σ
Δm_{21}^2 [10^{-5}eV^2]	6.9	6.0–8.4	5.4–9.5	2.1–28
Δm_{31}^2 [10^{-3}eV^2]	2.6	1.8–3.3	1.4–3.7	0.77–4.8
$\sin^2 \theta_{12}$	0.30	0.25–0.36	0.23–0.39	0.17–0.48
$\sin^2 \theta_{23}$	0.52	0.36–0.67	0.31–0.72	0.22–0.81
$\sin^2 \theta_{13}$	0.006	≤ 0.035	≤ 0.054	≤ 0.11

Maltoni et al '04



Fogli et al '05



2 σ ranges 95%

very precise (KamLAND) \rightarrow
very close to 1/3



$$\delta m^2 = 7.92 (1_{-0.09}^{+0.09}) \times 10^{-5} \text{ eV}^2$$

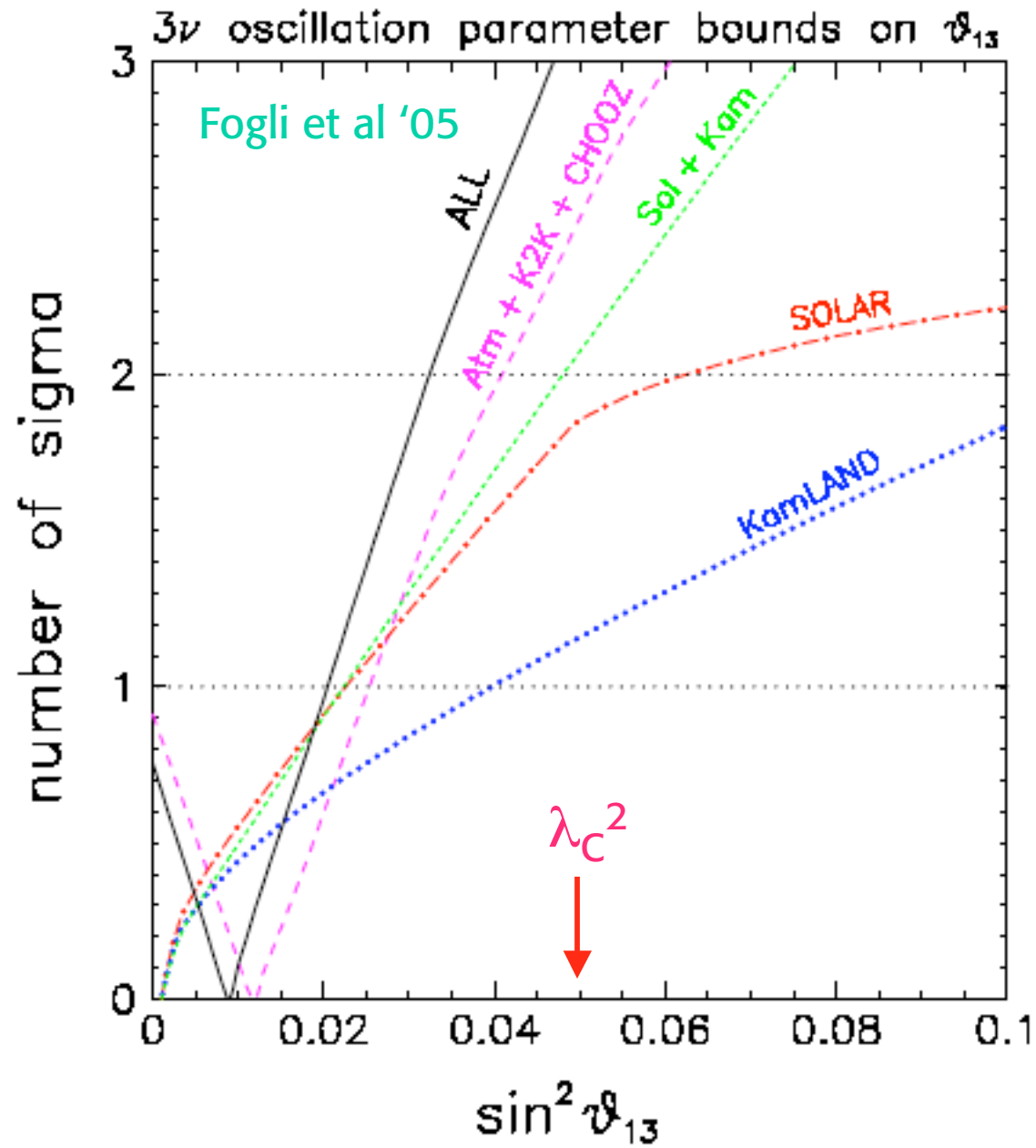
$$\Delta m^2 = 2.4 (1_{-0.26}^{+0.21}) \times 10^{-3} \text{ eV}^2$$

$$\sin^2 \theta_{12} = 0.314 (1_{-0.15}^{+0.18})$$

$$\sin^2 \theta_{23} = 0.44 (1_{-0.22}^{+0.41})$$

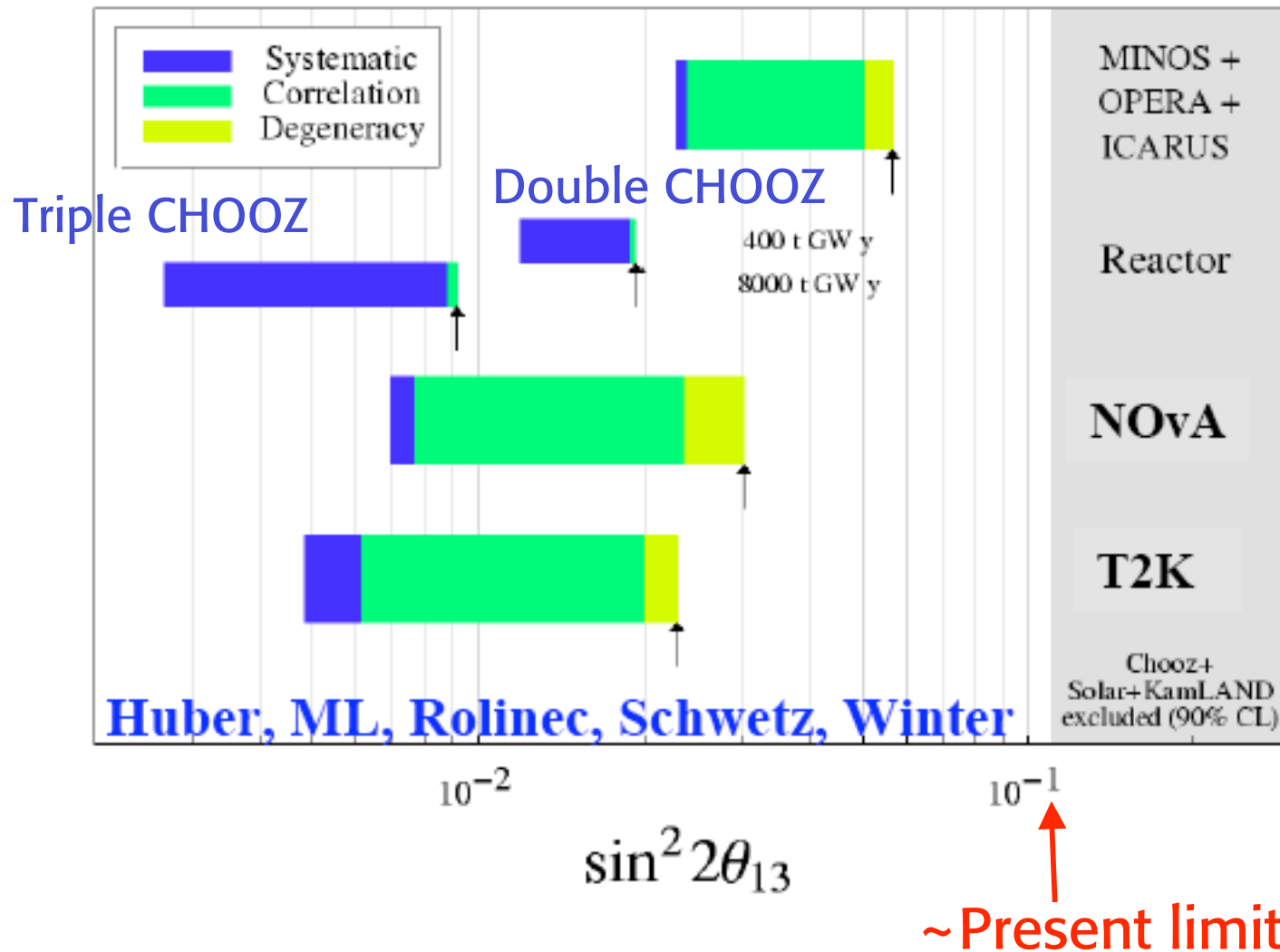
$$\sin^2 \theta_{13} < 3.2 \times 10^{-2}$$

θ_{13} bounds

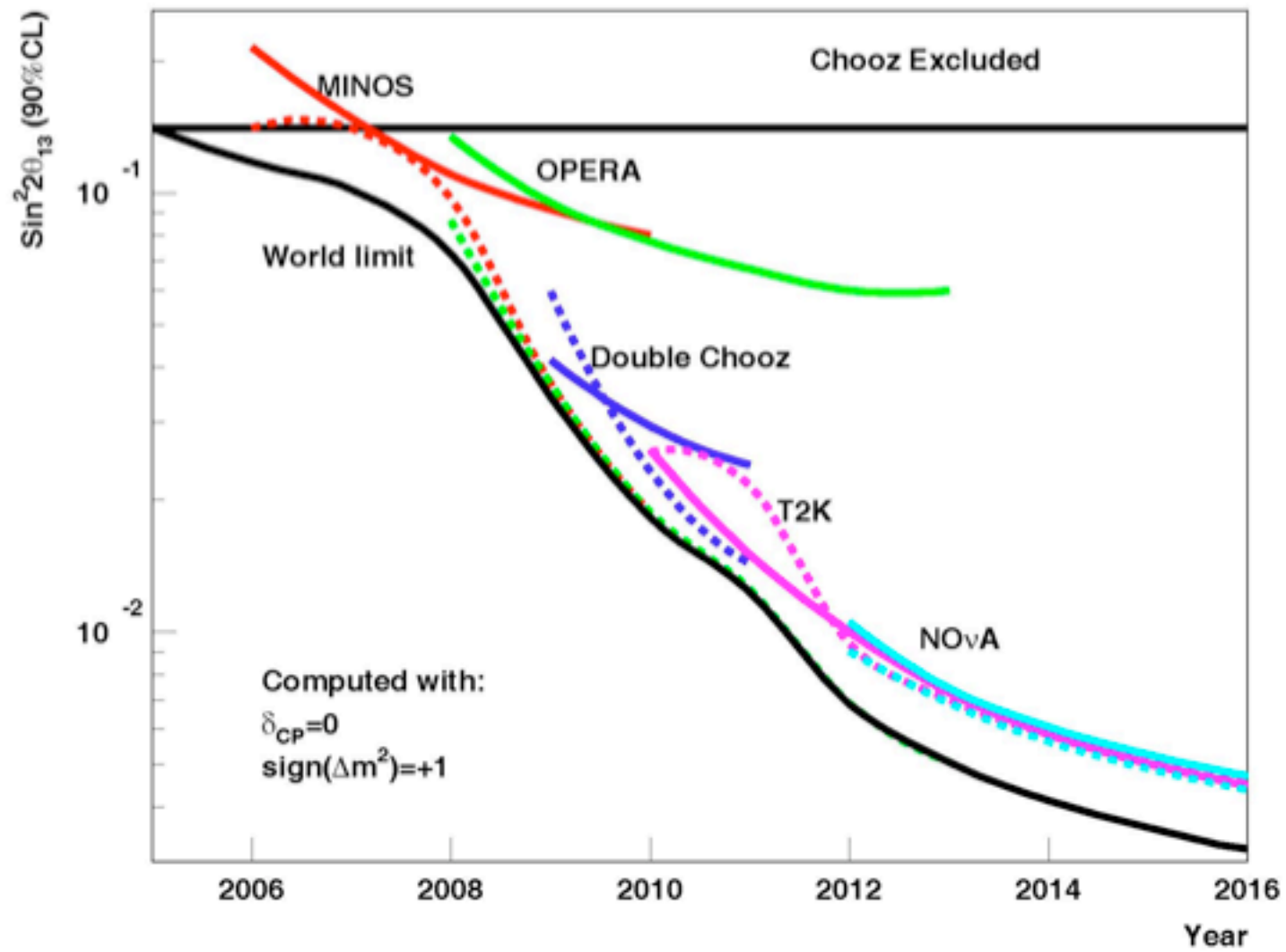


Measuring θ_{13} is crucial for future ν -oscill's experiments (eg CP violation)

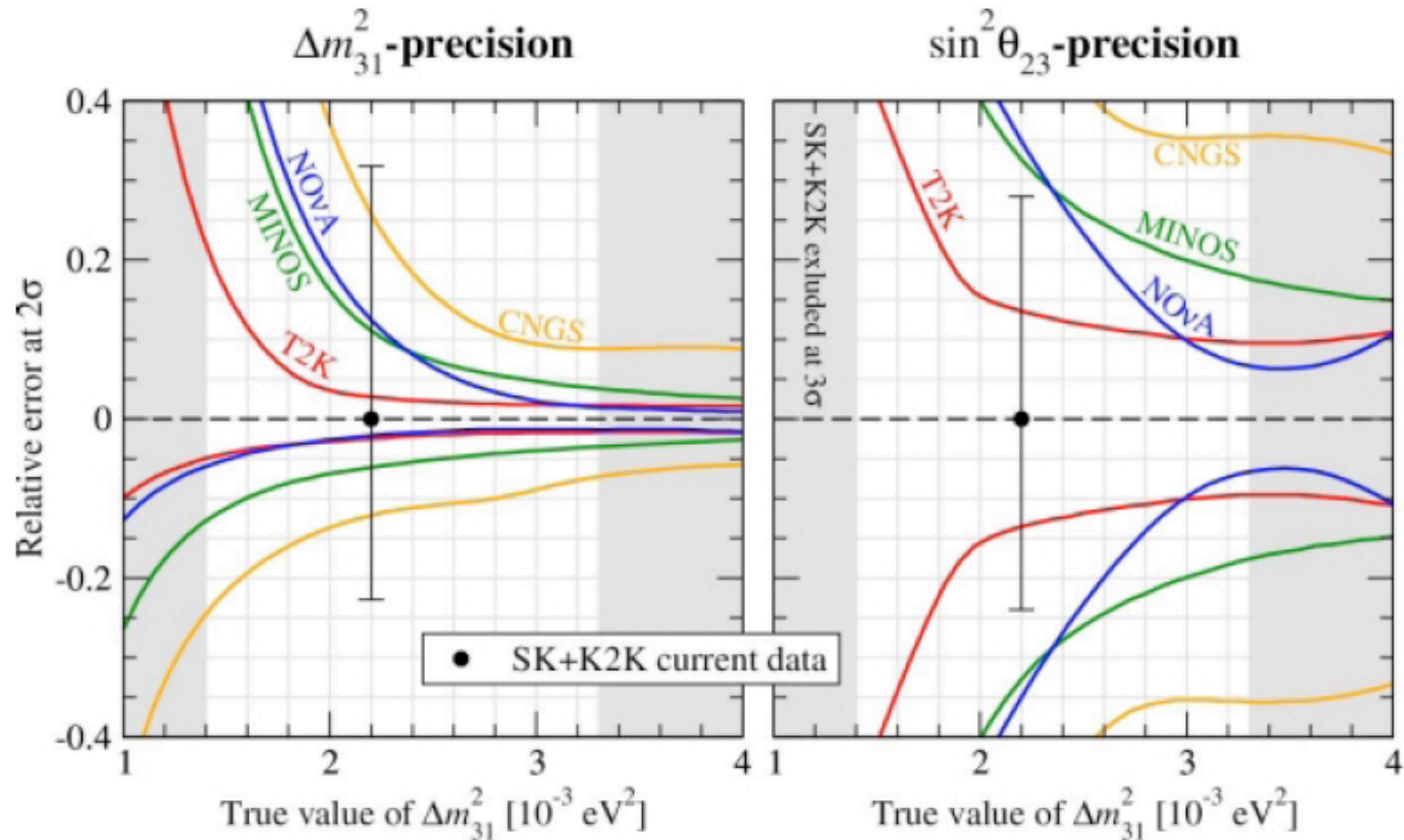
Sensitivity to $\sin^2 2\theta_{13}$ at 90% CL



A possible time map for $\sin^2 2\theta_{13}$



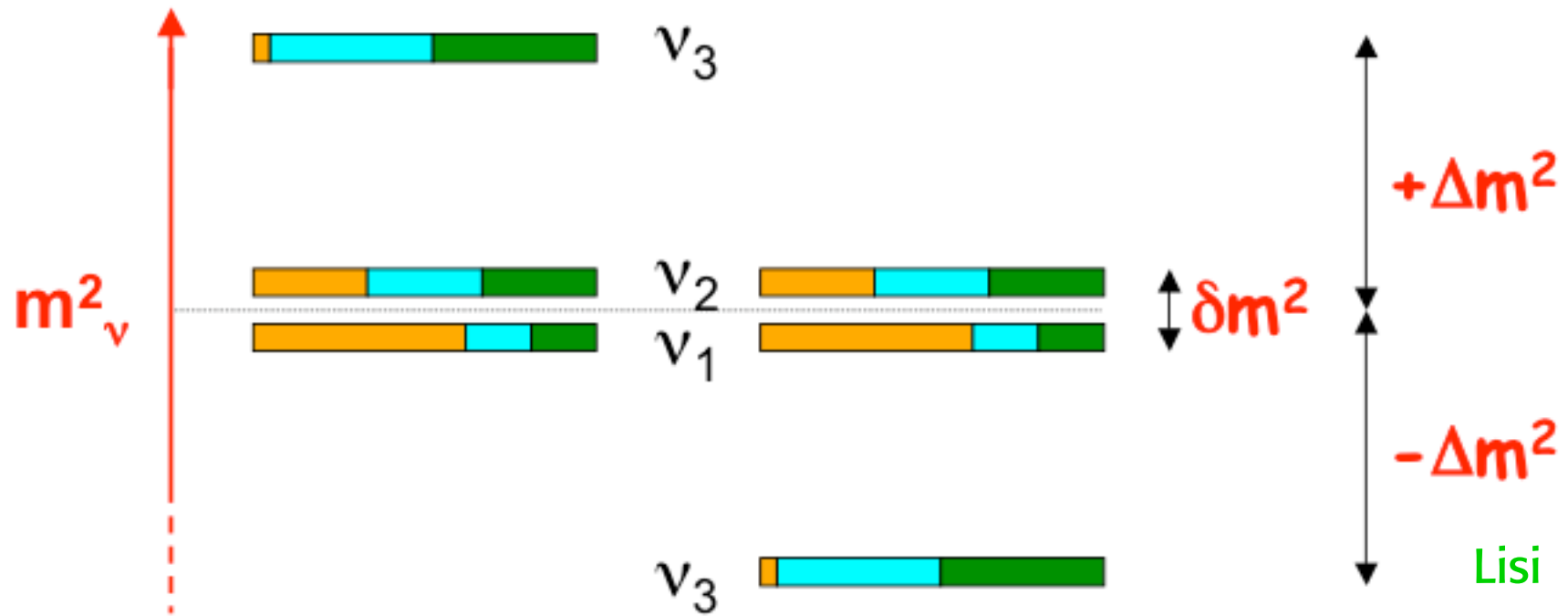
Improvement of Δm_{31}^2 and $\sin^2\theta_{23}$



Lindner



Abs. scale Normal hierarchy... OR... Inverted hierarchy mass² splittings



$\delta m^2 \sim 8 \times 10^{-5} \text{ eV}^2$
 $\Delta m^2 \sim 3 \times 10^{-3} \text{ eV}^2$

$\sin^2 \theta_{12} \sim 0.3$
 $\sin^2 \theta_{23} \sim 0.5$

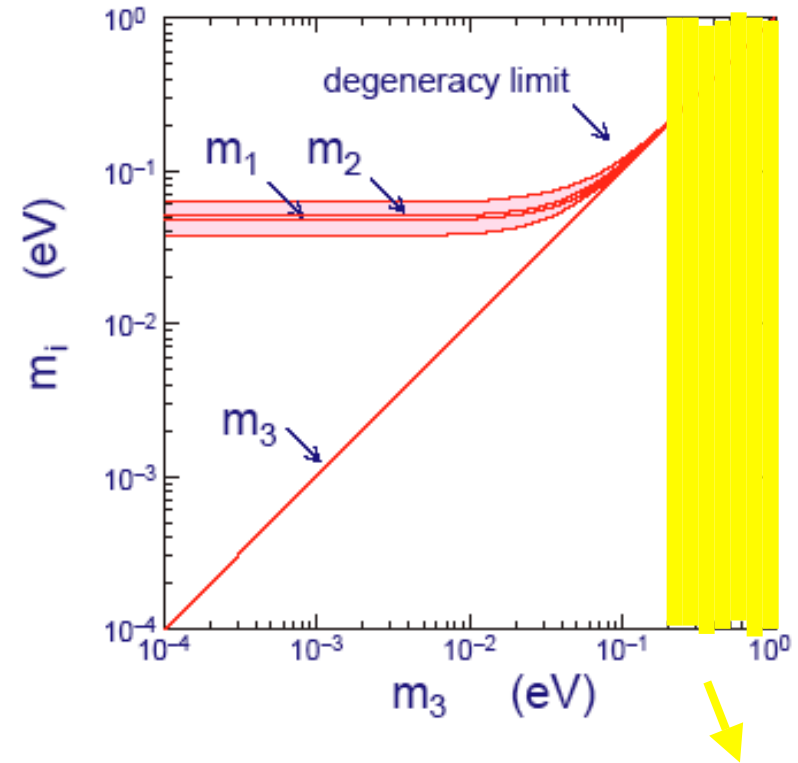
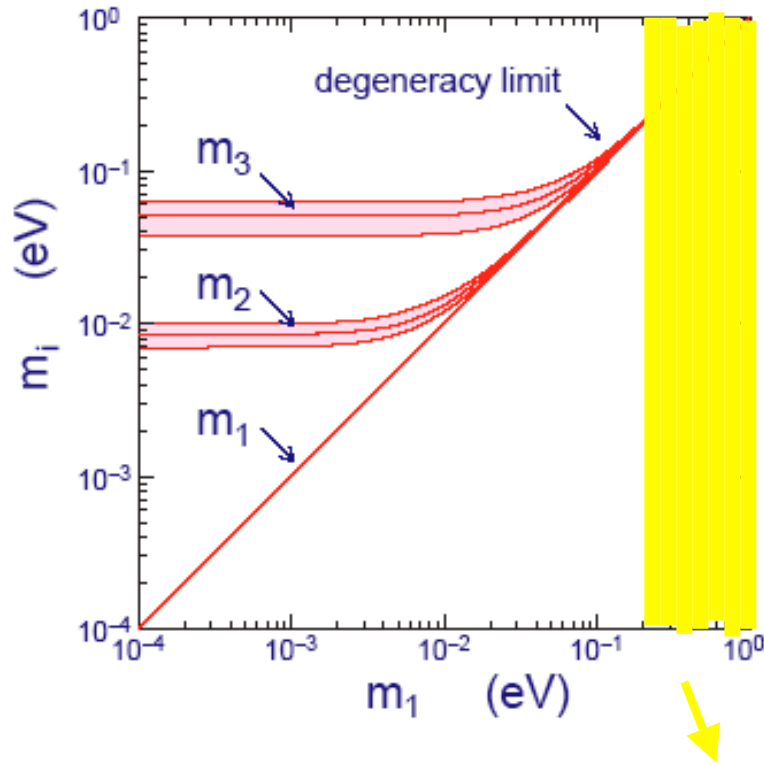
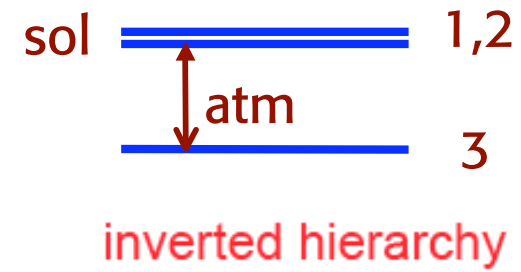
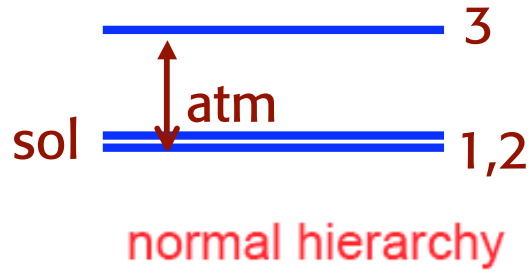
$m_\nu < O(1) \text{ eV}$

$\sin^2 \theta_{13} < \text{few}\%$

sign($\pm \Delta m^2$) unknown

δ (CP) unknown





cosmo
limit

cosmo
limit



Only moderate degeneracy allowed

$0\nu\beta\beta$ would prove that L is not conserved and ν 's are Majorana
 Also can tell degenerate, inverted or normal hierarchy

$$|m_{ee}| = c_{13}^2 [m_1 c_{12}^2 + e^{i\alpha} m_2 s_{12}^2] + m_3 e^{i\beta} s_{13}^2$$

Degenerate: $\sim |m| |c_{12}^2 + e^{i\alpha} s_{12}^2| \sim |m| (0.3-1)$

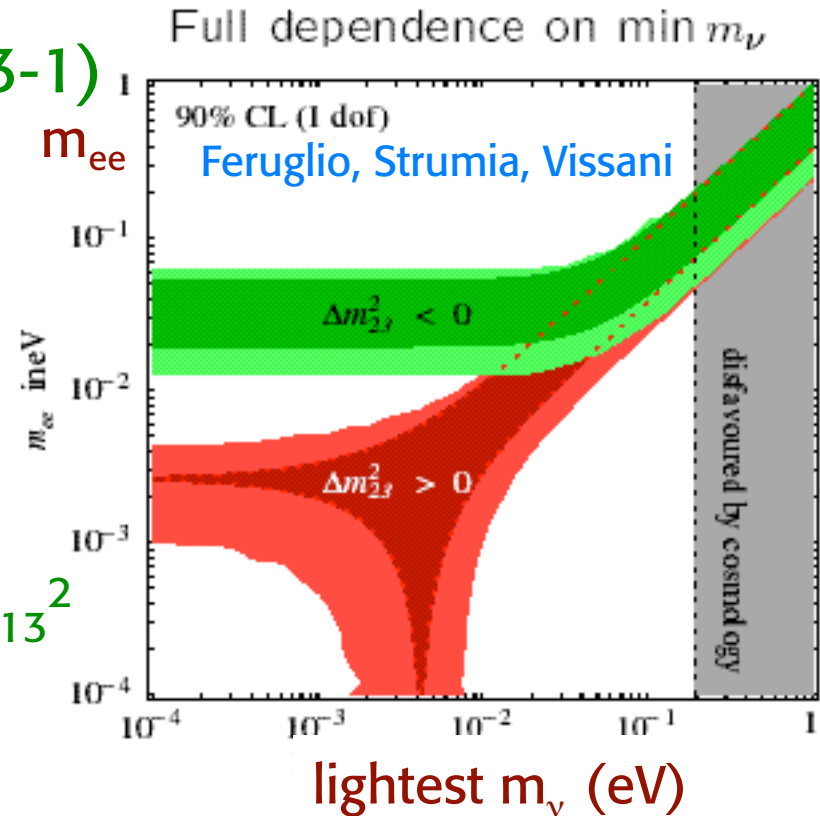
$$|m_{ee}| \sim |m| (0.3 - 1) \leq 0.23-1 \text{ eV}$$

IH: $\sim (\Delta m_{\text{atm}}^2)^{1/2} |c_{12}^2 + e^{i\alpha} s_{12}^2|$

$$|m_{ee}| \sim (1.6-5) 10^{-2} \text{ eV}$$

NH: $\sim (\Delta m_{\text{sol}}^2)^{1/2} s_{12}^2 + (\Delta m_{\text{atm}}^2)^{1/2} e^{i\beta} s_{13}^2$

$$|m_{ee}| \sim (\text{few}) 10^{-3} \text{ eV}$$



Present exp. limit: $m_{ee} < 0.3-0.5 \text{ eV}$
 (and a hint of signal????? Klapdor Kleingrothaus)



A most attractive possibility:

BG via Leptogenesis near the GUT scale

$T \sim 10^{12 \pm 3}$ GeV (after inflation)

Buchmuller, Yanagida,
Plumacher, Ellis, Lola,
Giudice et al, Fujii et al
.....

Only survives if $\Delta(B-L) \neq 0$ is not zero
(otherwise is washed out at T_{ew} by instantons)

Main candidate: decay of lightest ν_R ($M \sim 10^{12}$ GeV)

L non conserv. in ν_R out-of-equilibrium decay:

B-L excess survives at T_{ew} and gives the obs. B asymmetry.

Quantitative studies confirm that the range of m_i from
 ν oscill's is compatible with BG via (thermal) LG

In particular the bound
was derived for hierarchy

$$m_i < 10^{-1} \text{ eV}$$

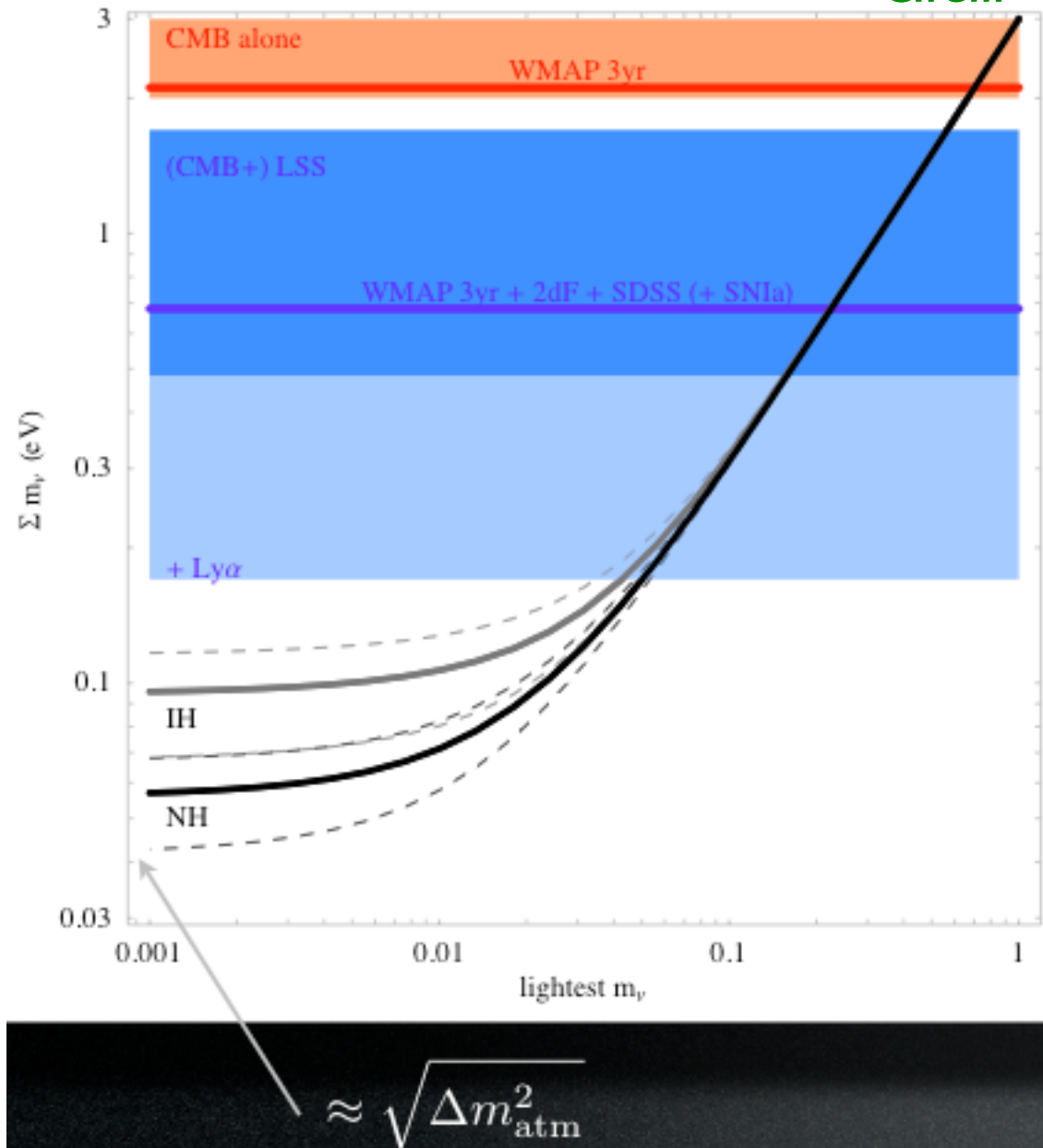
Can be relaxed for degenerate neutrinos
So fully compatible with oscill'n data!!

Buchmuller, Di Bari, Plumacher;
Giudice et al; Pilaftsis et al;
Hambye et al



Summary

Cirelli



Model building

Quality factors for models:

- Based on the most general lagrangian compatible with some simple symmetry or dynamical principle
- Should be complete: address at least charged leptons and neutrinos ($U_{P-NMS} = U_e^+ U_\nu$, and the gauge symmetry connects ch. leptons and LH neutrinos)
- As many as possible small parameters (masses and mixings) should be naturally explained as a consequence.
- The necessary vev configuration should be a minimum of the most general potential for a region of parameter space
- The stability under radiative corrections and higher dim operators must be checked
- Simplicity, economy of fields and parameters, predictivity



General remarks

- After KamLAND, SNO and WMAP.... not too much hierarchy is needed for ν masses:

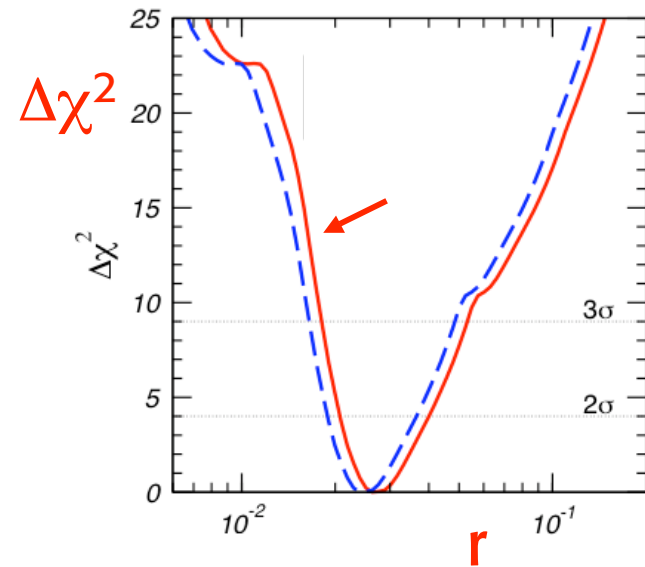
$$r \sim \Delta m^2_{\text{sol}} / \Delta m^2_{\text{atm}} \sim 1/30$$

Precisely at 2σ : $0.025 < r < 0.049$

or

$$m_{\text{heaviest}} < 0.2 - 0.7 \text{ eV}$$

$$m_{\text{next}} > \sim 8 \cdot 10^{-3} \text{ eV}$$



For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν



e.g. θ_{13} not too small!

- Still large space for non maximal 23 mixing

$$2\text{-}\sigma \text{ interval } 0.32 < \sin^2\theta_{23} < 0.62$$

Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small
probably accessible to exp.

Very small θ_{13} theoretically hard

"Normal" models: θ_{23} large but not maximal,
 θ_{13} not too small (θ_{13} of order λ_C or λ_C^2)

"Exceptional" models: θ_{23} very close to maximal and/or θ_{13}
very small
or: a special value for θ_{12}, \dots



Natural models of the “normal” type are not too difficult to build up

It is reasonable to attribute hierarchies in masses and mixings to differences in some flavour quantum number(s).

A simplest flavour (or horizontal) symmetry is $U(1)_F$

For example, some simple models based on see-saw and $U(1)_F$ work for all quark and lepton masses and mixings, are natural and compatible with (SUSY) GUT's, e.g $SU(5) \times U(1)_F$.

Larger flavour symmetry groups have been studied.

They are more predictive but less flexible.

The problem of the "best" flavour group is still open.

The most ambitious models try to combine (SUSY) $SO(10)$

⊕ GUT's with a suitable flavour group

Hierarchy for masses and mixings via horizontal $U(1)_F$ charges.

Froggatt, Nielsen '79

Principle:

A generic mass term

$$\bar{R}_1 m_{12} L_2 H$$

is forbidden by $U(1)$
if $q_1 + q_2 + q_H$ not 0

q_1, q_2, q_H :
 $U(1)$ charges of
 \bar{R}_1, L_2, H

$U(1)$ broken by vev of "flavon" field θ with $U(1)$ charge $q_\theta = -1$.
If $\text{vev } \theta = w$, and $w/M = \lambda$ we get for a generic interaction:

$$\bar{R}_1 m_{12} L_2 H \left(\frac{\theta}{M}\right)^{\Delta_{\text{charge}}} \quad m_{12} \rightarrow m_{12} \lambda^{\Delta_{\text{charge}}}$$

Hierarchy: More $\Delta_{\text{charge}} \rightarrow$ more suppression (λ small)

One can have more flavons (λ, λ', \dots)
with different charges (>0 or <0) etc \rightarrow many versions

